RECORDS
ARITHMETICK:
OR,
The Ground of Arts:

TEACHING

The perfect work and practise of Arithmetick, both in whole Numbers and Fractions, after a more easie and exact form then in former time hath been set forth:

Made by Mr. *Robert Record*, D. in Physick.

Afterward augmented by Mr. John Dee.

And since enlarged with a Third part of *Rules of Practise*, abridged into a briefer method then hitherto hath been published, with divers necessary *Rules* incident to the *Trade of Merchandise*: with *Tables* of the Valuation of all *Coynes*, as they are current at this present time.

By *John Mellis*.

And now diligently perused, Corrected, Illustrated and Enlarged; with an *Appendix* of figurative Numbers, and the extraction of their Roots, according to the method of *Christian Vrstius*: with Tables of Board and Timber measure; and new Tables of Interest, after 10. and 8. *per* 100. with the true value of Annuities to be bought or sold, present, Respited, or in Reversion: The first calculated by *R. C.* but corrected, And the latter diligently calculated by *Ro. Hartwell. Philomathemat.*

Scientia non habet inimicum nisi ignorantem.
Fide —— sed —— Vide

LONDON.
Printed by *James Flesher*, and are to be sold by *Edward Dod* at the signe of the Gun in Ivie-lane. 1654.

THat which my friend hath well begun
For very love to Common-weal,
Need not all whole to be new done,
But now encreaſe I doe reveal.

Something herein I once redreſt,
And now again for thy behoof.
Of zeal I doe and at requeſt,
Both mend and adde, fit for all proof.

Of numbers uſe, the endleſſe might,
No wit nor language can expreſs:
Apply and trie both day and night,
And then this truth thou wilt confeſs.

J. Dee.

The Books Verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to every man:
To pleaſe the beſt ſort would I fain,
The froward diſpleaſe ſhall I certain;
Yet wiſh I will though not with hope,
All ears or mouths to pleaſe or ſtop.

To the moſt mighty Prince, *Edward* the Sixt, by the grace of God, King of *England*, *France*, and *Ireland*, &c.

THE Excellency of mans nature being ſuch, as it is by Gods divine favour (moſt mighty Prince) not only created in highneſs of degree farre above all other corporall things, but by perfection, reaſon, and ſearch of wit, much approaching toward the image of God, as not only the holy Scriptures doe teſtifie, but alſo thoſe naturall Philoſophers, which exactly did conſider the nature of man, and namely the farre reach and infinite compaſſe of the words of the mind, were inforced to confeſſe, that man ſcarcely was able to know himſelf. And if he would duly ponder the nature of himſelf, he would find it ſo ſtrange, that it might ſeem unto him a very miracle: And thereof ſprang that ſaying; *Magnum miraculum eſt homo, maximum miraculum ſapiens homo.* For undoubtedly, as man is one of the greateſt miracles that ever God wrought, ſo a wiſe man is plainly the greateſt.

And therefore was it that ſome did account the head of a man the greateſt miracle in the world, becauſe not only of the ſtrange workmanſhip that is in it, but much more of the efficacy of reaſon, wit, memory, imagination, and ſuch other powers, and works of the mind, which can more eaſily conceive any thing in a manner then underſtand it ſelf. Amongſt all the creatures of God, it findeth none more difficult to be perceived then the ſame powers of it ſelf, whereby it doth conceive and judge: as it may be well conjectured by the diverſity of opinions, that the wiſeſt Philoſophers did utter touching the ſpirit of man, and the ſubſtance of it: whereof I now intend to make no rehearſall; but who ſo liſteth to read thereof, may find it largely

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set forth, not only in *Aristotle* his books *de Anima*, but also in *Galen* his book called *Historia Philosophica*: and again in *Plutarch* his work, *De Philosophorum placitis*, whose words are also repeated of *Eusebius* in the xv. book, Τῆς εὐαγγελικῆς προπαρασκευῆς, unto whom I remit them that have desire to understand intricate difficulty of knowing our own selves, as touching our best part, and that part whereby we deserve to bear the name of men.

This matter seemed so obscure and difficult in knowledge, that *Galen*, who for his excellent wisdome and judgment in naturall works, is called of many men a Miracle in Nature, yet in searching the nature and substance of the spirit of man, he not only confesseth himself ignorant, but counteth it plain temerity to attempt to find it. So farre above the hope of mans knowledge is that part, whereby man doth know and judge of things. And although the ignorant sort (which hate all things that they know not) do little esteem the profoundnesse of mans spirit and reason, the chief power and faculty of it: yet as there is a kind of fear and obedience of unreasonable beasts unto man, by the working power of God, so is there in those small reasoned persons a certain kind of reverence toward wisdome and reason, which they doe shew oftentimes, and by power of perswasion, are inforced to obey reason, will they nill they. And hereby it came to passe, that the rudenesse of the first age of man was brought unto some more civil trade, as it is well declared by *Cicero*, in the beginning of his first book, *De Inventione Rhetorica*, where he saith thus, *Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed pleraque viribus corporis administrabant. Nondum divinæ religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos; non jus æquabile quid utilitatis haberet, acceperat: ita propter errorem atque inscitiam cæca ac temeraria dominatrix animi cupiditas, ad se explendam viribus corporis abutebatur perniciosissimis satellitibus. Quo tempore quidam, magnus videlicet vir & sapiens, cognovit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam possit elicere, & præcipiendo meliorem reddere.*

reddere. Qui diſperſos homines in agris, & in tectis ſilveſtribus abditos, ratione quadam compulit in unum locum, & congregavit: & eos in unumquaq; rem inducens utilem atque honeſtam, primo propter inſolentiam reclamantes, deinde propter rationem atque orationem ſtudioſius audientes, ex fera & immanibus, mites reddidit, & manſuetos.

This long repetition of *Tully's* words will ſeem tedious to them that love but little, and care much leſſe for the knowledge of reaſon, but unto your Majeſtie (I dare ſay) it is a delectable remembrance, and unto me it ſeemed ſo pleaſant, that I could ſcarce ſtay my pen from writing all that mine eyes did ſo greedily read.

This ſentence of *Cicero* am I loath to tranſlate into Engliſh partly for that unto your Majeſtie it needeth no tranſlation, but eſpecially knowing how farre the grace of *Tully's* eloquence doth excell any Engliſh mans tongue, and much more exceedeth the baſeneſs of my barbarous ſtyle: yet for the fruit of my ſentence, I had rather unto my meer Engliſh Country men utter the rudeneſs of my tranſlation, then to defraud them the benefit of ſo good a leſſon, truſting they will ſo learn to love reaſon, that they will alſo gladly and greedily embrace all good Sciences, that may help to the juſt furniture of the ſame, when they conſider that informed reaſon was the only inſtrument, or at leaſt the chiefeſt means to bring men into civil regiment, from barbarous manners, and beaſtly conditions. "For the time "was (ſaith *Tully*) that men wandred abroad in the fields "up and down like beaſts, and uſed no better order in "feeding then they: ſo that by reaſons rule they wrought "nothing, but moſt of their doings did they atchieve by "force of ſtrength. At this time there was no juſt regard "of religion towards God, nor of duty towards man. No "man had ſeen right uſe of marriage, neither did any man "know their own children from other; nor no man had "felt the commodity of juſt Laws: ſo that through error "and ignorance, wilfull luſt, like a blind and heady ruler, "abuſed bodily ſtrength as a moſt mortall miniſter for "the ſatisfying of his deſire. At that time was there one "which not onely in power, but alſo in wiſdome was "great, and he conſidered how that in the minds of men

" was both apt instruments, and great occasion to the due
" accomplishment of most weighty affaires, if a man
" could apply them to use, and by teaching of rules frame
" them to better trade. This man with perswasion of rea-
" son gathered into one place the people that were wan-
" dring about the fields and lay lurking in wild cottages,
" and woods, and bringing them into one common so-
" ciety, did trade them to all such things, as either were pro-
" fitable or honest, although not without repining at the
" first, by reason then they had not been so accustomed be-
" fore: yet at length through reason and perswasion of
" words they obeyed him more diligently, and so of a wild
" and cruel people, he made them courteous and gentle.
Thus hath *Tully* set forth the efficacy of reason & perswasion, how it was able to convert wild people to mildnes, and to change their furious cruelnes into gentle courtesie: were it not now a great reproach in this our time (when knowledge reigneth so large) that men should shew themselves lesse obsequious to reason? Unless it may be thought that now every man having sufficient knowledge of himself, needeth not to hearken to the perswasion of others.

Indeed he that thinketh himself wise, will not esteem the reason of any other, be he never so wise; so that of such a one it may well be said: He that thinketh himself wiser then he is, may justly be counted a double fool. Wherefore such men are not to be permitted in open audience to talk, but must be put to silence, and be made to give ear to reason; which reason consisteth not in a multitude of words heaped rashly together, and applied for one purpose, but reason is the expressing of a just matter with witty perswasions, furnished with learned knowledge: such knowledge had *Moses*, being expert in all learning of the Ægyptians, as the Scriptures declare, and therefore was able to perswade the stubborn people of the Jews, although not without pain. Such knowledge, and such reasons did *Druys* shew, which was the first Law-maker of all the West part of Europe. Like reason and wisdome did *Xamolxis* amongst the Goths. *Lycurgus* unto the Lacedemonians. *Zeleucus* to the Locrians. *Solon* to the Athenienses, and *Dunwallo Molmutius* two thousand years past amongst the old Britains of this Realm.

Druys was son to king *Sarron*, and succeeded him in his kingdome.

Realm. And hereby came it to passe, that their Laws continued long, till more perfect reason altered many of them, and wilfull power oppressed most of them.

At the beginning when these wise men perceived how hard it was to bring the rude people to understand reason, they judged the best means to attain this honest purpose, to depend of learning in every kinde: for by learning (as *Ovid* saith) *Pectora mollescunt, asperitásque fugit*, Stout stomachs doe wax milde, and sharp fierceness is exilde. Therefore as *Zerosus* doth testifie, *Samon* that was the third King over all this West part of Europe, for to bring the people from beastly rage to manly reason, did erect Schools of liberall Arts, which took so good success, that his name continued in that sort famous about two thousand years after: for *Diodorus Siculus*, who was in the time of *Julius Cæsar*, maketh mention of the learned men of Gothes or Celtes, and nameth them *Samonides*, that is to say, *Samon his Scholars and followers.*

Among these Arts that then were taught some did inform the tongue, and make them able both to utter aptly their mind, and also to perswade; as Grammar, Logick, and Rhetorick, although not so curiously as in this time; some other did appertain to the just order of partition of lands, the true using of Weights, Measures & reckonings of all sorts of bargains, and for order of building & sundry other uses; those were Arithmetick and Geometry. Again, to incourage men to the honour of God, they taught Astronomy, whereby the wonderfull works of God were so manifestly set forth, that no mans tongue, nor pen can in like sort expresse his infinite power, his unspeakable wisdome, and his exceeding goodness toward man, whereby he doth bountifully provide for man all necessaries, not only to live, but also to live pleasantly. And so was their confidence in Gods providence strongly stayed, knowing his goodness to be such, that he would help man as he could, and his power to be so great, that he would doe nothing but that that was best. Besides these Sciences they taught also Musick, which most commonly they did apply partly to religious services, to draw men to delight therein, and partly to songs made of the manners of men, in praise of virtue and discommenda-

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This *Bardus Dryadius* the 5. King of the Celtes, reigned 60. years, and died 1832 years before Christ.

tion of vice, whereby it came to passe, that no man would displease them, nor doe any thing evil that might come to their hearing: for their songs did make evil men more abhorred in that time then any excommunication doth in this time. The posterity of these Musicians continue yet both in Wales and Ireland, called *Bardes* unto this day, by the ancient name of *Bardus*, their first founder.

And as these Sciences did encrease, so did virtue encrease thereby. Again, as those Sciences did decay, so virtue lost her estimation, and consequently was little in use: whereof to make a full declaration were a thing meet for a Prince to hear, but it would require a peculiar Treatise. Wherefore at this present I count it sufficient lightly to have touched this matter in general words, and to say no more of the particularity thereof, but only touching one of those Sciences, that is Arithmetick, by which not only just partition of lands was made, but also touching buying and selling, all assises, weights, and measures were devised, and all reckonings and accounts driven; yea by proportion of it were the true orders of justice limited, as *Aristotle* in his Ethicks doth declare, and the degrees of estates in the Common-wealth establisht; although that proportion be called Geometricall and not Arithmeticall, yet doth that proportion appertain to the art of Arithmetick, and in Arithmetick is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seeing Arithmetick is so many waies needfull unto the first planting of a Common-wealth, it must needs be as much required to the preservation of it also: for by the same means is any Common-wealth continued, by which it was erected and established. And if I shall in small matters in appearance, but indeed very weighty, put one example or two. What shall we say for the Statutes of this Realm, which be the only stay of good order in manner now? As touching the measuring of ground by length and breadth, there is a good and an ancient Statute made by art of Arithmetick; and now it shall be to little use, if by the same art it be not practised & tried. For the assise of bread and drink, the two most common and most necessary things for sustentation of man, there was a goodly ordinance in the Law made, which by ignorance

rance hath ſo grown out of knowledge, and uſe, that few men doe underſtand it, and therefore the Statute bookes wonderfully corrupted, and the Commons cruelly oppreſſed: notwithſtanding ſome men have written that it is too doubtfull a matter to execute thoſe aſſiſes by thoſe Statutes, by reaſon they depend of the ſtandard of the coyn, which is much chang'd from the ſtate of that time, when thoſe Statutes were made. Thus ſhall every man read (that liſteth) in the Abridgment of the Statutes, in the title of Weights and Meaſures, in the ſeventh number of the Engliſh Booke, where he ſhould have tranſlated a good ordinance which is ſet forth in the French Booke: but no marvell if the Abridgement doth omit it, ſeeing the great Book of Statutes doth omit the ſame Statute, as it hath done divers other very good laws. And this is the fruit of ignorance to reject and condemn all that it underſtandeth not, although they uſe ſome cloaks for it, but ſuch cloaks as being allowed, might ſerve to repell all good laws; which God forbid.

Again, there is an ancient order for aſſiſe of fire, wood and coals, which was renued not many years paſt; and now how avarice and ignorance doth canvaſe that Statute, it is too pitifull to talk of, and more miſerable to feel.

Furthermore, for the Statute of Coynage, and the ſtandard thereof, if the people underſtood rightly the Statute, they ſhould not nor would (as they often doe) gather an excuſe for their folly thereby: but as I ſaid, theſe Statutes by wiſdome and good knowledge of Arithmetick were made, and by the ſame muſt they be continued. And let ignorance no more meddle with the uſe of them, then it did with the making of them. Oh in how miſerable caſe is that Realm, where the miniſters and interpreters of the law are deſtitute of all good Sciences, which be the keyes of the lawes! How can they either make good lawes, or maintain them that lacke that true knoledge whereby to judge them? And happy may that Realme be accounted, where the Prince himſelf is ſtudious of learning, and deſireth to underſtand equity in all laws. Therefore moſt happy are we the loving ſubjects of your Majeſty, which may ſee in your Highneſſe not onely ſuch towardneſſe, but alſo ſuch knowledg of divers Arts, as ſeldome hath been

seen in any Prince of such years, whereby we are enforced to conceive this hope certainly, that he which in those years seeketh knowledge when knowledge is least esteemed, and of such an age can discern them to be enemies both to his roial Person, and to his Realms, which labour to withdraw him from knowledge to excessive pastime, and from reasonable study to idle or noysome pleasures, he must needs when he cometh to more mature years, be a most prudent Prince, a must just Governor, and a right Judge, not only of his subjects commonly, but also of the ministers of his laws, yea, and of the laws themselves: and to be able to conceive the true equity and exact understanding of all his Laws and Statutes, to the comfort of his good Subjects, and the confusion and reproach of them which labour to obscure or pervert the equity of the same Laws and Statutes. How some of these Statutes may be applyed to use, as well in this our time, as in any other time, I have peculiarly declared in this Book, and some other I have omitted for just considerations, till I may offer them first unto your Majesty to weigh them as to your Highnesse shall seem good: for many things in them are not to be published without your Highnesse knowledge and approbation; namely, because in them is declared all the rates of alloyes for all standards from one ounce upward, with other mysteries of Mint matters, and also most part of the varieties of coyns that have been currant in this your Majesties Realm by the space almost of six hundred years last past, and many of them that were currant in the time that the Romans ruled here.

All which, with the ancient description of England and Ireland, and my simple censure of the same, I have almost compleated to be exhibited to your Highnesse: In the mean season most humbly beseeching your Majesty to accept this simple Treatise, not worthy to be presented to so high a Prince, but that my lowly requeste to your Majesty is, that this amongst other of my Books may passe under the protection of your Highnesse, whom I beseech God most earnestly and daily, according to my duty, to advance in all honor, and Princely Regality, and to encrease in all knowledge, justice, and godly policy. Amen.

Your Majesties most obedient subject and servant,

ROBERT RECORD.

To the loving READERS,

The Preface of Mr *Robert Record.*

SOre oft-times have I lamented with my self the unfortunate condition of England, *seeing so many great Clerks to arise in sundry other parts of the world, and so few to appear in this our Nation: whereas for pregnancy of naturall wit (I think) few Nations doe excell* English-men: *But I cannot impute the cause to any other thing, then to the contempt, or misregard of learning. For as* English-men *are inferiour to no men in mother wit, so they passe all men in vain pleasures, to which they may attain without great pain and labour: and are slack to any never so great commodity, if there hang of it any painfull study, or travelsome labour.*

Howbeit, yet all men are not of that sort, though the most part be, the more pity it is: but of them that are so glad, not only with painfull study, and studious pain to attain learning, but also with as great study and pain to communicate their learning to other, and make all England (*if it might be*) *partakers of the same; the most part are such, that unneath they can support their own necessary charges, so that they are not able to bear any charges, in doing of that good that else they desire to doe.*

But a greater cause of lamentation is this, that when learned men have taken pains to doe things for the aid of the unlearned, scarce they shall be allow'd for their wel-doing, but derided and scorn'd, and so utterly discourag'd to take in hand any like enterprise again. So that if any be found (as there

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are ſome) that doe favour learning and learned wits, and can be contented to further knowledge, yea only with their word; ſuch perſons though they be rare, yet ſhall they encourage learned men to enterpriſe ſomething at the leaſt that England may rejoice of. And I have good hope that England will (after ſhe hath taken ſome ſure taſt of learning) not only bring forth more favourers of it, but alſo ſuch learned men, that ſhe ſhall be able to compare with any Realm in the world. But in the mean ſeaſon, where ſo few regarders of learning are, how greatly they are to be eſteemed that doe favour and further it, my pen will not ſuffice at full to declare.

Therefore, gentle Reader, whereas I doe upon moſt juſt occaſion judge, yea and know aſſuredly, that there be ſome men in this Realm, which both love & alſo much deſire to further good learning, & yet yet am not able to write their condign praiſe for the ſame, I think it better with ſilence to overpaſſe it, then either ſay too little of it, or to provoke againſt them the malice of ſuch other, wch do nothing themſelves that is praiſ-worthy, & therfore cannot abide to hear the praiſe of any other mans good indeed.

And conſidering their great favor unto learning, though I my ſelf be not worthy to be reckoned in the number of great learned men, yet am I bold to put my ſelf in Preſs, with ſuch ability as God hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man to help my Countrey-men, and will not ceaſe daily, (as much as my ſmall ability will ſuffer me) to endite ſome ſuch thing, that ſhall be to the inſtruction, though

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not of learned men, yet at the leaſt of the vulgar ſort, whoſe argument alwaies ſhall be ſuch as it ſhall delight all learned wits, though they doe not learn any great thing out of it.

But to ſpeak of this preſent Book of Arithmetick, I dare not, nor will not ſet it forth with any words, but remit it to the judgment of all gentle Readers, and namely ſuch as love good learning, beſeeching them ſo to eſteem it, as it doth ſeem worthy. And ſo either to accept the thing for it ſelf, either at the leaſt to allow my good endevour. But I perceive I need not uſe any perſwaſions unto them, whoſe gentle nature and favourable mind is ready to receive thankfully, and interpret to the beſt all ſuch enterpriſes attempted for ſo good an end, though the thing doe not alwaies ſatisfie mens expectation. This conſidered, did bolden me to publiſh abroad this little Book of the Art of Numbring, which if you ſhall receive favourably, you ſhall encourage me to gratifie you hereafter with ſome greater thing.

And as I judge ſome men of ſo loving a mind to their native Countrey, that they would much rejoice to ſee it proſper in good learning, and witty Arts: ſo I hope well of all the reſt of Engliſh-men, that they will not be unmindfull of his due praiſe, by whoſe means they are helped and furthered in any thing. Neither ought they to eſteem this thing of ſo little value, as many men of little diſcretion oftentimes doe. For who ſo ſetteth ſmall price by the witty device and knowledge of Numbring, he little conſidereth it to be the chief point, (in manner) whereby men differ from all brute beaſts: for

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as in all other things (almoſt) beaſts are partakers with us, ſo in numbring we differ clean from them, and in manner peculiarly, ſith that in many things they excell us again.

The Fox in crafty wit exceedeth moſt men,
A Dog in ſmelling hath no man his peer.
To foreſight of weather if you look then,
Many beaſts excell men; this is clear.
The wittineſs of Elephants doth letters attain,
But what cunning doth there in the Bee remain?
The Emmet foreſeeing the hardneſs of winter,
Provideth victuals in the time of ſummer.
The Nightingale, the Linet, the Thruſh, the Lark,
In Muſicall harmony paſſe many a Clark.
The Hedghog of Aſtronomy ſeemeth to know,
And ſtoppeth his cave where the wind wil blow,
The Spider in weaving ſuch Art doth ſhow,
No man can him mend, nor follow I trow.
When a houſe will fall, the Mice right quick
Flee thence before; can man doe the like?

Many things elſe of the wittineſs of Beaſts and Birds might I here ſay, ſave that another time of them I intend to write, wherein they excell in manner all men, as it is daily ſeen: but in number was there never beaſt found ſo cunning, that could know or diſcern one thing from many, by daily experience you may well conſider, when a Bitch hath many whelps, or a Hen many chickens: and likewiſe of other whatſoever they be, take from them all their yong ſaving only one, & you ſhall perceive plainly that they miſs none, though they wil reſiſt you in taking them away, and will ſeek them again if they may

may know where they be, but else they will never misse them truly; but take away that one that is left, and then will they crie and complain; and restore to them that one, then are they pleased again. So that of Number, this may I justly say, it is the only thing almost that separateth man from beasts. He therefore that shall contemn number, declareth himself as brutish as a beast, and unworthy to be counted in the Fellowship of men. But I trust there is no man so foul overseen, though many right smally doe it regard.

Therefore will I now stay to write against such, and return again to this my Book, which I have written in the form of a Dialogue, because I judge that to be the easiest way of instruction, when the Scholar may ask every doubt orderly, and the Master may answer to his question plainly. Why the Authour wrote in Dialogue-wise.

Howbeit I think not the contrary, but as it is easier to mend another mans work then to make the like; so there will be some that will find fault, because I write in a Dialogue: but as I conjecture those shall be such as doe not, cannot, or will not perceive the reason of right teaching, and therefore are unmeet to be answered unto, for such men with no reason will be satisfied.

And if any man object, that other Books have been written of Arithmetick already so sufficiently that I needed not now to put pen to the book, except I will condemn other mens writings: To them I answer, That as I condemn no mans diligence, so I know that no one man can satisfie every man: and therefore like as many doe esteem greatly other

books, so I doubt not but some will like this my Book above any other English Arithmetick *hitherto written; and namely, such as shall lack Instructors, for whose sake I have so plainly set forth the Examples, as no Book that I have seen hath done hitherto: which thing shall be great ease to the rude Readers.*

Therefore (gentle Reader) though this Book can be but small aid to the learned sort, yet unto the simple ignorant (which needeth most help) it may be a good furtherance and mean unto knowledge.

And though unto the King's Majesty privately I dedicate it Dedicate, yet I doubt not (such is His clemency) but that he can be content, yea, and much desirous, that all His loving Subjects shall take the use of it, and employ the same to their most profit. Which thing if I perceive that they thankfully do, and receive with as good will as it was written, then will I shortly with no lesse kindness set forth such Introductions into Geometry, *and* Cosmography, *as I have at times promised, and as hitherto in* English *hath not been enterprised, wherewith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.*

I will say no more, but let every man judge as he shall see cause. And thus for this time I will stay my Pen, committing you all to that true fountain of perfect number, which wrought the whole world by number and measure: he is Trinity in Unity, and Glory, Amen.

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Before

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j	1	*one*	rr	20	*twenty*
ij	2	*two*	rl	40	*fourty*
iij	3	*three*	l	50	*fifty*
iv	4	*four*	lr	60	*ſixty*
v	5	*five*	lrr	70	*ſeventy*
vj	6	*ſix*	rc	90	*ninety*
vij	7	*ſeven*	C	100	*a hundred*
viij	8	*eight*	CC	200	*2 hundred*
ix	9	*nine*	D	500	*5 hundred*
x	10	*ten*	DC	600	*6 hundred*
xj	11	*eleven*	M	1000	*a thouſand*
xij	12	*twelve*	MD	1500	*a thou. 5 hund.*
&c.	&c.	*&c.*	&c.	&c.	*&c.*

A Dialogue between the Maſter and the Scholar: teaching the *Art* and uſe of Arithmetick with Pen.

The Scholar ſpeaketh.

SIR, ſuch is your authority in mine eſtimation, that I am content to conſent to your ſaying, and to receive it as truth, though I ſee none other reaſon that doth lead mee thereunto: whereas elſe in mine own conceit it appeareth but vain to beſtow any time privately in learning of that thing that every child may, and doth learn at all times and hours, when he doth any thing himſelf alone, and much more when he talketh or reaſoneth with others.

Maſter. Lo, this is the faſhion and chance of all them that ſeek to defend their blind ignorance, that when they think they have made ſtrong reaſon for themſelves, then have they proved

proved quite contrary. For if Numbring be so common (as you grant it to be) that no man can doe any thing alone, and much lesse talk or bargain with other, but he shall still have to doe with number: this proveth not number to be contemptible and vile, but rather right excellent and of high reputation. fifth it is the ground of all mens affaires, in that without it no tale can be told, no communication without it can be continued, no bargaining without it can duly be ended, or no businesse that man hath be fully completed. These commodities if there were none other, are sufficient to approve the worthinesse of number. But there are other innumerable, farre passing all these, which declare number to exceed all praise. Wherefore in all great workes are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be inhaunsed? Why are Astronomers so greatly advanced? Because that by number such things they find, which else would farre excell mans mind.

Scholar. Verily, Sir, if it be so, that these men by numbring, their cunning doe attain, at whose great workes most men doe wonder, then I see well I was much deceived, and numbring is a more cunning thing then I tak it to be.

Master. If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude num-

ber,

ber, and answer to this question: How many years old are you?

Scholar. Mum.

Master. How many daies in a week? How many weeks in a year? What lands hath your Father? How many men doth he keep? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if number want, you answer all by mummes: How many miles to London?

Scholar. A poak full of plums.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking it maketh men dumb, so that to most questions they must answer mum.

Scholar. This is the cause sir, that I judged it so wise, because it is so common in talking every while: For plenty is not dainty, as the common saying is.

Master. Nay, for store is no sore, perceive you this? The more common that the thing is, being needfully required, the better is the thing, and the more to be desired. But in numbring as some of it is light and plain, so the most part is difficult and not easie to attain. The easier part serveth all men in common, and the other requireth some learning. Wherefore as without numbring a man can doe almost nothing; so with the help of it you may attain to all things.

Scholar.

Scholar. Yea, sir, why then it were best to learn the Art of numbring, first of all other learning, and then a man need learn no more if all other come with it.

Master. Nay, not so but if it be first learned, then shall a man be able (I mean) to learn, perceive and attain to other Sciences, which without it he could never get.

Scholar. I perceive by your former words, that Astronomy and Geometry depend much on the help of numbring: but that other Sciences, as Musick, Physick, Law, Grammar, and such like, have any help of Arithmetick, I perceive not.

Master. I may perceive your great Clerkslinesse by the ordering of your Sciences: but I will let that passe now, because it toucheth not the matter that I intend, and I will shew you how Arithmetick doth profit in all these, somewhat grossly, according to your small understanding; omitting other reasons more substantiall.

Musick. First (as you reckon them) Musick hath not only great help of Arithmetick, but is made, and hath its perfectnesse of it: for all musick standeth by number and proportion:

Physick. And in Physick, before the calculation of criticall daies, with other things which I omit, how can any man judge the pulse rightly, that is ignorant of the proportion of numbers?

Law. And as for the Law, it is plain, that the man

man that is ignorant of Arithmetick, is neither meet to be a Judge, neither an Advocate, nor yet a Proctor. For how can he well understand another mans cause, appertaining to distribution of goods, or other debt, or of summes of money, if he be ignorant of Arithmetick? This oftentimes causeth right to be hindred, when the Judge either delighteth not to hear of a matter that he perceiveth not, or cannot judge for lack of understanding: this cometh by ignorance of Arithmetick.

Now, as for Grammar, me thinketh you should not doubt in what it needeth number, sith you have learned that Nouns of all sorts, Pronouns, Verbs, and Participles are diffined diverſly by numbers: besides the variety of Nouns of Number, and Adverbs. And if you take away number from Grammar, then is all the quantity of syllables lost. And many other wates doth number help Grammar. Whereby were all kinds of Metres found and made? was it not by number? Grammar.

But how needfull Arithmetick is to all parts of Philosophy, they may soon see, that doe read either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations depend of Arithmetick. It is the saying of Aristotle, that *he that is ignorant of Arithmetick, is meet for no Science.* And Plato his Master wrote a little sentence Philosophy.

sentence over his School-house door, *Let none enter in hither* (quoth he) *that is ignorant of Geometry*. Seeing he would have all his Scholars expert in Geometry, much rather he would the same in Arithmetick, without which Geometry cannot stand.

Divinity. And how needfull Arithmetick is to Divinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much doe write of it. And if I should go about to write all the commodities of Arithmetick in civill acts, as in governance of Commonweals in time of peace, and in due provision and order of Armies in time of warre, for numbring of the Host, summing of their wages, provision of Victuals, viewing of Artillerie, with other Armour, beside the cunningest point of all, for casting of ground, for encamping of men, with such other like: And how many wayes also Arithmetick is conducible for all private Weals, of Lords and all Possessioners, of Merchants, and all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receivers, Stewards, Bailiffes, and such like, whose Offices without Arithmetick are nothing: If I should (I say) particularly repeat all such commodities of the noble Science of Arithmetick, it were enough to make a very great book.

Armies.

Scholar. No, no, Sir, you shall not need: For I doubt not but this, that you have said, were enough

enough to perſwade any man to think this Art to be right excellent and good: and ſo neceſſary for man, that (as I think now) ſo much as a man lacketh of it, ſo much he lacketh of his ſenſe and wit.

Maſter. What, are you ſo ſarre changed ſince, by hearing theſe few Commodities in generall? by likelihood you would be ſarre changed if you knew all the particular Commodities.

Scholar. I beſeech you Sir, reſerve thoſe Commodities that reſt yet behind unto their place more convenient: and if ye will be ſo good as to utter at this time this excellent treaſure, ſo that I may be ſomewhat inriched thereby, if ever I ſhall be able, I will requite your pain.

Maſter. I am very glad of your requeſt, and will doe it ſpeedily, ſith that to learn it you be ſo ready.

Scholar. And I to your authority my wit doe ſubdue, whatſoever you ſay, I take it for true. The duty of a Scholar.

Maſter. That is too much, and meet for no man to be believed in all things, without ſhewing of reaſon. Though I might of my Scholar ſome credence require, yet except I ſhew reaſon, I doe it not deſire. But now ſith you are ſo earneſtly ſet this Art to attain, beſt it is to omit no time, leſt ſome other paſſion cool this great heat, and then Perſeverance in ſtudy.

you leave off before you ſée the end.

Scholar. Though many there be ſo inconſtant of mind, that flitter and turn with every wind, which often begin, and never come to the end, I am none of this ſort, as I truſt you partly know. For by my good will what I once begin, till I have it fully ended, I would never blin.

Maſter. So have I found you hitherto indéed, and I truſt you will increaſe rather, then go back. For better it were never to aſſay, then to think and ſtie in the mid way: But I truſt you will not doe ſo; therefore tell me briefly: What call you the Science that you deſire ſo greatly?

Scholar. Why ſir, you know.

Maſter. That maketh no matter, I would hear whether you know, and therefore I aſk you. For great rebuke it were to have ſtudied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arſemetrick, and ſome Augrime.

Maſter. And what doe theſe names betoken?

Scholar. That if it pleaſe you, of you would I learn.

Maſter. Both names are corruptly written: Arſemetrick for Arithmetick, as the Greeks call it, and Augrime for Algoriſme, as the Arabians found it: which doth betoken

Αριθμητική.

ken the Science of Numbring: for *Arithmos* in Greek is called Number: and of it cometh Arithmetick, the Art of Numbering. So that Arithmetick is a Science or Art teaching the manner and use of Numbering: This Art may be wrought diverſly, with Penne, or with Counters. But I will firſt ſhew you the working with the Pen, and then the other in order.

Scholar. This will I remember. But how many things are to be learned to attain this Art fully?

Maſter. There are reckoned commonly ſeven parts or works of it.

Numeration, Addition, Subſtraction, Multiplication, Diviſion, Progreſſion, and Extraction of roots: to theſe ſome men adde Duplication, Triplation, and Mediation. But as for theſe three laſt they are contained under the other ſeven. For Duplation, and Triplation are contained under Multiplication, as it ſhall appear in their place: And Mediation is contained under Diviſion, as I will declare in his place alſe.

Scholar. Yet then there remain the firſt ſeven kinds of Numbring.

Maſter. So there doth: holde it if I ſhall ſpeak exactly of the parts of Numbering, I muſt make but five of them: for Progreſſion is a compound Operation of Addition, Multiplication and Diviſion. And ſo is the Extraction

traction of roots. But it is no harm to name them as kinds severall, seeing they appear to have some severall working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them, as seven kinds distinct: But now I desire you to instruct me in the use of each of them.

Master. So I will, but it must be done in order: for you may not learn the last so soon as the first, but you must learn them in that order, as I did rehearse them, if you will learn them speedily and well.

Schol. Even as you please. Then to begin: Numeration is the first in order, what shall I doe with it?

Master. First, you must know what the thing is, and then after learn the use of the same.

Numeration.

NUmeration *is that* Arithmetical *skill, whereby we may duly value, expresse, and read any* Number *or* summe *propounded: or else in apt* Figures *and* Places *set down any* Number *known or named.*

Scholar.

Scholar. Why? then me thinketh you put a difference betwæn the Value and the Figures.

Master. Yea so doe I. For the Value is one thing, and the Figures are another thing, and that cometh partly by the diversity of Figures, but chiefly in the places wherein they be set.

Scholar. Then must I know here 3 things, the Value, the Figure, and the Place.

Master. Even so. But yet adde Order to them as the fourth. And first mark, that there are but ten Figures that are used in Arithmetick; and of those ten, one doth signifie nothing, which is made like an o, and is privately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured. A Cypher. Figures.

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is their value.

i. ii. iii. iv. v. vi. vii. viii. ix.

But here you must mark, that every Figure hath two values: One alwaies certain, that it signifieth properly, which it hath of his form, and the other uncertain, which he taketh of his place.

A place is called the seat or room that a Figure standeth in. And lok how many A place.

Figures are written in one ſumme, ſo many places hath that whole number. And that muſt be called the firſt place, that is next the right hand, and ſo reckoning by order towards the left hand, ſo that that place is laſt that is next to the left hand. As for example, If there ſtood before you ſix men in a row, ſide by ſide, and you ſhould tell them as they ſtand in order, beginning with the man that were next to your right hand; then he that were next him ſhould be called the ſecond, and ſo forth to the furtheſt from your right hand, which is the ſixt and the laſt.

Scholar. Sir, I perceive you well: ſo might I reckon Letters or any other thing. As if I ſhould write eight Letters after this order, a, b, c, d, e, f, g, h. then muſt I ſay, h, is the firſt, g, the ſecond, f, the third, e, the fourth, d, the fifth, c, the ſixth, b, the ſeventh, a, the eighth.

Maſter. That is well done. And after the ſame ſort uſe hereafter, that what I declare by one example, do you expreſſe by another: and ſo ſhall I perceive whether you underſtand it or no. And ſo paſſe over nothing, till you perceive it well, and be expert therein.

Schol. I pray you how many of theſe places be there in all?

Maſter. There is no certain number of them, but they are ſometimes more, and ſometimes fewer, according to the ſumm that is expreſſed.

For

For so many as the figures are, so many are the places: and the last place is so called, not because it is last of all other, but it is the last of that present summe, and it may be the middle place in another summe.

Scholar. Me seemeth I perceive this very well, as touching the order of reckoning of the places: but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diversity of places, which you called the value uncertain. Value uncertain.

Master. But first let me hear whether you know perfectly the certain value, Value certain.

Scholar. Yes sir, as you wrote them, so I marked them.

Master. How write you then five?

Scholar. By this figure 5.

Master. And how six?

Scholar. Thus, 6.

Master. Write these three numbers, each by it self, as I speak them, vij, iiij. iij.

Scholar. 7. 4. 3.

Master. How write you these four other, ij. j. ix. viij.

Scholar. Thus (I trow) 2. 1. 6. 8.

Master. Nay there you misse: look on mine example again.

Scholar. Sir, true it is, I was to blame, I take 6 for 9, but I will beware hereafter.

Master. Now then take heed, those cer-

tain values every figure representeth when it is alone written without other figures joyned to him, And also when it is in the first place, though many other doe follow: as for example, This figure 9 is it, standing now alone.

Scholar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen if there were no Englishmen with you, you would reckon your self to be alone.

Scholar. So it is. Then 9 without more figures of Arithmetick betokeneth it. whatsoever other letters be about it.

Master. Even so, and so doth it if it be in the first place joyned with other, how many soever do follow, as in this example, 3679. You see 9 in the first place, and doth betoken nine as it were alone.

Scholar. I perceive that, and doth not 7 that standeth in the second place, betweén 7 & 6 in the third place, betoken vi. & so 3 in the fourth place betoken three?

Master. Their figures be as you have said, but their values are not so, For as in the first place every figure betokeneth his own value certain only, so in the second place every figure betokeneth his own value certain ten times: as in the example, 7 in the second place

is

is seven times ten, and is lxx. And in the third place, every Figure betokeneth his own value an hundred times, so the 6 in that place betokeneth vi. C. and in the fourth place every figure betokeneth his own value a M. times, as in the aforesaid number 3 in the fourth place standeth for 3 M. and in the fift place every figure standeth for his own value x M. times, and in the sixt place a CM. times, and in the seventh place a MM times, and in the eighth place x MM, so that every place exceedeth the former ten times.

Scholar. As thus: if I make this number at all adventures, 91359684. here are eight places. In the first place is 4, and betokeneth but four: in the second place is 8, and betokeneth ten times 8, that is 80. in the third place is 6 and betokeneth six hundred: in the fourth place 9 is nine thousand: and 5 in the fift place is x M times 5. that is fifty M. So 3 in the sixt place is a CM times 3 that is, CCCM. Then 1 in the seventh place one MM. and 9 in the eighth place, ten thousand thousand times 9, that is xc MM. But now I cannot easily nor quickly read it in order. A general Rule.

Master. That shall you practise by this means. First put a prick over the fourth figure, and so over the seventh. And (if you have so many) over the tenth, thirteenth, sixteenth, and so forth, still leaving two figures between each two pricks. And those two

rooms

Ternaries. rooms betwéen the pricks are called Ternaries.

Then begin at the laſt prick, and ſée how many Figures are betwéen him and the end, which cannot paſſe thrée, reckoning himſelf for one: then pronounce them as if they were written alone from the reſt, and at the end of their value, ſo many times thouſands as your numbers have pricks.

After that, come to the next thrée figures, and ſound them as if they were apart from the reſt, and adde to their value ſo many times thouſands, as there are pricks betweén them, and the firſt place of your whole number. And ſo doe every other thrée figures following, if you have more. As in example, 913509684 this was your number.

Put a prick over 9 in the fourth place, and over 1 in the ſeventh place, and then no more (for your places come not to ten) as thus: 91359684,

Now go to the laſt prick over 1, and take it and the figure 9 that followeth it, and value them alone.

Scholar. 91, that is xcj.

Maſter. So it is, then adde for the number of your pricks twice M.

Scholar. That is, xcj. thouſand thouſand.

Maſter. So it is. Then take the three other figures from one to the next prick, and value them.

Scholar.

Scholar. 352, that is, CCC. lii.

Master. Now adde for the one prick, that is between them and the first place, X.

Scholar. CCC. lii. thousand.

Master. Then come to the other 3 figures that remain.

Schol. 684, that is, vi C. lxxxiiii.

Master. Now have you valued all. And at the end of the last number you shall adde nothing, because there remaineth no prick nor number after it, yet store it in another number, as thus, 2308640891053̇40.

Schol. 2308640891053̇40, I have pricked them as you taught me, but I am in doubt whether I have done well or no, because of the Cyphers, for I remember you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a Figure in setting of the pricks; and again, I know not wherefore they serve.

Master. That will I tell you now. Indeed they are of no value themselves, but they serve to make up the number of places, and so make the Figure following them to be in a further place, and therefore to signifie the more value: as in this example 90, the Cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is 90. so that two Cyphers thrust the Figure following them into the third place, and so forth. The use of Cyphers.

Schol.

Scholar. Then I perceive in the example above, I have pricked well enough, for though that Cypher that is pricked signifie nothing: yet must he have the prick, because he came in the thirteenth place. Then will I prove to number that summe. First, there is 230, M, M, M, M. & then followeth 864, M, M, M. And what shall I now do? There is a Cypher in the third place, and no Figure after him, but they that I have reckoned.

Master. He did serve for them that you have already reckoned, to make them in a place further then they should be, if he were away, and therefore now ye shall let him go. And so doe alwaies when he occupieth that place next before any prick, which is the last of that Ternary, and a Cypher in the last place doth nothing.

Scholar. Then shall I say but 89, M, M.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my pricks spent, and yet remain 340, so that I must value them, CCC, xl. only.

Master. Now can you reckon after this sort: and remember that every such room, so parted, is called a Ternary, or Trinity, for you have numbred or valued the summe most truly, and by the aid of the pricks each denomination is distinct most plainly.

Trinity.

Denomination.

Scholar. What call you Denomination?

Master,

Maſter. It is the laſt value or name added to any ſumme. As when I ſay, an hundred two and twenty pounds: Pounds is the Denomination. And likewiſe in ſaying, 25 men: Men is the Denomination, and ſo of other: But in this place (that I ſpake of before) the laſt number of every Ternary is the Denomination of it. As for the firſt Ternary, the Denomination is Unites, and of the ſecond Ternary, the Denomination is thouſands, and of the third Ternary, thouſand thouſands, or Millions; of the fourth, thouſand thouſand thouſands, or thouſand millions: and ſo forth.

Scholar. And what ſhall I call the value of the three figures that may be pronounced before the Denomination, as in ſaying, 203000000. that is, two hundred three millions. I perceive by your words, that millions is the Denomination: but what ſhall I call CCIII. joyned before the millions?

Maſter. That is called the Numerator, or Valuer, and the whole ſumme that reſulteth of them both, is called the ſumme, value or number. Numerator. Summe or value.

Scholar. Now is there any thing elſe to be learned in Numeration? or elſe have I learned it fully?

Maſter. I might ſhew you here who were the firſt Inventors of this Art, and the reaſon of

of all these things that I have taught you, but that I will reserve till ye have learned over all the practise of this Art: lest I should trouble you with overmany things at the first.

Three kinds of numbers.

But yet this you must mark, that there are three kinds of Numbers, one called Digits, another Articles, and the third mixt numbers.

Digits.

A Digit is any number under ten, as these: 1. 2. 3. 4. 5. 6. 7. 8. 9.

Articles.

And 10 with all other that may be divided into ten parts just, and nothing remain, are called Articles, such as are 10. 20. 30. 40. 50. &c. 100 200. &c. 1000. &c.

Mixt.

And that number is called mixt, that containeth Articles, or at the least one Article, and a Digit, as 12. 16. 19. 21. 38. 107. 1005. and so forth: and for the more ease of understanding and remembrance mark this. The Digit number is never written with more then one figure, but the Article and the mixt number are ever written with more then one figure. And this they differ, that the Article hath evermore this Cypher 0 in the first place: and the mixt number hath ever there some Digit.

Scholar. By these last words I perceive it much better then I did before, and now (I think) I will never misse to know these three asunder.

Master. If you remember now all that I have said, you have learned sufficiently this first kind of Arithmetick called Numeration.

How

Howbeit I will exhort you now to remember both this that I have ſaid, and all that I ſhall ſay, and to exerciſe your ſelf in the practiſe of it; for rules without practiſe, are but a light knowledge, and practiſe it is that maketh men perfect and prompt in all things. Uſe maketh maſtery.

And as you have learned to gather and expreſſe the value of a ſumme propounded and ſet down before you, ſo muſt you practiſe to mark, note, and write down with apt figures and in due places, any number only named, or recited to you, or if your ſelf imagined; as for a proof. How note you, or write down this ſumme, five thouſand two hundred fifty and ſeven?

Scholar. This troubleth me now, whether I ſhould begin at the firſt, or at the laſt. For reaſon (me thinketh) ſhould cauſe me to begin at the firſt, and yet if I write it as you ſpeak it, I muſt begin at the laſt.

Maſter. When you know your places perfectly, you may begin where you liſt; but the more eaſe for your hand is to begin with the laſt, that is to ſay, as I did ſpeak them, yet for the more ſurety, a while you may begin with the firſt, repeating my words backward thus: ſeven, fifty, two hundred, five thouſand: or elſe ſounding them all by their digit, or value, as thus: ſeven, five, two, five; for that way is eaſieſt: But then muſt you look well whether there be any cypher in your ſumme, that be

may be ſet in his place: as if the laſt value of your ſumme (as you ſpeak it) be above 9, then is there a cypher in the firſt place. And if it be an hundred, or above, then is there two cyphers, one in the firſt place, and another in the ſecond, and ſo forth.

But becauſe this thing is ſuch that cannot be ſet forth without many words; I think beſt here now at the end of Numeration, to adde a Table eaſie and ready for the firſt exerciſe of it.

Lo this is the Table.

The

The left hand, or side. The right hand, or side. The names of Digits, Values certain, or Values.

The Denominators of the place or value uncertain.	Vnites.	Tenes.	Hundreds.	Thousands.	X. Thousands.	C. of Thousands.	Millions.	X. of Millions.	C. of Millions.	M. of Millions.	X. M. of Millions.
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8	8
Seven.	7	7	7	7	7	7	7	7	7	7	7
Six.	6	6	6	6	6	6	6	6	6	6	6
Five.	5	5	5	5	5	5	5	5	5	5	5
Four.	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cypher.	0	0	0	0	0	0	0	0	0	0	0
The order of places.	First.	Second.	Third.	Fourth.	Fifth.	Six.	Seventh.	Eighth.	Ninth.	Tenth.	Eleventh.

This Table (as yee may see) hath eleven Places, and in each of them are set all the Digits, whose certain value is written on the right hand of the Table, and the value uncertain on the left hand; so that by this Table you may learn both how to expresse any

Number that you list, if that it exceed not eleven places) that is to say, ℔C. thousand Millions, and so may you by help of it, value all summes proposed under the said number.

For example: take the summe that I proposed before which was five thousand, two hundred fifty and seven. And if you will expresse it, take the first number (as I speak it) which is five ℙ, whose valuer or certain value is 5, and his uncertain, value, or Denomination is ℙ. First, you shall seek at the right hand of the valuer 5. Then seek along under the title of Denomination toward the left hand till you find thousands, and under it, right at the foot of the Table, is the number of the place that is in the fourth, wherein you must write your Digit, or valuer 5.

Afterward come to the second part of the number two hundred, whose valuer is 2, and his Denomination ℂ. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand, till you come under ℂ. then look to the foot of the Table, and there you shall see the number of the place, that is to say, the third, wherein you must set your Digit 2.

Then do so by your other two numbers that remain, and you shall find 5 in the second place for your fifty, and 7 in the first place for your seven. And thus you may doe with other numbers.

Scholar. Maister, I thank you heartily. I per-

perceiue you ſeek to inſtruct me moſt plainly and briefly, and not to hide your knowledge with ſubtile words as many do. For this rule is ſo plain, that I can deſire it no plainer. And though it ſeem ſomewhat long, yet I perceiue it to be a ſure way.

Maſter. So it is, and though it be long, yet it is neither to long, neither to plain for young learners that lack practiſe: for this Table is inſtead of a Teacher to them that lack one. But now I truſt I haue ſaid enough of Numeration: which after you haue well practiſed, then may you learn forth.

Scholar. Yet I pray you in one thing to tell mée your iudgment: Why do men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Maſter. In that thing all men doe agrée, that the Chaldees, which firſt inuented this Art, did ſet theſe Figures as they ſet all their Letters, for they write backward, as you term it, and ſo do they read. And that may appear in all Hebrew, Chaldee and Arabick Books; for they be not only written from the right hand to the left, and ſo muſt be read, but alſo the right end of the book is the beginning of it, whereas the Greeks, Latins, and all Nations of Europe, do write and read from the left hand toward the right: and all their Books begin at the left ſide.

Scholar. That reaſon hath ſatisfied mée.

Maſter. It neither ſatisfieth mée, neither liketh mée well, becauſe I ſée that the Chaldees

and Hebrews do not so use their own Numbers, as at another time I will declare. But this plain reason may best satisfie you presently, that seeing in pronouncing of Numbers we keep the order of our own reading, from the left hand to the right: and again, we doe ever name the greater numbers before the smaller: It was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places containing the greater Numbers to proceed toward the left hand.

Scholar. This reason is to me so plain, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make mee fully ready and expert in it. And in the mean season I desire to learn the other kinds of Arithmetick.

Master. That is well said: but what should you next learn? can you tell?

Scholar. I remember you said, that Addition was next.

Master. Even so, and what that is, must you first know.

Addition.

Addition.

Addition is the gathering together and bringing of two numbers, or more into one summe. As if I have 160 Bookes in the Latine tongue, and 136 in the Greek tongue, and would know how many they bée in all, I muſt write theſe two numbers one over another, writing the greateſt number higheſt, ſo that the firſt figure of the one being under the firſt figure of the other, and the ſecond under the ſecond, and ſo forth in order.

When you have ſo done, draw under them a right line, then will they ſtand thus:

Now begin at the firſt places toward the right hand alwaies, and put together the two firſt figures of theſe two numbers, and lok what cometh of them write under them, right under the line. As in ſaying 6 and 0 is 6, write 6 under 6, as thus:

160	
136	
6	

And then go to the ſecond figures, and doe likewiſe: as ſaying 3 and 6 is 9, write 9 under 6 and 3, as here you ſée.

160	
136	
96	

And likewiſe doe you with the figures that be in the third place, ſaying, 1 and 1 be 2, write 2 under them, and then will your whole ſumme appear thus:

160	
136	
296	

So that now you see that 160, and 136, doe make in all 296.

Scholar. What? this is very easie to doe, mée thinketh I can do it even since.

There came through Cheapside two droves of Cattell: in the first was 848 sheep, and in the second was 186 other Beasts.

Those two summes I must write as you taught mé, thus: Then if I put the two first Figures together, saying: 6 and 8, they make 14, That must I write under 6 and 8, thus:

	848
	186
	14

Master. Not so: and here you are twice deceived. First in going about to adde together two summes of sundry things, which you ought not to do except you séek only the number of them, and care not for the things: For the summe that should result to that Addition, should be a summe neither of sheep, nor of other beasts, but a confused summe of both. Howbeit sometimes yée shall have summes of divers Denominations to be added, of which I will tell you anon: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 & 8, under 6, 8, which is impossible: for how can two figures of two places be written under one figure and one place?

Scholar. Truth it is, but yet I did so understand you,

Master.

Maſter. I ſaid indeed, that you ſhould write that under them that did reſult of them both together: which ſaying is alwaies true, if that ſumme doe not exceed a Digit: But if it be a mixt number, then muſt you write the Digit of it under your Figures, as you have ſaid before: and if it be an Article, then write o under them, and in both ſorts you ſhall keep the Article in your mind; and therefore when you have added your ſecond Figures, which occupy the place of tens, you ſhall put that one thereto, which you kept in your mind for though it were ten indeed, yet in that place it is but as one, becauſe that every one of that place is ten, for that it is the place of tens. And in like manner, if you have in the ſecond place ſo great a number that it amounteth above 9, then write the Digit, and reſerve the Article in your mind, ever adding it to the next place following, and ſo of all other places, how many ſoever you have. And if you have a mixt number when you have added your laſt Figures, then write the Digit under the laſt Figures, and the Article in the next place beyond them: ſo ſhall your number reſulting of Addition, have one place more then the numbers which you ſhall adde together.

A place.

Scholar. Now do I perceive you, and the reaſon of this is, (as I underſtand) becauſe that no one place can contain above 9, which is the greateſt Figure that is. and then all tens

or Articles must be put to the next place following: for every place (as I may sée) exceedeth the other place next before him by 10.

Now if it please you, I will return to my example of Cattell. But I remember you said I might not adde summes of sundry things together, and that I may sée by reason.

Master. Truth it is, if you séek the one summe of any thing, but if you only séek a bare summe, and have no respect to the thing, then were it better to name the summe only without any thing: as in saying 848, without naming sheep or any thing else. And likewise 186, naming nothing.

Now let mée sée how you can adde those two summes.

Scholar. I must first set them so that the two first figures stand one over another, and the other each one over his fellow of the same place; then shall I draw a line under them both. And so likewise of other figures, setting alwaies the greatest number highest, thus as followeth.

Then must I adde 6 to 8, which make 14, that is a mixt number, therefore must I take the digit which is 4, and write it under 6 and 8, keeping the Article 1 in my mind thus:

848	
186	
4	

Next that, I doe come to the second figures, adding them together, saying, 8 and 4 make 12, to the which I put the one reserved in my mind, and that maketh 13, of which

which number I write the Digit 3 under 8 and 4. and keep the Article in my mind, thus:

848
186
34

Then come I to the third Figures, saying, 1 and 8 make 9, and 1 in my mind maketh 10. Sir, shall I write the Cyper under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keep the Article in my mind.

Master. What needeth that, seeing there follow no more Figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last Figures, I must write the Cypher under them, and the Article in a further place after the Cypher thus:

848
186
1034

Master. So now you see, that of 848, and 186 added together, there amounteth 1034.

Scholar. Now I think I am perfect in Addition.

Master. That will I prove by this example. There are two Armies of Souldiers: in the one are 106800, and in the other 9400. How many are there in both Armies say you?

Scholar. First, I set them one over another, beginning with the first number on the right hand, thus:

106800
9400

But the nether number will not match the over number.

Master.

Maſter. That forceth not.

Scholar. Then do I adde 0 to 0, and there amounteth 0, that muſt I write under the firſt place thus:

106800	
9400	
0	

Maſter. Well ſaid.

Scholar. Then likewiſe in the ſecond place I adde 0 to 0, and there ariſeth 0, which I write under the ſecond place thus:

106800	
9400	
00	

Then I come to the third place, ſaying, 4 and 8 make 12, of which I write the Digit 2, and kéep the Article 1 in my mind, thus:

106800	
9400	
200	

Then I adde 9 to 6, which make 15, to that I adde the Article 1 that was in my mind, and it is 16, I write 6 under 6 & 9, and kéep 1 in mind, thus:

106800	
9400	
6200	

Maſter. Why doe you not write both figures, ſéeing you are come to the laſt couple of numbers.

Scholar. May, reaſon theweth mée, that I muſt adde that Article that is in my mind unto the next Figure, of the over ſumme, though there be no moze in the nether ſumme.

Maſter. That is well conſidered: then do ſo.

Scholar. Then ſay I, 0 in the over ſumme, and 1 in my mind maketh 1: that write I under

under 0. Then followeth there yet one more in the over ſumme, which hath none to be added to it, for there is none in the nether ſumme, nor yet in my mind: therefore I think I muſt write that even as it is.

Maſter. Yea.

Scholar. Then doth my whole ſumme appear thus:

	106800
	9400
	116200

Maſter. If you mark this you have learned perfectly the common Addition of all ſummes which are of one Denomination: ſo that yée obſerue this alſo, that in Addition you muſt haue two numbers at the leaſt: or elſe how can you ſay that you doe adde? And euer let the greateſt number be written higheſt, for that is the beſt way, though it be not neceſſary.

And forget not this, that (if you haue many numbers to adde together) you ſhall haue oftentimes an Article of a greater value then 10, ſometimes 20, ſometimes 30, ſometimes more, yea (peradventure) 100. Therefore as you did with the Article 10, ſo do with them, reſerving them in your mind, and adding to the number next following ſo many, as their valuer or value certain is: that is to ſay, 2 for 20, 3 for 33, 5 for 50, 10 for 100, 12 for 120, and ſo forth of other like. So that if the Article be 130, then muſt you ſet down the 0 and kéep 10 in mind, to bée carried to the next row of Figures or place if any ſuch happen

pen to come. For your better understanding take this example for all.

I would adde these thirteen summes into one, which I set after this manner: then doe I begin and gather the summe of the first row of Figures, which come to 107, (for I take 9 there ten times, and that is 90) then 9 and 8 is 17, that is in all 107 of which summe I write the 7 under the first row of Figures, and then for that 100 is tenne tennes, I keep tenne in mind, which tenne I must adde unto the next row of Figures, which are in the second place:

4889
4599
2290
3699
2299
4096
1099
3298
299
699
499
899
389

Which second row of Figures (when they are added together with that tenne that I had in my mind) make in all 125, of which summe write the Digit 5 under the second row, and then (for that 120 containeth twelve tennes) I keep twelve in mind to bee added to the third place or row of Figures; which being added together, make in all 60. the Cypher o I set down under the row of Figures in the third place:

And the Figure 6 I keep in mind to bee added to the row of Figures in the fourth place, which (when they are added together) make 29. The Figure or Digit 9 I set down under the fourth place. And because it is my last worke, I set down the 2 also that I have in my

my mind to the 9 in the firſt place; ſo thoſe ſummes doe make in all 29057.

¶ But (for your more eaſe in work) when you have an Addition of ſo many ſummes to bee added together, you were beſt part that ſumme into two or three parts, and work them ſeverally, and ſo put their Additions together, and this were the beſt thing you could doe when over many ſummes fall to bee added.

4889
4599
2190
3699
2199
4099
1099
3298
279
699
499
899
389
29057

Scholar. This ſeemeth ſomewhat hard, by the reaſon of ſo many numbers together.

Howbeit, I think (if I do often prove, even with the ſame example, either by working of it alone, or elſe by parting it as you ſaid even now) that I ſhall be able to do ſo ſhortly with any other ſumme.

Maſter. So ſhall you: For it is often practice that maketh a man quick and ripe in all things: but becauſe, as well in great ſummes as in ſmall, there may chance to be ſome erreur, I will teach you how you ſhall prove whether you have done well or no.

Scholar. That were a great help and eaſe.

Maſter. Begin firſt with the higheſt number, and then to all the other orderly, and adde them together, not having regard to their places, but as though they were all Unites: The proof of Addition.

and

and ſtill (as your number encreaſeth above 9) caſt away 9. Then go forth, ever caſting away 9 as often as it amounteth thereto: and ſo doe till you have gone over all the number that you intended firſt to adde; and whatſoever remaineth after ſuch Addition & caſting away of 9, write it in ſome void place by the end of a line, for the better remembrance: and thus is the firſt part of your work proved. Then ſecondly, put together the Figures that reſult of the Addition under the line, ſtill caſting away 9 alſo. And then that that remaineth write at the other end of the line; and if thoſe two Figures be like, then have you well done, but if they be unlike, then have you miſſed. As for example, in this preſent ſumme. The firſt Figure of the over line is 9, let him go, then 8 and 8 is 16, take away 9, there reſteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there reſteth 2. Then come to the next row, whoſe firſt and ſecond numbers are 9, therefore overpaſſe them both, & take the 5 to the 2 which did remain in the firſt row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that, go to the third line, whoſe two firſt numbers you may let paſſe, becauſe they are nines: then take the two Figures of 2, which (with the other two that remained, in the ſecond row) make 6. Then go to the fourth row, whoſe two firſt numbers let go,

and

and take the 6 to the 6 that remained, and that maketh 12: take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so go through all the other numbers, and you shall find that there remaineth 5, after you have cast away 9, as often as you can find it: therefore write 5, at the end of the line in a void place thus:

5———

Then gather all the Figures of the Totall summe, which is under the lowest line, and cast away 9, as often as you can find it; as thus: 7 and 5 make 12, take away 9, there resteth 3. to that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5———5

And then you see that these two Figures be like, whereby you may know that you have done well, and so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master. With a good will.

Schol. Then will I take one of your former examples, which was this:

First in the highest line 8 and 6 make 14, then 9 taken away, there remains 5. to which I adde the 1 that followeth, and that maketh 6: then come I to the second line, where I find first 4, which with 6 maketh

106800	
9400	
116200	

keth 10, from that I take 9, and there resteth one, the next Figure is 9, and therefore I let him alone, so find I 1 remaining, which I set at the end of a line, thus:

1———

Then I come to the Totall summe, and there I find that all the figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other end of the line, thus: 1— —1

And because they be like, I know that I have well added.

Master. So you know now both how to adde two summes or more together, and also how to prove whether you have done well or no: and now I will teach you how to adde summes of divers Denominations together: which thing can never be but when the one Denomination is such that it containeth the other certain times. And yet you shall adde them to the other, not after this sort (as you did them that were of one Denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of divers Denominations.

If you have a summe of divers Denominations, then look that you set every Denomination by himself, with some note or figure of his Denomination, as they are wont to be written. Then write your other summes so under that first, that every one bee set under the other of the same Denominations: As for Example, If your Denominations be

pounds,

pounds, shillings, and pence, write pounds under pounds, shillings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, me thinketh it needeth not to warn me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peradventure I should have been deceived in it.

Master. If you doe say it is plain, I will speak no more of it, but with an example make the matter to appear evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is all together: Therefore must I first set down my great summe, and then the other, every one under his Denomination agreeing to the greatest summe, as here you see with a line under them.

	li.	s.	d.
	22	6	8
	5	16	6
	4	3	0

Then must I begin at the smallest numbers (which must alwaies be set next to the right) and adde them together: and if the summe will make 1 or 2 or 3 of the next Denomination) then must I keep it in my mind till I come to that place, and under that first place must I note the residue (if there remain any of the same Denomination) but if there remain none, then need I to write under it nothing. And this is all that you must mark in this Addition: for all other things are like to the

manner of Addition before mentioned: *Therefore the chiefest point of this Addition is, to know the values of common Coines, and rated summes.* As how many shillings be in a pound, how many pence in a shilling, of which (and of other like things) I will instruct you hereafter in teaching of Reduction: But now I may not disturbe your wit from the thing that we are about.

Therefore let us return to that former example which I proposed of the Debtors: which summes when I had set orderly, they stood thus with a line under them.

Then to adde them into one summe, I must begin at the right hand where the smallest Denomination is, and adde them together, first saying, 6 and 8 make 14. Now seeing these 14 are pence, which contain one shilling and 2 pence: the 2 pence I set down under the line of pence, and the one shilling I keep in my minde to carry to the next row being the place of shillings.

li.	s.	d.
22	6	
5	16	
4	3	

Then doe I adde the shillings together, saying, 1 in my minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the second place which standeth for 10. make 26, which is 1 pound 6 s. The 6 s. I set down under the place of shillings, as appeareth in the example. And the 1 pound I keep to carry to the pounds.

li.	s.	d.
22	6	
5	16	
4	3	
6		

The

Then come I to the pounds, adding them all together saying, 1 that I keep and 4 make 6, and 5 make 10 and 2 make 12. The Figure or Digit 2 I set down right under that place or row of pounds where I gather them, and the Article 1 I keep to carry to the next place, saying, 1 in minde and 2 is 3, which three I set downe directly under the 2. And then appeareth my whole summe thus.

li.	s.	d.
22	6	8
5	16	6
4	3	0
32	6	2

And thus must you do with any such like summes whatsoever, whether they be money, weight or measure, which (if you practise divers summes) you shall be well acquainted with the feat of addition.

But now can you tell how to prove this Addition, or such other like of divers Denominations, and to try whether you have well done or no?

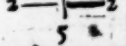
Scholar. I would I could.

Master. That shall you doe by this meane: You must make a Crosse which shall have as many lines as you have sundry Denominations in your Addition: As if you have but two Denominations, then you may take it thus: that the ouer part and nether part may serve for one Denomination. And if you have three Denominations (as pounds, shillings, and pence) then must you make

Proof of Addition of divers denominations.

three lines, thus: The upright line may serve for pounds, and the highest thwart line for shillings, and the lowest for pence: As for example, the sum which wee last wrought.

li.	s.	d.
22	6	8
5	16	6
4	3	0



For the proof of which, because it containeth 3 Denominations, I must make a crosse of 3 lines, as in the example before. Then I reckon first at the right hand the pence, 6 and 8 make 14, from which I take 12 for the next Denomination, that is to say, a shilling, and there resteth 2, which I must write at one end of the nether thwart line.

After that I gather the summe of the shillings 3, 16, 6, which maketh 25, to whom I put 1 that I took of the pence, and that maketh 26, from those I take 20, the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds 4, 5, and 2, which make 11. to them I adde the one that came of shillings, and they make 12, from whence I cast 9, and there resteth 3, that 3 I joyn to the 2 in the next place, and they make 5, which 5 I set at the Cross

also

also. And thus is my first part of my worke proved.

That done, I come to the total summe under the line, and examine it, beginning at the pence, where I finde but 2, and cannot take 9 from him: therefore I set him at the other end of the netherthwart line: then I come to the shillings, where I finde onely 6, which (because it is lesse then nine) I set it at the other end of the line of shillings, that is, the ouermost thwart line.

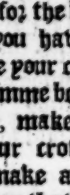
Last of all, of the 32 li. I take three times 9, which is 27, and there remaineth 5, which I write under the vpright line: either else I may reckon them simply without any respect of their valuation or place: saying, 2 and 3 make 5, which because it is lesse then nine, I set under the vpright line as before. Then I consider every number, comparing it to the number that is against it: and because I finde them to be every one like his match, I know that I have well done.

Scholar. This Crosse I perceiue doth serve for these 3 Denominations, pounds, shillings, pence: but what if I had l. s. d. ob. and qd?

Master. These lines as I have said, doe serve for 3 Denominations, such as they be, as here 3 doe serve for pounds, shillings, and pence: but if you have no pounds in your summe, then may they serve for shillings, pence, and halfe penies: yea, for d. ob. and q. or in weight for C. q. and l. or in measure for Elles,

Quarters, and Nayles, if you have no greater Denomination: so that you remember that the upright line serveth for the greatest Denomination, and the highest thwart line for the next, and the lowest for the least.

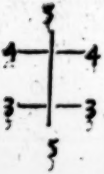
And so if you have foure Denominations, you must make your cross with so many lines: And if your summe be of more Denominations, make so many lines in your crosse. And thus will I make an end of Addition, saving that here (for the better understanding of this Rule) I have set you down certain examples both of money, weight, and measures, with their works and proofs.



Examples of Addition.

li.	s.	d.
23	10	4
45	6	8
37	2	9
25	13	6
131	13	3

li.	s.	d.
130	17	10
28	0	8
13	13	4
120	0	0
292	17	10

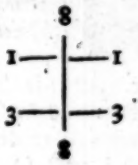


The proofs.

C

C.	q.	li.
34	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards.	q.	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3



Subtraction.

Scholar.

When have I learned the two first kindes of Arithmetick: *now (as I remember) doth follow Subtraction, whose name (me thinketh) doth sound contrary to* Addition.

Subtraction.

Master. So it is indeed: for, as Addition increaseth one grosse summe, by bringing many into one: so contrariwise, Subtraction diminisheth a grosse summe by withdrawing of other from it. *So that* Subtraction *or Rebating is nothing else but an* Art *to withdraw and abate one summe from another, that the remainer may appear.*

Scholar. What do you call the remainer?

Master. That you may perceive by the name.

Scholar. So me thinketh: but yet it is good to ask the truth of all such things, lest in trusting to mine owne conjecture, I be deceived.

Master. So it is the surest way. And, as I see cause, I will still declare things unto you so plainly, that you shall not need to doubt. Howbeit, if I doe overpasse it sometimes, (as the manner of men is to forget the small knowledge of them to whom they speake) then

then doe you put mee in remembrance your ſelfe, and that way is ſureſt.

And as for this word that you laſt asked me, take you this deſcription: The Remainer is a ſumme left after one Subtraction made, which declareth the exceſſe or difference of the two other numbers, as if I would abate or ſubtract 14 out of 18, there ſhould remaine 4, which is called the Remainer, and is the difference betweene thoſe two numbers 14 and 18. Remainer.

Scholar. I perceive then what Subtraction is: now reſteth to know the order how to worke it.

Maſter. That ſhall you doe by this means. First, you muſt conſider, that if you ſhould go about to rebate, you muſt have two ſundry ſummes propoſed: The firſt, which is your groſſe ſumme, (or ſumme total) and it muſt be ſet highеſt: and then the rebatement (or ſum to be withdrawn) which muſt be ſet under the firſt, (whether it be in one parcel or in many) and that in ſuch ſort, that the firſt figures be one juſt over another, and ſo the ſecond and third, and all other following, as you did in Addition: then ſhall you draw under them a line, and ſo are your ſummes duly ſet to begin your working.

Then begin you at the right hand (as you did in Addition) and withdraw the nether number out of the higher, and if there remain any thing, write that right under them beneath the line: and if there remain nothing (by

(by reason that the two Figures were equal) then write under them a Cypher of nought: And so doe you with all the other Figures, evermore abating the lower out of the higher, and write under them the Remainder still, till you come to the end: And so will there appear under the line what remaineth of your grosse summe after you have deducted the other sum from it, as in this example,

I received of your Father 48 s. of which I have laid out for you 36 s. now would I know what doth remain. And therefore I set my number thus in order. First, I write the greatest summe, and under him the lesser, so that the Figures at the right side be even one under another, and so the other, thus.

	48
	36

Then do I rebate 6 out of 8, and there resteth 2, which I write under them right beneath the line thus.

	48
	36
	2

Then I go to the second Figures, and do rebate 3 out of 4, where there remaineth 1, which I write under them right, and then the whole summe and operation appeareth thus.

	48
	36
	12

Whereby it appeareth, that if I withdraw 36 out of 48 there remaineth 12.

Scholar.

Scholar. Now will I prove in a greater summe, and I will subtract 2367924 out of 3468946, those summes I set in order thus:

3468946	
2367924	

Then do I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write under them. Then go I to the second figures, and withdraw 2 out 4, and there remaineth 2, which I set under them also, then I take 9 out of 9 and there resteth 0, which I write under them (for you say, that if the Figures be equal, so that nothing doth remain, I must write the Cypher 0 under them.)

Master. It was well remembred: now go forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1, which I write under them also. Then in the fifth place, I take 6 out of 6, and there resteth 0, (for it I write under them the Cypher 0.) Then in the sixth place 3 rebated from 4 there remaineth 1, which I write under them, and likewise in the seventh and last place, 2 taken from 3 there is left 1, which I write under them: so have I done my whole working, and my summes doe appeare thus. Whereby I see, that (if I doe rebate 2367924 out of 3468946) there remaineth 1101022.

3468946	
2367924	
1101022	

Master. This is well done. And that you may be sure to perceive fully the Art of Subtraction,

traction, let mée ſée how you can ſubtract 52984732 out of 8250003456.

Scholar. Firſt, I ſet downe the greateſt ſumme, and after that I write under it the leſſer number, beginning at the right ſide, and then my Figures will ſtand thus:

8250003456
52984732

Note. Then take I 2 from 6, and the reſt is 4, which I write under them. Then do I withdraw 3 from 5, and there remaines 2, which I write under them. Then take I 7 out of 4, but that I cannot, what ſhall I now do?

Maſter. Marke well what I ſhall tell you now, how you ſhall do in this caſe, and in all other the like: If any figure of the nether ſum be greater then the figure of the ſum that is over him (ſo that it cannot be taken out of the figure over him) then muſt you put 10 to the over figure, and then conſider how much it is, and out of that whole ſumme withdraw the nether figure, and write the reſt under them. Can you remember this?

Scholar. Yes, that I truſt I ſhall. Now then in mine example where I ſhould have taken 7 out of 4 and could not, I put 10 to that 4, which maketh 14, from it I take away 7 and there reſteth 7 alſo, which I write under them.

Maſter. So have you done well: But now muſt you marke another thing alſo: that (when-

(whensoeuer you do so put ten to any Figure of the ouer number) you must adde one still to the Figure or place that followeth next in the nether line: as in the example there followeth 4, to which you must put 1, and make him 5, and then go on as I haue taught you.

8250003456
52984732
018732

Scholar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3. but that cannot be; therefore I must put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written under them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the nether line, and that maketh 9, which I should take out of 0 and cannot; therefore I put thereto 10, and that maketh 10, from 10 I take 9, and there remains 1, which I write under them.

Thus do I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure above, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written under them.

Then come I to the nert figure which is 2, and to him I doe adde 1, which maketh 3, that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doe take 3, so there remaineth 7 to be written under them: likewise doe I put 1 to 5, that

that 5, and make it 15, from which I rebate 6 and there remaineth 9, which I write under them. Now have I spent all the nether Figures, and what shall I do more?

	8250003456
	42984732
	8197018724

Master. You should have added one to the next figure following (if there had been any) because you added 10 to the last figure before of the over line: but being there is no figure following, you must adde that one to the place following, and then deduct that one from the number above.

Scholar. Then shall I say, because I borrowed 10 to the over 5, I must put 1 in the next place beneath, that is under 2, then must I subtract that 1 from 2, and there resteth 1 to bee written under that in the ninth place. Now I have no more to subtract, for there is not any figure remaining beneath, neither yet any unite to be added, because I borrowed not 10 to the Figure last before: and yet is there 8 remaining in the over line, which I thinke (by reason) should be set at the end of the Figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed.

Scholar. But Sir, I beseech you, shall I alwayes when any number so remaineth alone, as thus 8 did, write him under the line straight

ſtraight againſt his own place?

Maſter. Yes, what elſe? whether they be one or many: and this well remembred, you have ſufficiently learned Subtraction: Howbeit, becauſe of certain things that might deceive you, if you did not take good heed to your working, I will propoſe to you another example of many numbers to be ſubtracted, as thus: I received of a friend of mine to keep 2869 Crowns, of which at one time I delivered him again 500, at another time 368, at another time 440, at another time 80, and another time 64, now would I know how many do reſt behinde. Therefore firſt I ſet down my groſſe ſum, 2869 *Crowns received.* and underneath it I ſet all the parcels thus, and under them a double line. Then firſt I begin at the firſt place, and gather together the ſum of all thoſe lines (ſave the overmoſt) in their firſt Figures: and ſo I doe with all the figures of the ſecond place, and ſo forth, as I did in Addition, ſave that I leave out the higheſt row of numbers (as the line warneth me) and that ſumme ſo gathered between the double line, is the ſumme delivered in all: which ſumme I doe afterwards ſubtract out of the higheſt row of numbers, and the remainder doe I ſet under the nethermoſt line:

500
368
440 *delivered.*
80
64

Note.

as

as for example:

I set the summes as before: then do I gather the first figures of all the places delivered together: where I finde but 4 & 8, that maketh 12, (for three Cyphers increase no summe in Addition, as you learned before) of the 12 therefore doe I write the Digit 2 between the double line and keep the Article in my mind, till I come to the second place, where I find 6, 8, 4, 6, that maketh 24, to them I put the Article in my mind, and it is 25, of which I write under the second place, and keep the Digit 2 in my mind for the third place, where I find 4, 3, 5, that makes 12, to the which I adde the 2 in my mind, and it maketh 14, thereof I write the 4 under the third place, and because there remains no more Figures to be added, I write the Digit in the fourth place, as you see in the Example, and so it appeareth, I have delivered in all a thousand four hundred fifty two Crowns.

2869	*Crowns received.*
500	*Delivered.*
368	
440	
80	
64	
1452	*Delivered in all.*
1417	*Rest behind.*

Then come I to the subtracting of this summe between the lines, for by Addition it is equall to the five parcells over it. Therefore I proceed to subtract it from the outermost summe, saying, 2 from 9 remains 7 to be written under them beneath the lowest line,

line. Then in the ſecond place I take 5 from 6 & there reſteth 1 to be written under them: Then in the third place, 4 from 8, reſteth 4: Laſt of all in the fourth place, 1 from 2 remaineth 1. And thus I ſée that after thoſe five ſummes are ſubtracted from 2869, the Remainer is 1417.

Scholar. This I perceive: but is there no ſhorter way and more ſpéedy?

Maſter. Yea, when you are a while exerciſed in it: for you may (as faſt as you can gather the numbers together) withdraw them out of the higheſt ſumme. But if in quantity thoſe numbers added together, exceed the higheſt ſumme, or upper number, then ſhall you (as before hath béen taught you) imagine to borrow 10, 20, or 30 moe, as néed ſhall require, and put them to the upper number, to help to further the abatement, reſerving or reſtoring the Articles that you borrowed to the next place again: and ſo ſtill go forward till you have ended your work: As for example. In the laſt ſumme propoſed, I gather firſt in the firſt place 4 and 8 that maketh 12, which 12 I ſhould deduct or take out of 9 in the upper number above the line, but I cannot: and therefore I adde unto 9 an Article of 10, and it maketh the upper number 19, from whence I take 12, then there reſteth 7. then for the Article 10 I adde to the next place of money delivered; ſaying, 1 that I bring and 6 make 7, and 8 make 15, and 4 make 19, and 6 make 25,

An abridgment of the former manner of Subtraction.

which 25 I should take out of 6 in the upper number, but I cannot. Therefore I adde 2 tens, or 20 unto 6 in the upper number, and that maketh 26, then 25 out of 26, resteth 1; then the tens which I borrowed, or have in mind, I adde to the nert row or summe delivered; saying, 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to bée added, the one that I borrowed, or have in mind, I rebate from 2 in the upper line, and there remaineth 1, which I set down in the remainer line: and so my summe appeareth (as before) to be 1417 Crowns.

Loe thus have you now a shorter way.

Scholar. I like both waies well: and I perceive both well: yet, as in the working it séemeth somewhat long, so in the other it leaveth very much (mée séemeth) to remembrance, and therefore may cause error quickly, except a man have a quick and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will give mé leave) I will try what I can doe in a like summe, to work it the shortest way: whereupon I would subtract out of 40301964, these three parcels.

There

Therefore I ſet them firſt in due order: then I gather the parcells of the firſt place, which are 8. 2. 1. that is 11, which I ſhould take or deduct out of

46301964	*Charge.*
20003428	*Diſcharge.*
10001432	
16101461	
43	

4, which is over him, but I cannot: therefore I adde an Article, or one ten to 4, which maketh 14, then 11 out of 14, there reſteth 3 to bee written under the firſt place between the two lines.

Then come I to the ſecond place, ſaying, 1 that I borrowed to have in my mind, and 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, and then 12 from 16 reſteth 4. which I write under the ſecond place between the two lines.

Then come I to the third place ſaying, 1 that I borrowed or have in mind and 4 make 5 and 4 is 9, and 4 make 13, which I ſhould take out of 9 that is over them, but I cannot: therefore I adde 10 to 9 which make 19, then 13 out of 19 reſt 6.

Then come I to the fourth place, ſaying, 1 in mind and 1 is 2, and 2 is 4 and 3 make 7, which becauſe it cannot bee taken from 1, I take it from 11. and there reſteth 4.

After that I come to the fift place, where are only three Cyphers, which make nothing,

unto which I adde 1 in mind, then should I take that (that is to say) 1 from the Figure over them, which is also a Cypher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 under them. Then in the first place I find but 1 & 1 in mind make 2, which I take out of 3 over him, and the remainer is 1: that must bée written betwéen the two lines in the first place. So I go to the seventh place, where I find only Cyphers, and in the grosse summe over them a Cypher also: therefore must I write the remainer (which is nothing) with a Cypher also. Then in the eight and last place, I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is over them, there will nothing remain. And that must be noted with a Cypher betwéen the two lines (as I have often said) and so have I ended my worke, and the figures stand as followeth.

Scholar. But Sir, I remember you taught mée that Cyphers should not come in the last place, for because they serve onely to increase the value of other Figures which follow them and serve not those figures that goe before them: and now in my Example I have set two Cyphers in the two last places.

Master. I commend you for your remembrance. And truth it is, you should not have set them here, but only because that I would make you plainly to perceive the Art of Subtraction,

traction. Therefore seeing that you doe now perceive it, whensoever you would write down a Cypher, lok whether any other figures bee yet behind: and if not, then let go the o also, for it needeth not to write him in the latter places where no other figure doth follow, except it bee (as I did now suffer you) to teach the use of Subtraction the plainer.

Therefore your figures must stand thus when the work is ended.

40301964	*Charge.*
20003428	*Dischar:*
10001432	
10101461	
194643	*Rest.*

Scholar. Sir, I doe think with that that you taught mee before, and by these two sums that you taught mee last also, that now I could Subtract any summe.

Master. So may you, if you have marked what I have taught you. But, because this thing (as all other) must be learned surely by often practise, I will propound here two Examples to you: wherein if you often exercise your self, you shall bee ripe and perfect to subtract any other summe lightly, for in them is contained all the observances of whole numbers. And because you shall perceive somewhat both how to doe it, and also whether it bee well done when you have proved to doe it; therefore have I written under them both the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354	*Paid.*	103145	*Paid.*
10249		102597	
163		101024	
20766	*Paid in all.*	02198	*Rest.*
9840	*Rest to pay.*		

Scholar. Sir, I thank you: but I think I might the better doe it, if you did shew me the working of it.

Master. Yea, but you must prove your self to doe some things without my aid, or else you shall not be able to doe any more then you are taught: And that were rather to learn by wrote (as they call it) then by reason. And again, there is nothing in these examples, or any other of whole numbers, but I have taught you the rules of them already.

Scholar. Then I trust by practise to attain the use of it. And is this all that I shall learn of Subtraction?

Master. Yea, saving that (as you have seen in Addition) there are numbers of divers Denominations, in which the working is not much unlike: yet (without some instructions to bee given of it) it might seem to a learner more difficult then indeed it is. Therefore I will briefly shew you the use of it only by an example or two.

A certain man owed to me: 14 l. 12 s. 8 d.
of which he paid me at one time 4 l. 6 s. 8 d.
at another time 3 l. *at another* 2 l. 3 s. 4 d.
and last of all 6 s. 8 d.

Now would I know what remaineth unpaid yet: therefore I set my summes thus, every one in their due place: as pounds under pounds, shillings under shillings, pence under pence.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar. Sir, I pray why doe you write 2 l. for the common speech useth rather to say, 40 s.

Master. We must here use the Denomination that is greatest in any summe, so that we may not write according as we use to speak, saying, 16 d. 18 d. or likewise 7 groats, 8 groats, 24 s. 40 s. 48 s. and such other: but we must write every Denomination, that is in any summe, by it self. Note how the penne differeth from the common order of Counters.

Namely, shillings and pounds. So must we write for the last summes now named, 1 s. 4 d. 1 s. 6 d. 2 s. 4 d. 2 s. 8 d. 1 l. 4 s. 2 l. 8 s. and so forth of other like.

Scholar. So that we may not write in Arithmetick, pence, when the summe amounteth to shillings, nor shillings, when the summe maketh pounds. Now (if it please you) end your example.

Maſter. When my ſummes are ſo ſet as I ſhewed, then (according to the rules of Addition) I gather all the particular ſummes which be paid mé into one totall ſumme, directly to be ſet under them between the two lines, not medling with the 14 l. 12 s. 8 d. as the line warneth me: therefore muſt I begin with the ſmalleſt Denomination, ſaying, 8, 4, 8, is 20 d. which maketh 1 s. and 8 d. the 8 d. I ſet down under the place of pence; and the 1 ſhilling I kéep in minde to carry to the next Denomination of ſhillings. Then come I to the ſhillings, and ſay, one that I bring or have in minde, and 6 is 7, and 3 is 10 and 6 makes 16, which, becauſe it containeth not 1 pound, I ſet directly under the place of ſhillings. Then come I to the pounds, whoſe parcels are 2, 3, 4, that is in all 9, that 9 doe I ſet down directly under the pounds: And ſo the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

li.	s.	d.	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	
4	16	0	*Reſt.*

Now for the worke of Subtraction, I muſt rebate that totall ſumme of Addition out of the

the highest number, that is to say, from the 14 l. 12 s. 8. d.

Therefore to perform the work I say, 8 d. out of 8 d. remaineth or resteth nothing, therefore in the place of the rest or remain, right under the Denomination, I set down 0. Then coming to the shillings, where I find 16, which should bee taken out of 12, but I cannot: therefore I imagine to borrow 1 of the next Denomination, that is, of the 14 l. and put that one pound so borrowed unto 12 s. that maketh 32 s.

Now 16 s. out of 32 s. resteth 16 s. which 16 s. I set down directly under the place of the rest.

Lastly, coming to the pounds, saying, one pound in mind that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remain, appear to bee 4 l. 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this work also?

Master. Yes, as in this last Example I will also shew you, for you may adde together the particular summes as

they

they are set in order, beginning with the pence, saying, 8, 4, 8, make 20 d. which 20 d. you should take out of the 8 d. above the line, but you cannot, therefore shall you borrow 1 of the next Denomination, that is to say, 1 of the shillings, and put it to the 8 d. that maketh 20 d. now 20, out of 20 d, resteth 0, which Cypher I set down directly under them.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

Then 1 shilling that I borrowed or had in mind, & 6 make 7, & 3 make 10, & 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I doe borrow 1 l. & put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly, I come to the pounds, saying, 1 l. in mind, or that I borrowed, and 2 make 3. and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appear as before to be 4 l. 16 s. 0 d.

Scholar. Then doe I perceive very well, and if there bee no other thing to bee learned in Subtraction, then may I come to Multiplication, for that you reckoned to bee next in order.

Master. Wee have done indeed with the Art of Subtraction, as touching the working.

But yet before wee go to Multiplication, I will

will instruct you how to examine your work, whether it be well done or not. For the performance whereof, if you mark what I said in Addition, you may easily perceive what is to be done for the proof of Subtraction, which is best made by the aid of Addition thus. Proof of Subtraction.

Draw under the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did subtract before, together, and write that that amounteth under the lower line: and if the summe that cometh thereof, be equall to the highest of the Subtraction, then is the Subtraction well wrought, or else not: As you may see for example in the summes set down before, and first in summes of one Denomination, whereof one was this.

Where the Number 52984732 is subtracted from 8250003456, & the Remainer is 8197018724 Now to prove whether it be truly wrought or not, I adde the Remainer and the number subtracted together, beginning at the right hand; and first I say 4 and 2 is 6 which is set under the line:

	8250003456
	52984732
	8197018724

Example in a sum of one Denomination.

The number given	8250003456
The number to Subtract	52984732
The Remainer	8197018724
The Proof	8250003456

Then again in the second place I say 2 and 3 is 5, which I write under, next that in the third

third place, 7 and 7 are 14, of which I write the Digit 4, and keep the Article 1 in my mind. Then in the fourth place 8 and 4 is 12 and 1 in my mind maketh 13, whereof I write down the Digit 3, and keep the Article 1 in my mind. Again in the fift place, 1 and 8 is 9, and 1 in my mind is 10. Whereof I set down 0 and keep the 1 in my mind. And so going on to the rest (as it is taught in Addition) when I have made an end, I see that the lowest line of numbers and the highest be alike: wherefore I know that I have well done.

So likewise the Proof is to bee made in numbers of divers Denominations: as for Example, in our summe of that kind which in the first form of working stood thus: (all the particular numbers to be subtracted, being drawn into one.)

Example in a sum of divers Denominations.

Where, in the title of pence, I find 8 and 0: the 8 I set down directly under in that of pence.

Then in the place of shillings I find 16 & 16 which make 32 shillings, wherein is contained 1 li. and 12 s. the 12 s. I set downe directly under them in the due places

li.	s.	d.
14	12	8
4	6	8
3	0	4
2	3	0
	6	8
Paid in all	9-16-8	
Rest	4-16-0	
Proof	14-12-8	

of

of ſhillings, and one pound I kéep.

Then coming to the pounds, I ſay 1 that I kéep, and 4 is 5, and 9 is 14, which 14 in due order I ſet down directly under them as this Figure ſheweth. And the whole ſumme is 14 l. 12 s. 8 d. agréeing with the upper number above. So I find the worke is good, and the Subtraction well wrought.

The ſame thing is to bée done for the latter form of Subtraction (where the particular ſummes are not gathered together into one groſſe.) For the Remainer and all the particular ſummes ſubtracted, being added together, if the ſumme that cometh thereof bée equall to the higheſt number above, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſums which ſtood thus,

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

Example of a proof in the latter form of Subtraction.

Firſt in the title of pence, I adde 8, 4, 8, that maketh 20 d. which containeth 1 ſhilling and 8 pence.

The 8 I ſet down under the loweſt line in the row of title of pence, and that 1 ſhilling I kéep to carry the next Denomination or place of ſhillings.

Then returning to the ſhillings, ſaying: one

one in mind, or that I keep and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 12 shillings, which amounteth to one pound, 22 s. the 12 s. I set down under the title of shillings: and 1 pound I keep or have in mind to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14; then doe I write 14 under the pounds, and so have I end'd the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I have well done.

And thus have I taught you the Art of Subtraction, and the means to prove whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multi-

Multiplication.

MUltiplication is an operation, whereby two ſummes produce the third: which third ſumme ſo many times ſhall contain the firſt, as there are Unites in the ſecond. And it ſerveth in ſtead of many Additions. As for Example: When I would know how many are 30 times 48. if I ſhould adde 48 thirty times, it would be a long worke. Therefore was this worke of Multiplication deviſed, which ſhall doe that at once that Addition ſhould doe at many times. Multiplication what it is.

Scholar. I perceive the commodity of it partly, but I ſhall not ſee the full profit of it till I know the whole uſe of it. Therefore Sir, I beſeech you, teach me the working of it.

Maſter. So I judge it beſt, but becauſe that great ſummes cannot be multiplied but by the Multiplication of Digits, therefore I think it beſt to ſhew you the way of multiplying them. As when I ſay, 9 times 8, or 8 times 9, &c. And as for the ſmall Digits, under 5 it were but folly to teach any rule, ſeeing they are ſo eaſie that every childe can doe it: but for the Multiplication of the greater Digits, thus ſhall you doe. Multiplication of Digits.

Firſt, ſet your Digits one right over the other,

other then from the uppermoſt downwards, & from the nethermoſt upward, draw ſtraight lines, ſo that they make a croſſe, commonly called Saint Andrews croſſe, as you ſée here. Then lok how many each of them lacketh of 10, and write that againſt each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I muſt write thoſe Digits thus.

The difference.

Digit difference.

8
X
7

Then doe I lok how much 8 doth differ from 10, and I find it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference.

8 2
X
7

After that I take the difference of 7 likewiſe from 10, that is 3, and I write that at the right ſide of 7, as you ſée in this example.

Digit difference.

8 2
X
7 3

Then doe I draw a line under them, as in Addition, thus.

Laſt of all, I multiply the two differences, ſaying, 2 times 3 make 6, that muſt I ever ſet under the differences, beneath the line: then muſt I take one of the differences (which I will, for all is like) from the other Digit (not from his own) as the lines of the Croſſe warn

warn me, and that that is left, must I write under the Digits. As in this Example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write under the Digits, and then there appeareth the multiplication of 7 times 8 to be 56. And so likewise of any other Digits, if they be above 5, for if they be under 5, then will their difference bee greater then themselves, so that they cannot bee taken out of them. And again, such little summes every childe can multiply, as to say, 2 times 3, or 4 times 5, and such like.

Digits difference.

8	2
X	
7	3
5	6

Scholar. Truth it is, and seeing me seemeth that I understand the multiplying of the greater Digits, I will prove by an example how I can do it. I would know how many are 9 times 6.

Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

Scholar. Then would I know how many are 6 times 9: therefore I set the Digits thus, and make the crosse thus,

9.	
X	
6	

Then doe I set their differences from 10 at the right hand, the difference of 9, which is 1, against it, and the difference of 6 which is 4, against it also, as in this example.

And under them draw a line, multiply the differences together, saying: 1 time 4 maketh 4, that 4 doe I write under them thus.

Then doe I take I one of the differences from the other Digit, as, 4 from 6, or else 1 from 9, and each waies there resteth 5, which I doe write under the Digits thus. And so appeareth the Multiplication of 6 times 9 to be 54. Thus I see the seat of this manner of multiplication of Digits.

Master, how might you go straight to the multiplication of great numbers, save that both for your ease and surety in working I will draw you here a Table, whereby shall appear the multiplication of all the Digits, and this is it that followeth.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

In which Table when you would know the product in any multiplication of Digits, seek your first or last Digit in the greater figures, and from it go right forth towards the right hand, till you come under the number of your second Digit, which is in the highest row, and then the number that is in the meeting of the rowes of little squares (which come directly from both your propounded Digits) is the Multiplication that ariseth of them. As if I would know by this Table the multiplication of 7 times 9. I seek first 7 in the greater figures, and then go right forth toward the right hand, till you come under 9 of the highest row, in which place where you so come under the other Digit (as here for example you come under 9) is alwayes contained the 63 come of product, which you look,

and that place wee term to bee in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle, where the rows of little squares directly proceeding from 7 and 9 doe meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater summes.

Scholar. This is very good and ready. And so may I find the multiplication of any Digits: but now how shall I doe in greater summes?

Master. *When you would multiply any* summe *by another, you shall mark that it is the meetest order to set the greatest* number *highest, which is the place of the* number *that must be multiplied: and likewise the lesser* number *under it, for that*

Multiplier

is the place of the Multiplier *or* Multiplicator, *that is to say, the* number *by which the* Multiplication *is made, and is in* English *alwaies put be-*

Times.

fore this word, Times: *in such speaking when I say 20 times 70. And the* number *that followeth this word* Times, *is that which must be* Multiplied.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser under it, as in Addition. And under them must I draw a line. As for example,

If I would multiply 264 by	264
29, I must set them thus.	29

¶ Of which number thus set down to bee multiplied, may bee formed a question, as thus, There are 29 men, and each man hath

264

264 Lambs. The question is, how many Lambs they have in all.

To the performance whereof, I must multiply every figure of the high row, by every figure of the neather row: and that that amounteth I must set under the line, as thus.

First I doe multiply 4 by 9, saying, 9 times 4 (or 4 times 9 which is all one) and that maketh 36, as the Table before of Digits doth declare, of that 36 I must write the 6, that is the Digit, under the 9, and the Article 3 keep in mind to carry to the next place.

	264
	29
	6

When come I to the second figure of the higher row, which is 6, and say: 9 times 6 make 54, and with the 3 in my mind make 57, the 7 I set down under the 2, and 5 I keep in mind.

	264
	29
	76

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in mind, maketh 23: wherefore because it is the last work of the Multiplier, I set it down in order as you see:

	264
	29
	2376

And so have I ended the first figure of the Multiplier. Wherefore I give it now a fine rasb with my pen.

When begin I with the next figure, and multiply it into all the higher figures, as thus,

264
28
2376

First, 2 times 4 maketh 8, that 8 doe I write under the second place: for determining the Digit or first figure of the Multiplication that amounteth of the figure of the higher number, must be set under the Multiplyer of it, the other in their order toward the left hand.

Scholar. I understand you thus, that the Digit of the summe amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplyer, must be set under the first place: and that that amounteth of the same first figure by the second Multiplyer, must be set under the second place, and so of the other, if there be more Multiplyers.

Master. So mean I indeed: and if there amount but a Digit, then must it be set under the Multiplyer.

And now to go forth: I multiply by the same 2, the second figure of the higher row, which is 6, saying, two times 6, make 12, wherefore I write the Digit 2 under the third place, and the Article 1 I keep in mind.

264
28
2376

When doe I multiply the last figure of the higher summe by that same 2, saying, two times 2 is 4, and with the 1 that I have in mind maketh 5, which I write under the fourth place. And so

so have I ended the whole Multiplication: wherefore I also give the 2 a dash with my pen, thus ; and so I doe ever as soon as I have dispatched any Digit by which I multiply: and the summes stand thus.

264
29
2376
528

Then must I draw a line under all those summes that mount of the multiplication, and must adde all them into one summe, as in the Example you may see.

264
29
2376
528
7656

Where in the first place I find but 6, and therefore write I it under the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keep one in my mind, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264, by 29 and that is the just number of the Lambs that 29 men had.

Scholar. If there be no more to be observed in it, then can I doe it, I suppose, as by this example I shall prove.

¶ There is a piece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards square there is in all this piece of ground: which numbers I set down with the greater above, & the lesser under, as you see.

1365
236

Then doe I multiply 5 by 6, saying, 6 times 5 make 30, of which I write the Cypher in the first place, and the Article 3 I doe keep in minde to carry to the next place.

1365
236
8

Then doe I by the same 6 multiply the second figure of the higher summe, which is 6, saying, 6 times 6 make 36, and 3 in my mind make 39, of which I write the 9 under the second place and the Article 3 I keep in mind.

1365
236
90

Then doe I multiply the third figure, which is 3 by the same 6, and that maketh 18, and 3 in my mind make 21. The 1 I set down, and keep 2 in mind.

1365
236
190

Then come I to the last figure of the higher summe, and multiply it by 6, saying, 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I write under the fourth place. And so have I ended the first Multiplier, and dash him sleightly with my pen.

1365
236
8190

Then begin I with the second Multiplier, and say, first 3 times 5 that maketh 15, of which I set the 5 under the second place, because that the Multiplier is there, and the Article 1 I keep in my mind.

Then

Then come I to the second Figure that is 6, and multiply it by 3, which maketh 18, & with one in mind, maketh 19: then I set down under the third place, and 1 I keep in mind.

Then come I to the third Figure, which is 3, and multiply it by 3, saying, 3 times 3 make 9, and with 1 in mind, make 10, the Cypher I set under the fourth place, and the Article 1 I keep in mind.

And then coming to the least figure 1, I multiply it by 3: and it maketh 3, and with the 1 in mind, it maketh 4, which 4 I set in the fift place, and then have I ended two of the Multipliers, and the summes stand as you may see in the latter end of the page going before, and then I give 3 his dash.

	1365
	236
	8190
	095
	1365
	236
	8190
	095
	1365
	236
	8190
	4095

Then come I to the third Multiplier, and multiply it into every figure of the higher summe, and first I say, 2 times 5 make 10, of which I set the Cypher under the Multiplier in the third place, and the Article 1 I keep in minde.

1365	
236	
8190	
4094	
0	

And so multiplying the second figure 6 by that

that same 2, there amounteth 12, & 1 in my mind maketh 13, whereof I write the Digit 3 under the fourth place, and the Article 1 I keep in mind:

```
 1365
  236
 ----
 8190
4095
 30
```

Then doe I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and the one in mind make 7, which 7 I set down under the fift place, as appeareth by the example.

```
 1365
  236
 ----
 8190
4095
730
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```
 1365
  236
 ----
 8190
4095
2730
```

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the sixt place, and then doth the summe stand thus.

And so have I ended the whole Multiplication.

But now (as you taught mée) to know what this whole summe is, I must adde all those parcels together, and then under the line will appear, as you may sée, the grosse or totall summe is, 322140. Whereby I know there is so many yards square in that piece of ground.

```
 1365
  236
 ----
 8190
4095
2730
 ----
322140
```

Master. This is well done.

Scholar. Then mée thinketh I could call it well done, when I know, whether I had well done or no.

Master.

Maſter. It is to be proved by 9 as Addition was, but the ſureſt proof is by Diviſion, and therefore I will reſerve that proof by Diviſion, till you have learned the Art of Diviſion. And anon I will ſhew you how it is commonly proved.

¶ *But firſt, for your further inſtruction in this exerciſe of* Multiplication, *I will with one example trie your cunning, and ſo make an end: And the* Queſtion *is this, I would know how many daies it is ſince the* Nativity of our Lord and Saviour Jeſus Chriſt, *unto this year* 1630. *which to perform: you muſt* multiply *this preſent year* 1630. *by the daies in one whole year, which are* 365.

Scholar. Now for that you have given me ſo much light into the queſtion, you ſhall ſee I will handſomely finiſh the work, for according to your former inſtructions, I ſet them down with a line under them, thus;

1630
365

Then ſay I, 5 times 0 is 0, which I ſet down under the firſt place, as here appeareth. Then ſay I, 5 times 3 make 15, the Digit 5 I ſet down in the ſecond place under 3, and the Article 1 I keep in mind to be added to the next Multiplication. Then ſaying five times 6 make 30, and 1 in mind 31, the 1 I ſet down in the third place, and 3 I keep in mind. Then coming to the laſt Figure, I ſay once 5 is 5, and 3 in mind make 8, that 1 doe I ſet down above

the

the fourth place: and thus have I ended my first Multplier, and therefore I give it a dash with my Pen.

Then come I to the second Multiplier, which is 6, and doe likewise multiply it into the upper number, saying, 6 times 0 is 0, which I set down in the second place, right under his Multiplier: then say I, 6 times 3 make 18, the 8 I set down under the third place, & 1 I kéep in mind. Then say I 6 times 6 make 36, & 1 I kéep in mind make 37, the Digit 7 I set down in the fourth place, and 3 I kéep in mind: Then say I 6 times 1 is 6, or once 6 is 6, and 3 in mind make 9, which I set down next, & so have I ended two Multipliers: wherefore I dash the 6 with my pen.

Then I begin to multiply the third Multiplier into the over number, saying, 3 times 0 is 0; the 0 I set down in the third place right under his Multiplier: Then say I, 3 times 3 make 9, which I set down in order next: then say I, 3 times 6 is 18, the 8 I set down, and 1 I kéep. Lastly, I say, once 3 is 3 and 1 I kéep is 4, which I set down orderly next: And so have I ended the Multiplication, and my figures stand thus.

		or thus
	1590	1590
	369	369
	7950	
	9540	
	14310	14310
	4770	477
	594950	594950

Master;

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe it will be 594950; which is the grosse or totall summe of that Multiplication, and declareth the number of daies since our Lord and Saviour his incarnation, unto the end of 1630 *yeares, besides* 407 *daies, and twelve houres for leap yeares.*

Scholar. This is marvellous, me thinke, that such great matters may so easily be atchieved by this Art, which heretofore I ever thought had been impossible, as infinite sorts of people are of that mind.

Master. Truly it is so, that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to justifie.

Scholar. The manner of Multiplication I perceive, if there be no more in it.

Master. *Yes, there are other forms and helps for ease and shorter labour of the work of* Multiplication, *but I will remit them till you have a little tasted* Division, *where also the like help into* Division *may be used: and so therefore under one example for both, will I shew you both ease in* Multiplication, *and also in* Division.

But lith the other formes and workings doe nothing differ from these works in effect, but only in setting of the numbers, I will over-passe them till a more meet place and time.

time. And now will I instruct you in Division, so that you think your self sufficiently to perceive what I have taught you.

Scholar. Yes Sir, I thank you, but I do not perceive how to examine my work, to try whether I have well done or no: therefore as you promised me ere while, I pray you first shew me how I shall prove it.

Master. *That is commonly used by the proof of 9, as you learned before in* Addition, *saving that it differeth from that form in divers respects. As for example.*

First, must you make a crosse after this manner. X

Proof of Multiplication.

Then must you examine your summe that should bee multiplied, and lok what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and lok what remaineth thereof, if it bee under 9, write at the higher part of the crosse: but if it bee above 9, then take thence 9 as often as yee can, and write the rest at the head of the crosse: As for example, wee will prove the example you put forth of the piece of ground that contayned 1365 yards in length, and 256 yards in breadth.

Therefore first I cast away all the nines from the summe to be multiplied, saying, 5

and

and 6 make 12, caſt away 9 reſt 2: then 3, and 2 make 5, and 1 is 6, that 6 I write at one ende of the croſſe thus.

Then doe I examine the Multiplyer which is 236, wherein when the 9 is caſt out, there remaineth 2, that 2 therefore I ſet at the other ende of the croſſe.

Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then reſteth 3, which 3 do I ſet at the head of the croſſe. Then do I examine the groſſe ſumme, amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining; that 3 I ſet at the fot of the croſſe, and then I ſee it to agrée with the other 3 at the top of the croſſe, and ſo know I that I have done well: for if they two did differ, then were my worke vaine, and the Multiplication falſe.

This is the common proof: but the moſt certain proof is by Diviſion, of which I will anon inſtruct you.

Scholar. Sir, what is the chief uſe of Multiplication?

A ſure proof of Multiplication.

Maſter. The uſe of it is greater then you can yet underſtand: howbeit, theſe plain commodities it hath, that if you would reſolve any great and whole value into many ſmall and leſſe portions, as if you would change pounds

pounds in shillings and pence, or any other greater or smaller parcels by Multiplication, ye shall doe it speedily and easily. Also if you should need to adde one summe to it self, or to any other often times, you shall doe it by Multiplication much more speedily, readily, easily, and surely, then by often and sundry Additions. Take you these commodities gladly thereof for an answer at this time, and hereafter I will more abundantly make you to perceive the use of it.

Division

Diviſion.

Scholar.

WEll Sir, then in Diviſion *I pray you to inſtruct me. But me thinketh by the name of it, that it ſhould be all one with* Multiplication: *for I call that* Diviſion, *when any thing is parted into divers and many parts.*

Maſter. You take it as it is taken commonly: howbeit, if you mark well, you ſhall perceive that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrarywaies, it bringeth many parcels into few, but yet ſo, that theſe few taken together, are equall in value to the other many: for by Diviſion, pence are turned into ſhillings, and ſhillings into pounds: As for example, of 120 ſhillings, it maketh 6 pounds, ſo are 120 turned into 6, which is a ſmaller number: but then if you conſider the Denominators, you ſhall ſee that they are ſuch, that one of the latter is equall to 20 of the firſt, and ſo in value the ſummes are one, though in number they doe differ, and the latter ſumme is the leſſer, and ſo it is alwaies in Diviſion, howbeit, yet in the

 worke-

working the ſumme is parted by another, and thereof doth it take the name.

Scholar. I think I ſhall better underſtand the reaſon of the name when I know the uſe of the worke, therefore now would I gladly learn that.

Division what it is.

Maſter. *Diviſion is a diſtributing of a greater ſumme by the Unites of a leſſer*: Or, *Diviſion is an Arithmeticall producing of a third number, in reſpect of two propounded numbers; which third number ſhall ſo often contain an unite, as the greater of the two propounded numbers can contain the leſſer.* So that as Multiplication did ſeem to ſerve in ſtead of many Additions, ſo Diviſion may ſeem to be in place of many Subtractions: Becauſe that third number briefly expreſſeth how many times the leſſer of your two propounded numbers may be ſubtracted from the greater: as in practiſe will more largely appear. Therefore (as you may perceive) unto Diviſion are required three numbers: The firſt, which ſhould bee divided, and that muſt (generally) bee the greater: And the ſecond, by which the other muſt bee divided, and that is (generally) the leſſer, and is called the Diviſion: And the third, which anſwereth to the Queſtion (How many times?) and therefore is called the Quotient.

A general rule for placing the figure.

The firſt muſt bee firſt written, and the ſecond ſo ſet under it, that the laſt figure of the lower number bee right under the laſt of the

the higher, contrariwiſe to the work of other kinds of Arithmetick: for in them the two firſt figures were ſet ever met one under the other; but in Diviſion, the laſt figures muſt be ſet met, except it chance ſo that the laſt figure of the Diviſor, be greater then the laſt of the higher number, for then you ſhall ſet the laſt of the Diviſor under the laſt ſave one of the higher number, as for example: An excepion.

If you ſhould divide 365 (which are the ſumme of the daies of a year) by 28, which are the daies of a common Moneth, then ſhould you ſet them thus.

365	
28	

But if you ſhould diviſe thoſe 365 daies by 52, which is the number of weeks in one year, then ſhould you ſet them thus.

365	
52	

Likewiſe, if I would divide the ſame 365 by 4, which is the ſumme of the quarters of years, then muſt I ſet them thus.

365	
4	

Scholar. Sir, this doe I underſtand, but now how ſhould I doe to divide the one by the other?

Maſter. You muſt begin with the laſt figure next the left hand, and ſee how many times the laſt Figure of the Diviſor may be taken out of the laſt Figure of the other Number, and that ſhall you note within a croked line toward your right hand: As for example, I would divide 365 by 28,

then set I those two summes thus.

365 (
28

And I lok how many times I may finde 2 (which is the last Figure of the Divisor) in 3, (which is the last of the number to be divided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1. and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, and deface or cancell the 3 and the 2, then will the Figures stand thus.

Quotient number.

	1
	365 (1
	28

Then come I to the next figure of the Divisor, and take it likewise so many times out of the figures that be over it, and lok what both remain, that I must write over them, and cancell them, as in this example.

Therefore now doe I take once 8 out of 16, and there remaineth 8, which I must set over the 6, and cancell or crosse out the 16, and the 8 of the Divisor: and then will the figures stand thus. And so I have once wrought.

	28
	365 (1
	28

Scholar. So I perceive that you take the nether figure, not only out of the other that is right over him, but out of that with the other also that remaineth

remaineth before, and are written toward the left hand.

Maſter. So muſt you doe: for you muſt ſo take the Diuiſor out of the ouer number, that there remain not ouer it ſo great a ſumme as it ſelf is, for then were your worke in vaine.

But yet again here muſt you marke, that when you ſéek how many times the laſt Figure of the Diuiſor may bée found in the number ouer him, that you lok alſo whether you may as often find all the Figures following in thoſe that are abode them (conſidering all the remainers, if there bée any) if not, take your Quotient leſſe by one, and then proue again, and ſo ſtill you finde a méet Quotient: and by that méet Quotient muſt you alwaies multiply your Diuiſor, and ſet the product under your Diuiſor, ſo that the firſt figure ſtand vnder the firſt figure of your Diuiſor, and the ſecond vnder the ſecond, and ſo forth: and then ſubtract that product from the number to bée diuided that ſtandeth directly ouer it, as you haue ſéen mée doe.

When you haue thus wrought once, then muſt you begin again, and write your Diuiſor anew, nérer toward the right hand by one place, as in this example, you ſhall ſet 2 vnder 8, and 8 vnder 5, thus.

28	
365	(1
288	
2	

When (as before) ſéek how many times you may

take your Diuisor out of the number ouer him now,

Scholar. That may I doe here 4 times.

Master. Truth it is, that you may finde 4 four times in 8: but then mark whether you can finde the Figure following so many times in the other that is ouer him. Can you finde 4 four times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Mark how to consider this kinde of Remainer.

Master. Well then, 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which summe I finde 8 three times also: and therefore I take 3 as a true Quotient, and write it within the crooked line of the Quotient before the 1, thus:

	2
	28
365	(13
	288

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8 and write ouer it the 2 that doth remain, thus:

	2

Then doe I take 8 as many times out of 25, saying, 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and ouer the 5 set 1 thus:

Or you might (after you found 3 to be a fit Quotient) straight-way have multiplied the whole Diviſor 28, by that at once: which giveth 84, which being ſet under 28, and duly ſubtracted from 85, of the number divided, giveth 1, the remainer of the whole Diviſion, as before you had. Work which way you liſt, here you may ſée alſo the form.

2	
281	
369	(13
288	
2	

And now have I done with the dividing, for I cannot find my Diviſor 28 no more in the over ſumme.

Scholar. No except you would part the 1 that remaineth into 28 parts.

Maſter. That is well ſaid, and ſo muſt wée doe in ſuch caſes, when there remaineth any thing: but I will let that paſſe now, and will make you perfect in Diviſion of whole numbers, and will hereafter teach you particularly of broken numbers called Fractions. Now if you doe perceive the order of Diviſion, then doe you divide this ſumme 136280 by 452.

Scholar. Firſt, I ſet down the number that ſhould bée divided; then doe I ſet the Diviſor under the laſt Figure of the over number. Then will it bée thus,

136280	
452	

Maſter. Can you take the laſt of your Diviſor (which is 4) out of one, which is the laſt of the over number?

Scholar. I had forgotten, becauſe the laſt of the Diviſor cannot bee taken out of the laſt of the over number in ſo much as it is the greater, therefore muſt I ſet the Diviſor one place more forward toward the right hand, thus.

	136280
	452

And then muſt I lok how often I may finde the laſt figure of the Diviſor (that is 4) in 13, which I may doe 3 times, therefore doe I ſay, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my ſummes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and over the 3 I ſet the 1 that remaineth, and then the Figures ſtand thus.

	1
	136280 (3
	432

Then I multiply the ſame Quotient into every Figure of the Diviſor, and withdraw the ſumme that amounteth out of the numbers over them, as firſt I ſay, 3 times 5 make 15, which I take from 16, and there reſteth 1, I cancell therefore 16 and 5, and write over the 6 that 1 that remaineth, thus.

	11
	136280 (3
	452

Then doe I ſay likewiſe, 3 times 2 make 6, which I take out of 12 and there reſteth 6,

there-

therefore I cancell the 12 and the 2 over, and then I write the 6 that remaineth thus.

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 116
136280 (3
 452
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Then should I set forward the Divisor into the next place toward the right hand thus.

```
 116
136280 (3
 4522
  45
```

Master. But you may sée that over the 4 is no figure, therefore I must set the Divisor yet forwarder by another place.

And mark, whensoever it chanceth so, that you should set forward the Divisor, and that it cannot stand there, because there is no number over the last place, or if there bée any, it is lesser then the last figure of the Divisor, then must you remove the Divisor, yet once again: and because that his first place of removing served not to subtract him so much as once, therefore you shall write in the Quotient a Cypher, and if you should by chance néed to doe so oft times, for every time write a Cypher in the Quotient. The reason of this will I shew you hereafter.

Scholar. Then must I set my summes thus.

```
 116
136280 (30
 4522
  45
```

And because I removed the Divisor, so that I overskipped one place, I must write a Cypher in the Quotient: and then must I séek a new Quotient, as

in

in this example I must say; How many times 4 is there in 6? (and sith it can be but once) therefore doe I write 1 in the Quotient: and then say I, 1 time 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 over them, thus:

2
2163
136280 (301
45222
455
4

Then say I again, once 5 out of 28, remaineth 23: I let the 2 stand as it did, and over that 8 I set 3, cancelling the 8 and the 5 under it, thus:

Master. You might as well have said, once 5 out of 8, and so remaineth 3, but now go forward.

Scholar. Then once 1 out of 0 cannot be: What shall I now doe?

Master. Borrow of the next number that is behind (for there is 230) and doe as you learned in Subtraction in a like case.

Scholar. Then must I borrow 1 out of the 3 comming behinde next, and make that 0

to

to bee 10, and then take 3
2 out of 10, and there reſt-
eth 8: and becauſe I bor-
rowed 1 of the 3, I muſt
cancell the 3, and write 2
over it, then doth the fi-
gure ſtand thus.

	22
	11638
136280	(301
	45222
	457
	4

Maſter. Now haſt you done, and yet remaineth 228, and your Quotient ſheweth you that if you divide 136280 by 452, you ſhall find your Diviſor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I divided 365 by 28, the Quotient was 13, and 1 remained: whereby I knew that in a year (which containeth 365 daies) there are 13 moneths, reckoning 28 daies (or 4 weeks) juſt to a moneth and 1 day more.

Scholar. Why then doe we call a year but 12 moneths?

Maſter. Of that at a more convenient time will I fully inſtruct you: but now it is not convenient to intangle your mind with other things then doe directly pertain to your matter. Therefore if you remember what you have heard, you have learned a ſhort manner of Diviſion, which I would have you often to practiſe, ſo that you may be perfect in it, and hereafter I will ſhew you certain other proper points touching it.

Scholar. Then I pray you tell me how

I shall examine and trie my work, whether I have done well or no, that though no man bée by mée to tell mée, yet I may perceive it my self.

Proof of Division.

Maister. Some men (yea and commonly most) doe trie it by the Rule of 9, as in all the other kinds, save that their order is; First, they cast away 9 as often as they can out of the Divisor, and that that remaineth they set at one side of a Crosse, as in our first example the Divisor was 28, from which you may take 9 three times, and 1 remaineth: which they set by a Crosse, thus:

X 1

Then they likewise examine the Quotient, (which in our example is 13) and from thence they cast away 9 as often as they can, and the remainder they set at the other side of the Crosse, and then they multiply together those two remainers: and to it that amounteth they adde the remainer of the Division if there were any, from that whole summe they withdraw 9 as often as they can, and the rest they set at the head of the Crosse, as in our example, the Quotient is 13, from which take 9, and there remaineth only 4, & therefore must you set 4 at the other side of the Crosse, thus:

4 X 1

Then multiply 4 by 1, and it yeldeth but 4, thereto adde the remainer of the Division (which was 1) and it will be 5, which summe doth

both not amount to 9, and therefore muſt be ſet wholly at the head of the Croſſe, as you ſée here.

And this number on the head of the Croſſe, is the firſt proof, to which if you find another like in the number that was diuided, then you haue done well.

Therefore now ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foot of the Croſſe: and if it be equall to that in the head of the Croſſe, then haue you done well, elſe not.

As in our example the whole ſumme was 365, which maketh 14. from that take 9, and there reſteth 5, which ſet at the foot of the Croſſe, thus.

And you ſhall ſée that they agrée: therefore haue you well done.

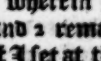
Now will I likewiſe examine our ſecond example, where the Diuiſor was 452, which maketh 11; from thence I take 9. and the 2 that remaineth I ſet at the right ſide of the Croſs, thus.

Then examine I the Quotient, which was 301, where I find but only 4: that I ſet at the other ſide of the Croſſe, thus:

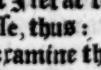
Then I multiply 4 by

2, and

2, and it maketh 8: to that I adde the remainer of the Diviſion (which was 228, and it maketh 12) and they two make 20, wherein I finde twice 9, and 2 remaining: that 2 muſt I ſet at the head of the Croſſe, thus:



Then I examine the whole number to be diviſed, which was 136280, where I finde twice 9 and 2 remaining, which I ſet at the foot of the Croſſe, thus:



And becauſe it doth agree with the figure at the head of the Croſſe, I know that the Diviſion was well wrought.

The Proof of Diviſion more certain by Multiplication.

Maſter. This is the common proof: Howbeit, the more certain working is by the contrary kind: as to proue Diviſion by Multiplication, thus:

Multiply the quotient by the Diviſor, and if the ſumme that amounteth, bee equall to the ſumme that ſhould bee diviſed, then haue you well diviſed: elſe not.

Howbeit, this muſt you mark, that if there remained any thing after the Diviſion, that muſt you adde to the ſumme that amounteth of the Multiplication. As in our firſt Example our Quotient was 13, and the Diviſor was 28: Now multiply the one by the other, and the ſumme will be 364: to that if you adde the 1 that remaind after the Diviſion;

then

then will it be 365, which was the ſumme that ſhould be divided: and therefore I know that I have well done.

Scholar. Now will I prove the ſame in the ſecond example, whoſe Diviſor was 452, and the Quotient 301: theſe doe I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280, which was the whole ſumme to be divided: and therefore I perceive that I have well done.

Maſter. This is the ſureſt way to examine Diviſion by Multiplication: and contrariwiſe, the ſureſt proof of Multiplication is by Diviſion.

And therefore (according to my promiſe) now will I ſhew you how you may prove Multiplication by Diviſion.

When you have ended Multiplication, and would know whether you have well done or not, ſet the groſſe ſumme that amounteth of the Multiplication ouermoſt, and divide it by the Multiplier: and if the Quotient be the ſame number that ſhould be multiplied, then have you well wrought, elſe not, as in that example where we multiplied 264 by 29, the groſſe ſumme was 7656. Proof of Multiplication by Diviſion.

Now if you will know whether that Multiplication be true, you ſhall divide that 7657 by the Multiplier 29, & you ſhall perceive that the Quotient will be 264, and that is a token that you have well wrought.

Scholar.

Scholar. By your patience I will proue that, and firſt ſet down the groſſe ſumme and the multiplier, not after the rule of Multiplication, but after the rule of Diviſion, for now that number is become the Diviſor, that was before the Multiplier, I ſhould ſet them therefore thus,

	7656
	29

Then ſhall I ſéek how many times 2 in 7, that may bée thrée times and one remaineth: but then may not 9 be found ſo often in 16? therefore muſt I take a leſſer Quotient, that is to ſay 2: then ſay I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then do I cancell 7 and 2, and ouer 7 I write 3, and in the Quotient I ſet 2: ſo the Figures ſtand thus.

3	
7656	(2
29	

Then ſay I further, two times 9 make 18, which I abate out of 36, and there reſteth 18: then cancell I 3, and ouer him ſet 1, and likewiſe I cancell 6 and 9, and ouer them I ſet 8: ſo that thus ſtand the Figures.

1	
38	
7656	(2
29	

Then I ſet forward the Diviſor by one place, and ſéek a new Quotient, that is to ſay, how many times 2 are in 18, which I finde to be 9 times: but then can I not finde 9 ſo many times in 5: but therefore I take a leſſer Quotient, as to ſay 8: but yet that is to great: for if I take 8 times 2 out of 18, there remaineth

remaineth but 2, and I cannot finde 8 times 9 in 25: therefore yet I take a lesse Quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seek a lesser quotient, as to say 6, then say I, 6 times 2 make 12, that I take out of 18, and there remaineth 6, so I cancell 18, and the 2, & write 6 over 8, thus:

Then say I forth, 6 times 9 maketh 54, that take I out of 65, & there remaineth 11, and the figures stand thus:

Then must I set forth the Divisor again, & seek a new quotient, which will be 4: for though I may find 2 in 11, 5 times, & 1 remain, yet I cannot find 9 so often in 6, therefore I set the figures thus:

And the 4 in the quotient I multiply into the figures of the divisor, saying, four times 2 makes 8, which I take out of 11, and there rests 3, therefore I cancell the 11, and the 2, and set 3 over the first place of 11 thus:

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing,

16
38
7656 (26
299
2
1
16
381
7656 (26
299
2
1
381
7656 (264
2999
22
1
163
381
7656 (264
2999
22

so that the Quotient of this Division also (where 7656 is divided by 29) is 264: which doth declare, that if 264 bee multiplied by 29, the summe will bee 7656. And thus I perceive now both Multiplication is proved by Division: and Division also by Multiplication.

Master. Now have I ended the five common kindes of Arithmetick: For (as touching Mediation, Duplation, Triplation, and such other) they are no severall kindes of Arithmetick, but are contained under the other. For Mediation is contained under Division, and is nothing else but dividing by 2: and so are Duplation and Triplation contained under Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation, is multiplying by 3, of which I will onely propose an example, for the Rules you have heard already.

An example of Mediation.

If you would mediate, or divide into 2, this summe 4531010, you shall set 2 for the Divisor, and work as you learned before, as thus:

```
4531010 ( 2
```

Then I finde 2 in 4 two times, therefore my Quotient must be 2: so I cancell 4 and 2, and remove the Divisor forward thus, as the work requireth, and as before in Division hath been declared.

```
4531010 ( 2165505
 zzzzzzz
```

Which mediation or division by 2 being finished,

finished, you shall have for your Quotient 216505, which is the half of 433010, as you may trie by Duplation; for double that Quotient, or multiply it by 2, and the same number will amount. Duplation

I will no longer tarry about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certain easie forms both of Multiplication and of Division. And first of Multiplication.

If you would therefore multiply any summe by 10, you shall need to doe no more but adde a Cypher before his first place; as for example, 36 multiplied by 10, make 360. Easie forms of Multiplication.

Likewise if you would multiply any summe by 100, put two Cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This doe I well perceive, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as to our matter now. look, as I have told you, that you both remember it, and also often practise it.

And now you have learned how to multiply easily by 10, 100, 1000: and of like manner may you doe with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 3000, and such like, where there is one Cypher in the first place,

or many orderly in the firſt places, you ſhall take away thoſe Cyphers, and multiply the ſumme only by the other figure or figures (if they be many) and then at the beginning of the ſumme that amounteth, you ſhall ſet ſo many Cyphers as you took away.

Example of 2873, which I would multiply by 300. Firſt, I omit the 2 Cyphers from the Multiplier, and I multiply the ſumme by three only that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or took away, and then is it 861900. And that is the ſumme that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beſide the Cyphers, I muſt only take away the Cyphers, and multiply by the other figures, as I learned before: As if I would multiply 93648 by 25000, I ſhould take away the three Cyphers, and multiply the ſame by 25, and then at the beginning of that totall ſumme ſhould I adde the 3 Cyphers again.

Maſter. Even ſo: but if it chance the number that ſhould bee multiplied, or both the ſummes, as well the number that ſhould bee multiplied as the multiplier, to have Cyphers in their firſt places evermore omit the Cyphers & work by the reſt. But remember to reſtore as many Cyphers to the amounting ſumme as you bated before. As in this example: 30200 ſhall bee multiplied by 206, I ſhall only take away two Cyphers from the greater

number

number, and then multiply 302 by 206, and afterward adde the two Cyphers again. But if I would multiply the same 30200, by 2060 I shall not only take away the two Cyphers from the number that should be multiplied, but also I may take away the one Cypher from the Multiplier, and then must I adde three Cyphers to the summe that amounteth: but take heed that you take away no Cypher that cometh after any signifying figure, as in the last example, you must not take away that in the fourth place of the higher number, neither that in the third place of the multiplier: howbeit, yet thus you may doe: If one Cypher or more come in the last of your summes, you may multiply the other figures, and overskip them: but so, that you give every figure his due place: as thus, I will multiply 3026 by 2004, therefore I set them thus.

	3026
	2004
	12104
	52

And thus I doe multiply them. First 4 times 6 make 24, I set 4 under the first place, and keep the two still in my minde. Then say I again: 4 times 2 make 8, and the 2 that is in my minde maketh 10. I set down the Cypher 0, and keep the Article 1 in my minde. Then four times 0 is 0, and the 1 in my minde maketh 1, I set down the figure 1, and say again, 4 times 3 is 12, I set down 2, and keeping the 1 still in my minde (having no more places of the upper number to multi-

tiply it withall) I put it down next 2 in the fifth place.

But now when I come to the next place (being a Cypher 0) I let it go, because it multiplieth nothing, and likewise the second Cypher.

But then when I come to the 2, and multiply it into the 6 of the over number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the multiplication bee set right under the Multiplier, and the other orderly toward the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as this Example sheweth.

3026
2004
12104
6052
6064104

Which is indeed wrought so much the sooner and shorter by over-skipping of the two Cyphers: which otherwise, (if the same example were wrought at length) it would have had two workings more, as by the same example here also set down doth appear.

3026
2004
12104
0000
0000
6052
6064104

Scholar. Sir I thank you, for I see great ease in this way of Multiplication: and (if you can shew mee such like in Division) you shall greatly farther mee.

Master.

Master. Yes, I will teach you some easie rules in Division also, and first this: If you would divide any summe by 10, you shall only with your Pen make a square line between the first figure of your summe and the second, and then have you done: for the whole number that followeth the line, standeth for the Quotient, and the figure that is before the line, is the remainer: As for example, 3648 divided by 10. Where 364 is the Quotient, and betokeneth that so many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be divided into 10, but by breaking it into Fractions, wherewith I will not meddle yet. Easie forms of division. 364 | 8

And so likewise if you would divide any summe by 100, with your Pen you shall cut away the two first figures, and if you would divide by 1000, you must cut away the three first figures, and so of any other Divisor, whose last figure is 1 and the other Cyphers, look how many Cyphers the divisor hath, and so many figures at the beginning shall you cut away with the square line, and they stand alwaies for the remainer, because they are lesse then the divisor, and cannot be divided by it, and the other figures that are behinde the line stand for the Quotient.

But now if your Divisor have any other figure in his last place then 1, and in all his other places have Cyphers, look how many

Cyphers they be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that figure that is in the last place, as if it were the whole divisor.

Example of 64284. which I would divide by 300, here must I cut away the two first figures, (for so many cyphers my divisor hath) and must divide the rest by 3, which is the figure in the last place of the divisor. First, therefore I part away the two first figures, and the summe standeth thus:

64|284 (2
3| 00

Then doe I divide 642 by 3, & the quotient is 214: for in 6 I finde twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, both make 12, wherein I finde 3 four times: And this is a ready way to turn shillings into pounds: for sith one pound doth contain 20 shillings I must divide the whole number of shillings by 20. Therefore easily doe it, I see that my Divisor hath one Cypher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I doe mediate or divide by the 2 other figures or summe that followeth.

Scholar. I will put an Example

If you would divide 64287 shillings, by 20: that is to say, If I would turn so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that is 6428 by 2, so shall the Quotient bee 3214,

whereby

whereby I know that 64187 shillings make 3214 pounds, and 7 shillings remaining.

Master. Now prove by Multiplication whether you have well done or no.

Scholar. The Quotient is 3214, which I doe multiply by the divisor 2, & it doth amount to 6428.

Master. Hereby you may perceive not only that you have well done, but also how by Division you may turn shillings easily into pounds: and contrariwise by Multiplication you may turn pounds into shillings.

But here shall you see amongst divers men divers forms of such division: but if you mark what I have told you, you shall perceive easily all the wayes. For some men doe not cut away so many of the first figures of the summe that they would divide, as there are Cyphers in the first places of the Divisor: but they set all their Cyphers orderly under the first places of the Number that they would divide; and then with the other figure or figures, (if there bee many) they divide the rest of their summe.

Another manner of the Abridgment.

Example. If they would divide 725931 by 3400, they doe set the summes thus:

7:5931	
34	00

And then doe they divide orderly till they come to the Cyphers: for there they stay and end their work, as in this example.

They seek how often 3 may bee found in 7, which is two times, and 1 remaining: there-

therefore they set 2 in the quotient, and cancell 3, and 7, and over 7 they set the 1 that remaineth, thus:

Then doe I goe forth, saying, two times 4 maketh 8, which they take out of 11, and there remaineth 4, thus:

Then remove they the Divisor forward, and seek how often 3 may bee found in 4, which is but once, & 1 remaineth, then set they 1 in the Quotient, and cancell 3 and 4, and ober them they set that 1, thus:

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancell the 4 and 5, and set 1 ober them, thus:

Then set they forth the Divisor again, and seek how many times 3 are in 11, which they finde three times, and 2 remaining: so they set 3 in the Quotient, and cancell 11 & 3, and ober them set 2, thus:

Then doe they multi-

ply

ply 4 by 3, which maketh 12, that withdrawn they out of 29 and there resteth 17, of which the 7 must be set over the 9, and the 1 over the 2, thus:

1	
12	
2417	
725931	(213
3444	
33	

And now are the two Cyphers next enduing, so that the Divisor can no more be set forward, and therefore is the division ended, and the remainer is 1731.

Now the Quotient which is 213, doth declare, that if you divide 725931 by 3400, you shall finde it therein 213 times, and there remaineth 1731: so shall you finde it, if you work as I taught you, by cutting away the two first figures, because of the two Cyphers.

But this must you mark (as you may perceive by this last example) that if there be left any other Remainer in the summe that was behind the square line, that the Remainer must be set to the latter end of the first remainer, which was cut away with the square line: as if you would divide 725931 by 3400, after the form that I taught you, then would your summes appeare, thus:

Note.

1	
12	
2417	
7259	31 (213
3444	
33	

So that 17 which remaineth after the line,

line, must bee set to the 31 (that was cut away with the line) in higher places, as you see here: where that 17 with the 31, doe make 1731.

Scholar. Sir, is there no other form of Division in practise but this?

Master. Yes verily, there are other forms in practise, but because I love brevity, I will declare only one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall loving friend, Master Henry Bridges, wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus.

6)72

Write the Divisor in a loose paper, to remove at pleasure.

Then if you please you may write the divisor in a loose paper that it may more easily without cancelling or defacing of the work bee applied to, and removed from the dividend at pleasure; then apply your Divisor 6 to 7, the first figure of the dividend, and inquire how oft it may be had in 7, and seeing 6 is but once in 7, set 1 in the Quotient line thus:

6)72(1

Then multiply the divisor 6, by the quotient 1, and set the Product 6 under 7, thus:

6)72(1
6

Then draw a line under 6, and subtract

6 out

6 out of 7, setting the remainer 1 under 6, thus:

6) 72 (1
6
1

Then bring down the next figure of the Dividend, and set it with the Remainer 1 under the line, thus:

6) 72 (1
6
12

And bring the moveable divisor 6 under the 2, and as before enquire how oft 6 is in 12, and finding it to be twice in 12, set 2 in the quotient, thus:

6) 72 (12
6
12
12
0

And multiply 6 by that new Quotient 2, setting the Product 12 under the other 12, and subducting it out of the upper number, there resteth nothing. And since the Unites of this Product doe stand under the Unites of the dividend, the division is ended: otherwise you should proceed as before, bringing down the next figure; removing the divisor, dividing, multiplying, subducting, &c.

Scholar. This is very easie, but if there be greater numbers propounded, is the operation the same?

Master. If the numbers be greater or lesser, the work is the same without any difference, as shall appear by this example.

Divide

Divide 7890 by 33.

First set them thus: 33)7890(2

then bring the Divisor under 78, and see how oft it is there found, which is twice, and therefore set 2 in the Quotient, by which multiply the Divisor 33, and set the Product 66 under 78, and subduct it out of it thus.

Then bring the next figure 9 down, and set it with the Remainer 12, it maketh 129, and removing the Divisor 33 thereto, enquire how often 33 is contained in 129, and I finde it but thrice, (though at the first it made a shew of more) therefore set 3 in the Quotient, and multiplying 33 by 3, set the product under 129, subducting that product out of the number above, and proceed as before.

33) 7890 (239 1/3
66
129
99
300
297
3

Then shall you finde the Divisor 9 times in the Remainer: therefore setting 9 in the Quotient, multiply, and subduct as before, and at the last you shall finde only 3 remaining, which must be set above a line after the Quotient, and the Divisor under, as above appeareth.

Scholar. Is there no more difficulty in the whole Rule?

Master.

Maſter. Not any, although your Number be not ſo great, as before I have ſaid.

¶ And here will I make an end of Diviſion, (ſaving that I doe requeſt you to exerciſe your ſelf well herein by many ſummes, till you have attained ſome expertneſſe therein.)

For the reaſons and concluſions thereof are ſo many, and ſo available for all ſorts of men whatſoever; that if I ſhould ſpeak of the infinite uſes thereof, I ſhould rather lack words then matter. And therefore recommending it to your judgment hereafter, upon your further travell into the Art, I will here end this Treatiſe, repreſenting unto you one example, or ſimple queſtion of Diviſion and Multiplication, in ſtead of many, which is this:

A queſtion of ſhooting in Ordnance.

There are four braſſe Peeces: The firſt of them at a ſhot ſpendeth 9 pounds of powder, the ſecond ſpendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed againſt the battery of a Hold, and there is allowed by the Maſter Gunner, 700 pounds of powder to bee ſpent by theſe four Peeces, in this aſſault. The queſtion is two-fold: The firſt, how many ſhot each Peece ſhall juſtly make about with this 700 pounds of powder? And laſtly, how many pounds of powder ought juſtly to be allowed to each Peece for his true proportion?

Scholar.

Scholar. Why Sir, you make me smile, to bear me in hand, that these two demands may be simply resolved by Multiplication and Division.

Master. Truly that they may, and that you may by and by work your self with a little labour: first adde together their quantities of powder, that is 9 pounds, 5 pounds, 4 pounds and 2 pounds, all which make 20: Divide the 700 pounds of powder by that 20, and your Quotient giveth 35, as here appeareth, which sheweth for most certainty that they shall make just 35 shoots about.

1	
700	
200	(35
2	

Scholar. Sir, all this have I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To trie the truth of the same, multiply the first piece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of powder. Multiply also the second piece that spends 5 pounds by 35, and you shall find 175 pounds his allowance: then 4 by 35, and you shall finde 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall finde 70 pounds his allowance. All which four particular summes you shall adde together by Addition, as here appeareth, and it maketh just 700 pounds, and so is the question truly resolved.

315	
175	
140	
70	
700	

Scholar.

Scholar. Truly Sir, theſe excellent concluſions doe wonderfully more and more make mée in loue with the Art.

Maſter. It is an Art, that the further you trauell, the more you thirſt to go on forward. Such a Fountain, that the more you draw, the more it ſpringes: and to ſpeak abſolutely, in a word, (excepting the ſtudy of Diuinity, which is the ſalvation of our ſouls) there is no ſtudy in the world comparable to this, for delight in wonderfull and godly exercíſe, for the ſkill hereof is well known immediatly to haue flowed from the wiſdome of God into the heart of man, whom he hath created the chief image and inſtrument of his praiſe and glory.

Scholar. The deſire of knowledge doth greatly incourage mée to bée ſtudious herein: and therefore I pray you ceaſe not to inſtruct mée further in the uſe hereof.

Maſter. With a good will. And now therefore for the further uſe of theſe two latter, that is, Multiplication and Diuiſion, I will briefly ſhew you the ſeat of Reduction.

Reduction.

Reduction what it is.

Reduction is, by which all ſummes of groſſe Denomination *may be turned into ſumms of more ſubtile* Denomination. And contrariwiſe, all ſummes of ſubtile Denomination may be brought to ſummes of groſſer Denomination.

Groſſe denomination.

Scholar. What call you groſſe Denomination, and ſubtile Denomination?

Maſter. That I call a groſſe denomination, which doth contain under it many other ſubtiler or ſmaller: as a pound (in reſpect to ſhillings) is a groſſe Denomination: for it is greater then ſhillings, and containeth many of them. And ſhillings (in compariſon to pounds) are a ſubtile Denomination, for becauſe they are leſſer then pounds, and many of them are contained in one of the other: and ſo likewiſe of other things: whatſoever thing is compared to other, if it be greater and containeth many of them, it is a groſſe Denomination, but if it bée leſſer (ſo that many of them are in the other) then are they called the ſubtile Denominations: whereby you may perceive that one Denomination may bée called a groſſe Denomination and alſo a ſubtile (that is to ſay, a great and ſmall) in divers compariſons. For ſhillings compared to pounds, are a ſubtile or ſmall Denomination: but compared to pence, they

Subtile denomination.

they are a groſſe, or great Denomination.

Scholar. Now I underſtand the name, I pray you teach me the uſe.

Maſter. The uſe is eaſily learned, if you remember what you have learned before. For if you will reduce any ſumme of a groſſe Denomination into a ſumme of a ſmaller or ſubtiler Denomination, you muſt conſider how many of that ſubtiler Denomination, doe make one of the groſſer Denomination, and by that number or numerator doe you multiply the ſumme. As if you would reduce 30 pounds into ſhillings, you muſt conſider that in a pound are included 20 ſhillings, therefore multiply the one 20 by the other 20, and there will amount 400 whereby you may know that in 20 pounds are contained 400 ſhillings. Likewiſe, if you would reduce 30 ſhillings into penee, conſidering that in a ſhilling are 12 pence, you muſt multiply 30 by 12, and it will be 360, whereby you may finde that in 30 ſhillings are contained 360 pence. And thus may you reduce any groſſe Denomination into a more ſubtiler, by Multiplication, if you know how many of the leſſer doe make the greater: of which thing I will anon give you a briefe Table for the moſt accuſtomed kindes of Money, Weights, Meaſures, and Time, and ſuch like: whereby you may know how often each ſubtile Denomination is contained in the groſſer, when you ſhall need it for the foreſaid kinde of Reduction. And alſo the ſame ſhall ſerve you, if you would

To reduce groſſe Denomination to ſubtile.

reduce any ſumme of a ſubtiler Denomination, into a ſumme of a greater Denomination. For in ſuch Reduction you muſt conſider (as in the other form) how many of the ſmaller doe make the greater: and by that number you muſt divide the other ſumme, and the Quotient will declare how many of the greater Denomination are comprehended in that ſumme: as for example: If you would know how many ſhillings are contained in 3240 d. conſider that 12 pence doe make 1 s. you muſt divide that 3240 by 12, and your Quotient will be 270, whereby you may know that ſo many ſhillings are in 3240 d. But if you would know further, how many pounds are in theſe 270 ſhillings, ſeeing that every pound containeth 20 ſhillings: divide that 270 by 20, and it will be 13 and 10 remaining, whereby you may know, that in 3240 d. (or 270 ſhillings) are 13 pounds & 10 ſhillings. For evermore the Remainer muſt be named by the name, or Denomination of that ſumme that was divided, which in this place were ſhillings. And thus may you doe with any other kindes of Denominations.

¶ Wherefore, to the intent you may have certain light or knowledge in moſt common Coynes, Weights and Meaſures, (which is the chief and principalleſt thing in traffick to be known) I have in each Reduction, as they come in order, ſet down certain inſtructions incident thereunto. And firſt I have hereunto added

added this Table, wherein is comprehended, not only our currant and common coynes, but also the most part of the usuall coynes of Christendome, with their just weights and value currant in this Realm of England, intending at the latter end of my Addition to this Book, to write of the ordinary Money used in divers places, and their common values currant for traffick, with the manner of their exchanges from place to place, &c.

A Table of the names, and now valuation of the most usuall Gold-coyns throughout Christendome, with their severall weight of Pence and Grains: and what they are worth of currant English money, this present year, 1630.

The names & titles of the Gold.	The weight in Pence,	Grains	The value in Shill.	Pence.
Great Soverain.	10	9	33	0
Double Sover. K. H.	8	1	22	0
Double Sov. of Q. E.	7	7	22	0
Royall.	4	23	16	6
Half Royall.	2	1 d.	8	3
Old Noble.	4	6	14	8
Half Noble.	2	3	7	4
Angell.	3	8	11	0
Half Angell.	1	16	5	6
Salute.	2	5	6	11 ob.
3 parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob.
Half George Noble.	1	11	4	11 q.
First Crown K. H.	2	9	6	11 ob.
Base Crown K. H.	2	0	5	6
Sover. K. H. best.	2	14	11	8 ob. q.
Soverain K. H.	4	0	11	0
Edward Sover.	3	15 d.	11	0
Elizabeth Sover.	3	15 d.	11	0
Elizabeth Crown.	1	9	5	6
Half Crown.	0	19	2	9
Unite.	0	12	22	0

Doubl

Double Crown.	3	6	11	0
Britain Crown.	1	1	5	6
Thistle Crown.	1	7	4	4 ob. q.
Half Crown.	0	19 d.	2	9
Crosse Dagger.	3	6 d.	11	0
Half Crosse Dagger.	1	15	5	6
Rose Royall.	0	21	33	0
Spur Royall.	4	10 d.	16	0
The Angell.	2	23 d.	11	0
Half Angell.	1	11 d.	5	6

All the severall pieces of gold heretofore mentioned, are set down according to their valuation by the Kings Majesties Proclamation for Gold, Dated the 23 of November, 1611.

A Table of forain Gold-coyn, according to their ancient valuation and severall weight in Pence and Grains.

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Unicorn of Scotl.	2	10	6	0
Scottish Crown.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crowns	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Riders.	2	2	3	6
Philips Royall.	2	10	10	0
Philips Crown.	2	5	5	0
Collen Gilden.	2	2	4	8
New And. Gild.	2	2	5	0

The names & titles of the Gold.	The weight in Pence,	Grains	The value in Shill.	Pence.
Flanders Noble.	4	10	12	0
Half Flan. Noble.	2	6	5	0
Flan. Angel. beſt.	3	6	3	0
Flan. Royall orke.	3	10	10	0
Carolus Gilden.	0	12	3	6
Flanders Royall.	2	6	5	0
Saxon Gilden.	2	2	4	8
Flanders Crown.	2	5	6	0
Philips Gilden.	2	3	4	2
Half Phil. Gild.	1	1	2	1
Golden Lion.	2	16	7	8
3 parts of gold. Lion.	0	21	2	5
¾ parts of gold. Lion.	1	19	4	11
Davids Gilden.	2	2	4	0
Horn Gilden.	1	12	4	11
Old undre Gilden.	2	3	4	10
Cruſa. long croſſe.	2	6	6	0
Cruſa. ſhort croſſe.	2	6	6	2
Milreyes.	4	20	1	4
Half Milreyes.	2	10	6	8
Portague 1 ounce.	2	16	68	0
Golden Caſtile.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Double Piſtolet.	4	9	11	8
Single Piſtolet.	2	4 d.	5	10
Ducket of Florēce.	2	5	5	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Double duc. of Rome.	4	13	12	8

It is to be understood (gentle Reader) that whereas in these Tables, the weight is called by the name of a penny, it is not meant a peny of silver-money, but a peny of Gold-smiths weight, which containeth 24 Barley-corns. Concerning which see Troy weight in folio 133.

So if a man have not the weight wherewith to weigh any piece of gold, he may doe it with barley-corns, being drie, and as it is said, folio 133.

The prices of Gold which the bringers in of forain Gold shall receive at the Mint, according to the Kings Majesties Proclamation, Dated the 14th of *May*, *Anno* 1612.

For an ounce of *French* crowns being 22 Karacts fine.	3 li. 6 s.
For every ounce of *Spanish* Pistolets, being 21 Karacts, 3 grains and a half fine.	3 li. 6 s.
For Duckets of *Spain*, being 23 Karacts, 1 grain fine at least the ounce.	3 li. 8 s. 8d.
For Milreas Crusado long cross, Crusado short cross, the ounce.	3 li. 6 s. 2 d.
For *Hungary* Duckets being 23 Karacts, 1 grain fine at least the ounce.	3 li. 9 s. 2 d.
For the Checkeen of *Venice*, being 23 Karacts, 1 grain fine at least the ounce.	3 li. 10 s.

For

For *Barbery* gold being 23 Karects and di. grain fine, at the leaſt the ounce. } 3 li. 9 s.

¶ *And if the ſaid Barbary Gold be of leſſe fineneſs, abatement to be made according to the rate.*

For Sultains being 23 Karects, 1 grain fine at leaſt the ounce. } 3 li. 8 s. 8 d.

For all other Gold, being 22 Karects fine the ounce. } 3 li. 6 s.

¶ *And being finer, a greater price according to that rate, and being courſer, a leſs, ſo that the bringer in ſupply the leſſe fine, with the more fine, in ſuch ſort, that in the totall it makes good the ſame rate of 22 Karects fine.*

The price of Silver, which the bringers in of forain Silver ſhall receive at the Mint, according to the Kings Majeſties aforeſaid *Proclamation.*

For the ounce of *Spaniſh* Silver money of *Civill.* } 5 s.

For the ounce of *Mexico* money. } 4 s. 10 d.

For

For Ingots of Silver, being 11 ounces, 2 d. weight fine according to the Standard of *England*, the ounce. — 5 s.

¶ *And for other Silver of more fineneſs, a better price according to that rate, and for courſer a leſſe: ſo that the bringer in ſupply the leſſe fine with the more fine, in ſuch ſort, that in the totall it makes good the ſaid rate of* 11 *ounces,* 2 *pence weight fine according to the Standard of* England.

Of Silver Coynes currant in this Realme.

The *Edward* crown of 5 s.
The *Edward* half crown of 2 s. 6 d.
The *Edward* ſhilling, half ſhilling, and the three pence.
Philip and *Maries* ſhilling, and half ſhilling.
The *Mary* groat, and *Mary* two pence.
Queen *Elizabeths* ſhilling, 9 d. 6 d. 4 d. 3 d. 2 d. 1 d. three farthings, and half peny.

HEre would I now expreſſe the values of ſundry other Coynes of divers Countries, but for three cauſes I now refrain.

The firſt and chiefeſt is, becauſe they are not currant by the Statutes of this Realm.

Another cauſe is, by reaſon they are ſo uncertaine, that they bée never long at one rate

rate. And again, they are so different in so many places, that it were matter enough for a great Book to speak sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in use, and first of France.

French Coynes. The most common Money are Deniers, Soulx, and Frankes: 12 Deniers make 1 shilling, 20 Soulx make 1 Frank: so that as you see these three kindes are like in the rate to pence, shillings and pounds with us; but that this is the difference, that their Denier is but the ninth part of our peny, and so their Soulx (commonly called Souses) go 9 to our shilling, and 9 of their Franks to an English pound of money. So that three of their Franks make a Noble. And by those three you may practise how to reduce French money into English money, according as I have set forth here following.

2160 Deniers make 240 d. or 20 s.
3240 Deniers make 360 d. or 30 s.
8352 Deniers make 928 d. or 3 li. 17 s. 4 d.

2160 Soulx make 240 shillings. And so of other in like rate.

As for the rest of their Coynes I omit them till hereafter, that you have some understanding in broken numbers.

Flanders Coynes. But now as for the Coynes of Flanders, they be so changeable, that you must know them from time to time: else you cannot reduce them into our money certainly: but yet that

that you have an example of their money to exercise you withall, you shall take those that be most common: as Stivers both single and double, Groats Flemish, Carolus and Gyldens. A Flemish Groat is a little above 3 farthings English. A single Stiver is 1 d. ob. q. half farthing. The double Stiver Carolus is 4 d. ob. half farthing. Then there is also the Carolus Gylden, which is worth 10 Stivers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Stivers.

So that if you would convert Flemish money or any other kind of money whatsoever it bee, justly into Sterling, you must reduce it first into the smallest part of English money that is in that Coyne. As for example: If I would reduce 368 double Stivers into English money (considering that a double Stiver containeth 3 d. farthing) you shall first look how many farthings be in the double Stiver, and you shall find them 13, therefore multiply the summe of the Stivers by 13, and then have you their value in farthings, which is 4784. Now if you divide that by 4, then there will appear the number of pence: but better it were to divide it by 48 (for so many farthings are in one shilling) and then will the Quotient declare the summe of the shillings.

Likewise if you would reduce any summe of single Stivers into English money, you must multiply the summe first by 13, and then have you reduced them into a certain summe, that

is

is to wit, half-farthings, which ſumme if you divide by 8, then will amount the ſumme of pence: or if you divide it by 96, the ſumme of ſhillings will appear.

But mark this in all Diviſion: when ye doe reduce to bring one Denomination into another, if there be any remainer after the Diviſion, that muſt bee named by the Denomination of the groſſe ſumme that was divided. As for example, I would bring 254 farthings into pence, therefore I divide that 254 by 4 (for ſo many farthings make a peny) and the Quotient is 63, which is the ſumme of the pence, and then remaineth yet 2, which are farthings ſtill, as one may prove by dividing.

Note well.

And this muſt bee marked in all Diviſion, namely, when it is done for Reduction.

¶ Touching Danks money, they have their Soulx, whereof 20 is a Liver: which is 2 ſhillings ſterling. They have alſo their Graſh whereof 80 make a Gilden, which is 4 ſhillings ſterling. They have alſo Dollars, and their common or old Dollar is 35 Graſh. New Dollars they have which be divers, ſome valued at 24 Graſh, ſome at 26, and ſome at 30. And thus much I thought good to adde to the Author, touching Danks money.

Danks money.

Concerning Spaniſh money, whereof the moſt common are Cornadoes, Marveides, Marveide. 4 Marveides make a Ryall, and

Spaniſh money.

11 Ryalls

11 Rials make one Ducket, ſo the Ducket containeth 374 Marveides, which is about 5 ſhillings 10 pence ſterling. Therefore if you would convert 124 pounds 5 ſhillings ſterling into Duckets, conſider that pence is the leaſt value or Denomination named in this queſtion: therefore reduce 124 pounds 5 ſhillings into pence, and it maketh 29820 pence: which if you divide by pence that a Ducket is worth, (which is 70) you ſhall have for your Quotient 426 Duckets, your deſire.

In Venice they have Bettes, Souldyes, Lieures. 5 Bettes make an Engliſh peny, 60 Bettes a ſhilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice which is a pound ſterling. Venice money.

Thus much have I ſaid of Money: Now will I ſhew you in like ſort the deſtinction of weights. Weights.

After a Statute made anno 11 *H.* 7. *there ought to be but one ſort of weight: as* 24 *Barley-corns drie, and taken out of the midſt of the Ear, do make a peny weight,* 20 *of thoſe peny weights make an Ounce: and* 12 *Ounces a pound of Troy weight, by which is weighed Bread, Gold, Silver, Pearl, Silk, and ſuch like.* But commonly there is uſed another weight called Haberdupoiſe; in which 16 Ounces make a pound. Therefore when you would reduce Ounces into pounds, you muſt conſider whether your weight bée Troy weight or Haberdupoiſe: and if it bée Troy weight you muſt Troy weight. A peny weight. An ounce. A pound Troy. Haberdupoiſe weight.

divide

divide your ounces by 12, to bring them to pounds, but if it be Haberdupoise, you must divide them by 16. Now again, there be greater weights, which are called a hundred, half a hundred, and a quartern, and also a half quartern, &c.

A hundred weight.

Scholar. Why? so there may be reckoned 26 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they have not common weights made to their rate as the other have. And again, these that I did name, are not just in number as they seem by their name: for an hundred is not just 100, but is 112 pound. And so the half hundred is 56, the quarter 28, and the half quarter 14. And these be the common weights used in most things that are sold by weight.

Howbeit there are in some things other names, as in Wooll 28 pound is not called a quartern, but a Todde: and 14 pound is not named half a quartern, but a Stone, and the 7 pound half a Stone. Other names because they differ in many places, and agree in few, I let them passe.

Woollweights. Todde. Stone.

But a Sack of Wooll by the Statutes is limited to be 26 stone.

Sack of Wooll.

¶ Now in Cheese, though it be sold by the hundred, and by the stone in some places, yet the very weights of it are Cloves, and Weyes. So that a Clove containeth 8 pound, and a

Cheeseweights.

Wey

a wey 32 Cloves, which is 256 pound, that is 12 score and 16 pound, and so much weigheth the wey of Suffolk cheese, and the like is or should be the Barrel of Suffolk butter.

The wey of Essex cheese containeth six score and sixteen pound: and so much is also the barrel of Essex butter.

Moreover this weight is used by the Apothecaries in their Physicall composition, and mixture of medicine, wherein the least is a grain.

And { 20 *Graines* / 3 *Scruples* / 8 *Dragmes* / 16 *Ounces* } make { *A Scruple* / *A Drachm* or *Dragm* / *A Ounce* / *A Pound.* } thus charactered { ℈ / ʒ / ℥ / ℔ } The Apothecaries weights.

Now of weights are made other measures both for grain & liquor. For a pound in Troy weight, maketh a pint in measure, so that 8 pound or 8 pints do make a gallon: half a gallon is named a pottle, and half a pottle is called a quart, which containeth two pints: Now above a gallon the next measure is a firkin: then the tertian, a kilderkin, or half a barrel, and a barrel: And by these measures are sold commonly Ale, Beer, Wine, and Oyle, Butter and Soap, Salmon, Herrings, and Eels. Measures for liquor. A Pint. Gallon. Pottle. Quart. Firkin. Tertian. Kilderkin. Barrell.

But as these be unlike things, so the measures of their vessels do differ, for the measures of them all are as followeth.

Of Ale { *The firkin* / *The kilderkin* / *The barrell* } *containeth* { 8 / 16 / 32 } *Gallons.* Ale measure.

Beer measures.

Of Beer { the Firkin / the kilderk. / the barrell } containeth { 9 / 18 / 36 } Gallons.

Sope measures.

Sope measures, both Firkin, Kilderkin, and Barrell should be equall to Ale measure.

Moreover, the Statutes doe limit the weight of every of those three vessels being empty.

{ A barrell / Half a barrell / A firkin } to weigh empty { 26 / 13 / 6 1/2 } pounds.

Herrings.

Herrings also sold by the same measures that Ale and Soap be sold by.

Herrings are sold by the tale, 120 to the hundred, 10 thousand to the Last.

Salmon & Eeles.

Salmon and Eeles have a greater measure.

Salmon & Eeles { the butte / the barrell / half barrell / the firkin } holdeth { 84 / 42 / 21 / 10 1/2 } gallons.

Notwithstanding, some Statutes did limit Eele vessels equall with Herring vessels.

Wine measures.

Now as for Wine vessels they are seldome smaller then Hogsheads, which are of 63 Gallons: Every Hogshead, is two Barrels: yet there are many other wine vessels, but of them all see this Table; and mark the measures one by another.

Of

Of wine and oyle	the runlet	holdeth	18½ Gallons.
	the barrell		31½
	the hogshead		63
	the tertian		84
	the pipe		126
	the tunne		252

But you shall mark that there be other kindes of Tertians: for there be Tertians (that is to say) Thirds of pipes, of hogsheads, and of barrels, as well of other things as of wine. Tertians.

Also Malmseyes, and Sack, &c. the butt Tun is not called a Pipe, but rather a Butte. A Butte.

And thus much have I thought meet to tell you at this time.

Scholar. And is that alwaies true?

Master. I have told you how it should be, but how it is, I may not say: how they doe differ daily from their just measure, that Gaugiers can tell you better then I. But I will let this passe now, and speak briefly of the other measure.

And as of weights there did spring the liquid measures (whereof I spake last) so of the same springeth drie measures as Pecks, Bushels, Quarters, and such like, whereby are measured corn and like grains, also Salt, Lime, Coals, and other like. And this is the order and quantity of them. Drie measures.

A peck is the measure of two Gallons.

A Buſhel. A Quarter. A Wey.

A buſhel } containeth { *four pecks.*
A quarter } { *eight buſhels.*
A wey { *ſix quarters.*

Theſe are the common names & meaſures, but in divers places there be divers ſorts.

Strike. Cornock.

The Buſhell in many places is two buſhels, but then is that buſhel there called a ſtrike: and in ſome places half a quarter is called a Cornock. But theſe diverſities are to many to tell you briefly them all: and again, ſith they are againſt the Law and Statutes, I count them unmeet to be uſed.

Meaſure to mete length, bredth, & thickneſs.

But now remaineth yet another kinde of meaſures, whereby men mete length, breadth, and thickneſſe, and theſe are an Inch, a Foot, and ſuch other: whoſe names and quantities this Table ſheweth.

An Inch. Foot. Yard. Ell. Pearch. Acre.

3 *Grains of barley in length make an inch.*
12 *Inches* } *make* { *a foot.*
3 *Foot* } { *a yard.*
3 *Foot and* 9 *Inches* } *make* { *an Ell.*
5 *Yards and a half* } { *a Pearch.*
1 *Pearch in breadth, and* 40 *in length, doe make a rodde of land, which ſome call a rood, ſome a yard land, ſome a farthendele.*
2 *Farthendels* } *make* { *half an acre of ground.*
4 *Farthendels* } { *an acre.*

¶ *More,* 40 *rods in length doe make a furlong,* 8 *furlongs make an Engliſh mile, which containeth* 320 *Perches.*

So that an Engliſh mile, grounded upon the

the Statute, is in length 1760 yards, 5280 foot, and 63360 Inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foot to a pace.

Here might I tell you many things elle touching measures, and also how to reduce strange measures to our measures, but because it cannot be well done without the knowledge of Fractions, which as yet you have not learned, I will let them passe till another time, that I have taught you the knowledge of broken numbers.

Scholar. But yet Sir of the parts of time, I pray you tell mée somewhat. The parts of time.

Master. You know that a naturall day hath 24 hours, and every hour hath 60 minutes. It néedeth not to tell you, that 7 daies make a week, and 4 weeks make a common moneth, and 13 moneths make a year, lacking one day, and certain hours and minutes: but of that I shall instruct you hereafter. A day. An hour. Week. Moneth. Year.

Here will I make an end of Reduction *for this time, which though it be counted no kinde severall of* Arithmetick, *you see it is no lesse needfull to be known, or easier to be done, then any of the other.*

Scholar. Marry Sir, it séemeth unto mée much harder then any other sort, for it requireth the knowledge of so many things: but now Sir when you sée time, I am ready to learn forth, as much of Reduction as you have taught mée, I remember; but and if I doe at any time forget, I shall have re-

course to the Tables which you set forth for me.

Master. So doe you: for it will not be remembred without exercise. But in as much as you understand so much as we have intreated of, I will now instruct you in Progression.

Progression.

Progreſsion.

Although until this day the moſt part of Writers have defined Progreſsion *as a compendious kinde of* Addition, *yet truly it is not ſo: for* Progreſsion (*as the very nature of the word doth inform any man*) *is a going forward and proceeding in* numbers, *and that regularly and orderly, whoſe place is aptly choſen to be very neer, or rather next after the* expoſition of the four principall parts of Arithmetick: *for in it after a moſt eaſie manner, are all the four former parts exerciſed and practiſed: and not onely* Addition, *as cuſtomably is done. Which cuſtome hath been the cauſe, why it hath ſo ſpecially been named a kind of* Addition, *and defined to be a quick and brief* Addition *of divers* ſummes, *proceeding by ſome certain and reaſonable order.* What Progreſsion is.

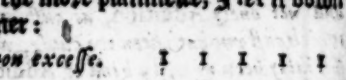
You ſhall alſo underſtand there are infinite kinds of Progreſsions, **but for you (as yet) two are ſufficient to be exerciſed in, of which the one I call** Arithmeticall, **and the other** Geometricall.

Arithmeticall Progreſsion *is a rehearſing or placing down of many* numbers, number *after* number, *in ſuch ſort, that between every two next* numbers *rehearſed or placed down, the difference, diverſity, or exceſſe, be equall and alike.* Arithmetical Progreſsion.

Scholar. Sir I thank you for that you have both opened unto me what Progreſſion is truly, and alſo why it is here placed.

But I pray you with an example make plain your definition.

Maſter. Examples cannot want, ſeeing all reaſonable creatures naturally uſe the order of one kinde of Arithmeticall progreſſion (which therefore is alſo named naturall) whenſoever they diſtinctly doe count or number any multitude by one, ſaying, 1,2,3,4,5,6, whereby the proceeding from number to number, and every one ſurmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the ſame to be Arithmeticall progreſſion. And for the more plainneſſe, I ſet it down in this manner:

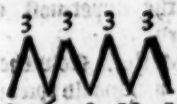


Scholar. This is moſt evident. And I think that I am able to tell you now of any progreſſion Arithmeticall propounded, what is that common exceſſe or difference whereby it proceedeth, if this order be kept in it.

Maſter. What ſay you of 3, 6, 9, 12, 15?

Scholar. They exceed each other by 3: And that may I ſet down in ſuch evident order, as you did your example of naturall Progreſſion, in this wiſe.

The common exceſſe. 3 3 3 3



The Progreſsion. 3 6 9 12 15

Maſter. And doe you not alſo now perceive, that the whole Table of Multiplication may be made by the order of progreſsion Arithmeticall? either if you will begin at the firſt number of any of them on the left hand, & ſo proceed right overthwart: or at any of the firſt numbers of the upper row, & go directly downward.

Scholar. I pray you let me conſider the thing a little, and I will anſwer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceive it now very well, for the common exceſſe or difference between any two next, is continually as much as the firſt number of every row, either from the left hand overthwart taken, or from any of

of the uppermoſt overthwart rowes downward.

To know the totall ſumme of an Arithmeticall Progreſſion.

Maſter. Now then, if of any ſuch Progreſſion, you would ſpeedily know the totall ſumme much quicklier then by common Rules of Addition: firſt tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their ſumme downe by it ſelf: as in this Example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7, as you may ſee: therefore ſet down 7 in a place alone, then adde together the firſt number and the laſt, as in this Example; Adde 2 to 14, and that maketh 16, take half of it, and multiply by the 7 which you noted for the number of the ſame places, and the ſumme that amounteth, is the ſumme of all thoſe figures added together: As in this Example 8 multiplied by 7, maketh 56, and that is the ſumme of all thoſe figures.

Scholar. That will I worke by another Example. I would know how much this ſumme is, 5, 8, 11, 14, 17, 23, 26, 29, I tell the places and there are 9, that I note. Then I put the firſt number 5, and the laſt 29 together, and they make 34, I take the half of it, that is 17, and multiply by 9, and it maketh 153. That you ſay is the ſumme of all the numbers.

Maſter. So ſhall you find it if you trie it.

Scholar. How ſhall I trie it?

Maſter. By your common Addition, for

it

if you adde all the parcels together, you ſhall ſee the ſame ſumme amount, if you did worke well. And that manner of Addition trieth all kinds of ſumming any Progreſſion.

Scholar. Then can I ſumme any Progreſſion, if the number of the parcels be odde. But what if they be even, as in this example, 1, 2, 3, 4. 5. 6. 7. 8?

Maſter. When the number of the parcels is even, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the half of the number of the places, doe you multiply it: as in our example, the parcels are 4, that I note: then adding the firſt ſumme to the laſt, there amounteth 9, that doe I multiply by half of the parcels, that is, by 4, and it maketh 36, which is the ſumme of the parcels.

But if you will take one Rule for theſe both, doe thus: Multiply the half of the one by the other whole, and the ſumme will amount all one. For ſometime it chanceth that the number of the parcels be odde, ſo their half cannot be taken: and that ſometime it chanceth the Addition of the firſt number and the laſt, doe bring forth an odde number, ſo that half of it cannot be taken: but they will never bee both odde. A generall rule.

Scholar. Then I perceive this, if there bee no more belonging to it.

Maſter. As accuſtomably it hath been taught, this hath been the chiefe and onely exer-

exercise in Progreſsion uſed. But that you may perceive how divers waies, and to how great profit ſo ſimple a thing (as this Arithmeticall progreſsion is) may be conſidered and uſed, I will here propound you ſix Propoſitions: of which four of them were invented by a friend of mine, and never before thus publiſhed: and the two firſt were never to my knowledge written of but by three men.

Scholar. This doth greatly encourage me to be attentive unto your worke, ſeeing I ſhall not only be inſtructed at your hands in the common known Rules of this excellent Art, but beſides that, ſo abundantly in other new Rules enformed, as my very entrance ſhall ſeem to paſſe a great many mens further ſtudy, and longer continuance. Therefore Sir, I beſeech you, let me know your ſix Propoſitions.

Maſter. Theſe they are.

1. *To know the laſt* number *without proceeding by continual* Addition, *till you come unto it, ſo that the common exceſſe, the firſt* number, *and the* number *of the places be known.*

2. *The firſt* number *of the* Progreſsion *and the laſt being known, with the common exceſſe to finde the* number *of the places.*

3. *The exceſſe being given, and the* firſt *or* laſt, *to know the quantity of any middle* number, *whoſe place is given from the* firſt *or* laſt.

4. *The* totall ſumme *being given, and the* firſt *and* laſt, *to find out the* number *of the places.*

The

The totall ſumme *of any* Arithmeticall Progreſſion *being given, and the firſt and laſt, to finde out the common exceſſe.* 5.

The totall ſumme *being given, and the mutuall exceſſe with the* number *of the places, to give the firſt or laſt* number *of the ſame* Progreſſion. 6.

Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*, but theſe are ſufficient, to give you occaſion to think, that Rules of Knowledge and Arts are infinitely capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written: And yet in my ſimple fantaſie, theſe things offer themſelves (in manner) to be ſtudied for about Progreſſion, therefore I pray you to proceed to the Rules anſwering to theſe Propoſitions.

Maſter. I will orderly for every of theſe 6 Propoſitions give you Rules, and with every one an Example, unleſſe the plainneſſe of eaſineſſe need no farther exemplifying.

For the ſolution of the firſt multiply the exceſſe by a number *leſſe by* 1 *then the* number *of the places, and the off-come adde to the firſt* number, *ſo you ſhall have the laſt* number, *which is ſought for.* 1 Propoſition.

As for example. If there were ſeven places in a Progreſſion Arithmeticall, whoſe continuall encreaſe or mutuall exceſſe were 4, and the firſt number were 5, and I would know what the laſt and ſeventh number is: I multiply 6 which is one leſſe then 7, (the number

number of the places) by 4, thereof cometh 24, which I adde to 5, that maketh 29 and that is the laſt number which I deſire to know. And this you may ſtraightway proue by continuall proceeding from 5, till the ſeuenth place, encreaſing euery one by 4, as thus

5 9 13 17 21 25 29.

Loe here, the laſt being alſo the ſeuenth, is 29.

Scholar. I perceiue already one good property in this Rule, which in all workes is to bee deſired, that is, it will eaſe one from great labour, if a Progreſſion were propounded of a hundred or two hundred places, or moe: And alſo it is very eaſie to worke, and moſt neceſſary for the totall ſumme finding, in a very long Progreſſion.

Maſter. It is true, and therefore now let mee ſee if you can anſwer mee this queſtion by this propoſition.

A Merchant buyeth 50 pounds of Spices, and agreeth to pay for the firſt pound 4 pence, for the ſecond 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The queſtion is, how much hee muſt pay for the laſt pound, and then how much the 50 pound cometh to?

Scholar. According to the propoſition, I multiply 49 (which is leſſe by one then the number of the places) by the exceſſe, which is 3,

to

to the product 147, I adde the firſt number which is 4, it maketh 151 pence, the price of the laſt pound. Now I adde 4, the price of the firſt pound, to 151 the price of the laſt pound, it maketh 155, which I multiply by half the number of the places, which is 25 the Product, 3875 pence is the totall ſumme or price of the 50 pounds of Spices, as appeareth.

49 Places 1 leſſe.	151 laſt
3 exceſſe	4 firſt
147	155
4 firſt	25 half places
151	775
	310
	3875 totall ſumme which amounteth to

li. s. d.
16——2——11

Maſter. It is truly wrought.

Scholar. Then I intreat you to proceed to your ſecond propoſition.

Maſter. *The ſecond* Rule *is this. From the laſt ſubtract the firſt, the* remainder *divide by the common exceſſe, to the* Quotient *adde* 1, *and you have the* number *of the places, which you would know: as in this* Progreſſion. 2 Propoſition.

6 11 16 21 26 31

If I know onely 6 and 31, and that they encreaſe

encreaſe by 5 then according to the Rule, from 31 I ſubtract 6, there remaineth 25; which 25 I divide by 5 (the common exceſſe) the quotient cometh forth 5, to which I adde 1 that maketh 6: and ſo many are the places, as you ſee.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I could not remember it.

3 Propoſition.

Maſter. *The third* Propoſition *may alwaies thus be ſolved. Multiply the* exceſſe *by a* number *leſſe by one, then the diſtance of the place is from the* firſt, *or the* laſt number given: *the offcome adde to the* firſt, *if the* diſtance *be reckoned from the* firſt, *and the* firſt *alſo known, or ſubtract from the* laſt, *if the diſtance be from the* laſt counted, *and the* laſt given *alſo, and that which cometh forth, either in that* Addition to the firſt, *or* Subtraction from the laſt *is the* number *ſought.* As for Example, *I propound you this* Progreſſion.

8 15 22 29 36 43 50 57

And for the apt conſidering the manner of this queſtion, I will note over every place his diſtance from the firſt, and under every place his diſtance from the laſt, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the exceſſe whereby this Progreſſi-

on

ſion ſtandeth, be known to be 7, and the firſt numbers given, being 8, I would know what number ſtandeth under 4, that is to ſay, in the fourth place. I multiply 7 by 3 (which is leſſe by 1 then the number of the place propoſed) that yeldeth 21, to which I adde 8, the firſt number, ſo cometh 29: which I ſay to belong to the fourth place, as you ſee in the Example it alſo doth: or if in the third place from the laſt, you would know what number in this Example ſhould ſtand, the laſt number being known to be 57, and the common exceſſe 7, then by 2 which is leſſe by 1 then the place propounded, I multiply 7, that giveth 14, which appertaineth to the third place inclu ſively reckoned from the laſt, and ſo my Example giveth you.

Scholar. I perceive right good uſe of this Rule: for if I had forgotten what the firſt number were, and remember ſtill but the laſt the common exceſſe, and the number of the places, then might I come by the knowledge of my firſt number again.

And me thinketh, that it differeth not much from the firſt Propoſition, ſaving that which you make here a middle number, there was made the laſt: & alſo in this point it differeth, that in it the laſt was only ſought, and no conſideration had in numbring the places from the laſt, as here I mark in your numbers noted under your Progreſſion.

Maſter. And think you not, the middle

numbers of Progreſſion ſtanding of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt by the common exceſſe, or abating from the laſt continually the common exceſſe, as the very finall numbers in a ſhorter Progreſſion would doe?

Scholar. Yes Sir, that I think right well, & therefore I am glad of this new framed propoſition, and the manner of working of it.

4 Propoſition. Maſter. *The rule of the fourth is this. Adde the firſt and the laſt together, and by the off-come divide the totall* ſumme. *Double the* Quotient, *and that will be the* number *of the places.*

Scholar. Then if in a Progreſſion, whoſe ſumme were 207, and the firſt number 12, and the laſt 57, if I adde 57 and 12 together, that maketh 69, and by it I divide 207, the Quotient will be 3, which I double; and ſo I have 6, and ſo many muſt be the number of the places that this Progreſſion ſtandeth on.

Maſter. Whether it bee ſo or no, how will you trie?

Scholar. Half 6, which is 3, being multiplied by 69, muſt make 207, the totall ſumme; if 6 bee the number of the places. For ſo the whole work of your Rule in ſumming any Arithmetical Progreſſion did enform mee. I will then multiply 69 by 3, thus.

	69
	3
	207

It cometh forth juſtly.

Maſter.

Maſter. I muſt much herein commend your promptneſſe both in memory and in well applying your Rule: although in manifeſt words it did contain no ſuch matter.

Scholar. Sir, I pray you hear me frame one example or more.

Maſter. I am well pleaſed, ſo that ye bee ſhort, for you make mee more longer here then willingly I would have bin: but I cannot perceive how I could have omitted any thing as yet, without your great lack thereof.

Scholar. If I had received 85 pounds of certain men, but of how many I have forgotten, yet I remember that the firſt gave me 7 pound, and the laſt 27 pound, and every payment after other did riſe by a like ſumme. And the man from whom I received this money, conditioned with me, that of every payment I ſhould have twelve pence for my labour: now unleſſe I can by Art finde the truth of this caſe, I am like to loſe the moſt part of my reward. A queſtion of money.

Maſter. I perceive you can handſomely frame an example, which ſhould concern your own gain: I pray you let mee ſee how you would doe juſtice in this point.

Scholar. I adde the firſt and the laſt together, that maketh 34: by which I divide 85, thus:

x		
27		17
85	(2	
34		34

Why how now, Sir, here is a remnant of 17, in which 34 cannot bee

had: ſo that now I am in the briers for doubling of my Quotient, and farewell then both my Juſtice and a good lump of my gains.

Maſter. Yée are never the farther from the matter, though it fall into a Fraction. For you ſhall underſtand that the Fraction which of any ſuch work proceedeth, is ever half of one ſuch, as the unites of the Quotient before are. And that you may trie, if you double that which ſo remaineth, for then it will be equall to your Diviſor, as if ye double 17, (the remnant) it maketh 34, and your Diviſor alſo was 34, this noteth the remainder to be half of one.

Scholar. Now I am glad of this hard Example: For with it I have a generall rule for the Fraction that may hap in this work. So that the Quotient being two and a half, I double that, it maketh 5, therefore ſhould my gain be 5 ſhillings. And to be ſure (by your leave) I will trie it, for I will multiply half of 34: (which is the firſt and laſt number joyned together) by 5, thus.

	17
	5
	85

It is moſt true (I ſée) that I ſhould leeſe nothing by the former working.

5 Propoſition.

Maſter. *The fift Propoſition hath this Rule appertaining unto it: By the fourth Rule finde the* number *of the places, that being done, from the laſt ſubtract the firſt, and the reſidue divide by a* number *leſſe by* 1, *then the* number *of the places, and the* quotient *will ſhew the exceſſe, which is ſought for.*

An

An example hereof ſhall be this: If ye had disburſed 685 pounds to a certain number of men, you neither can tell how many they were, or how much the ones money exceeded his next before, but you are ſure that the exceſſe was equall between every two next: and alſo you remember that the firſt had 19, and the laſt 180 pounds, how would you finde the number of the name and the exceſſe continually obſerved in the ſucceſſion of their payments? Example.

Scholar. Your Rule doth plainly bid, firſt to finde the number of the places, which I will do according to the fourth rule: I adde 19, and 118 together, thus.

```
 118
  19
 ---
 137
```

By this 137, divide 685 thus.

Séeing there is no Fraction but a whole number, being 5, I double that, and then muſt the number of the places bée 10. Now from the laſt I ſubtract the firſt, as 19 from 118, thus: And ſo remaineth 99.

```
 13
685 (5
137

 118
  19
 ---
  99
```

This 99 I divide by a number leſſe by one then the number of the places, and ſéeing the places were 10, I divide 99 by 9, thus:

```
 99
99 (11
```

The quotient is 1, and ſo was the exceſſe, if I have followed your rule right.

Master. You have wrought every part of this question both well in order, and truly in the practise of your Rules.

Scholar. I will then set it down also formably, so that the number of the places, the excesse, and the totall summe may straight appear, as your first example stood.

The common excesse.	11	11	11	11	11	11	11	11	11	11
	19	30	41	52	63	74	85	96	107	118

The progression.

That the places be 10, and that from the first to the last, the common excesse is 11. I perceive most evidently: but whether the totall summe be 685, I have not yet proved, which I will now doe: I adde 19 and 118 together, that maketh 137: I multiply that by half the number of the places, thus.

	137
All things agree most exactly, so that I am perfect enough in these Rules, if I forget them not again.	5
	685

Master. Use maketh all things perfect.

6 Proposition.

Your sixt rule is this: By the number of the places divide the totall summe, double the quotient, and that will be the first and last joyned in one summe: Then by a number lesse by 1, *then the number of the places, multiply the excesse, that off-come subtract from the first doubled quotient, and the half of the residue is the first number. The last number you may diversly finde*

out

out, as by the firſt of our 6 Rules, *or by ſubtracting this firſt number from the* ſumme *which here contained both the firſt and laſt joyntly,* (*or thirdly*) *by continuall adding the exceſſe.*

Scholar. I pray you make this ſomewhat more plain with an example.

Maſter. If every moneth in the year (counting them now as 13) you gained clearly 40 ſhillings more then you did the moneth next going before, and at the years end you finde the whole gain 5720 ſhillings, but ye remember not how much either the gaine of the firſt moneth or the laſt was, by this Rule it may be tried out. Example of gain.

Scholar. So that here yée ſéem to apply the 13 moneths to thirtéen places, the 40 ſhillings every one more then the other next before it to be the common exceſſe, and 5720 ſhillings to the totall ſumme.

Maſter. It is true: by 13 then I divide 5720 in this manner.

	1	
	13	
I double this Quotient,	1720	(440
ſo have I 880 for the firſt	1333	
and the laſt ſumme joyned	11	
together, by 12 which is		
leſſe by one then the num-		

her of the places; I multiply 40 (the common exceſſe) ſo cometh 480.

	40
	12
	80

This 480 I ſubtract from 880, ſo remaineth 400: half whereof is the firſt number which wee deſired to know: that is 200.

	4
	480

And as for the laſt number, I can giue you it three waies. As by the firſt of my ſix rules I multiply the exceſſe, by a number leſſe by 1 then the number of the places, as 40 by 12 that giueth 480, which I adde to the firſt, being 200 ſo ſhall the laſt be 680.

The ſame ſumme cometh forth if ye ſubtract 200 from 880.

And thirdly, If I begin at 200, and ſo proceed, encreaſing by 40, I ſhall at the thirteenth place haue 680, as thus:

200 240 280 320 360 400 440
480 520 560 600 640 680

Scholar. I thank you moſt heartily for theſe ſix Rules. Now if it bee your pleaſure I would hear and learn ſomewhat of Progreſſion Geometricall.

Maſter. There are yet very many Rules and Propoſitions which fall into this Arithmetical Progreſſion.

And for the uſe and practiſe of them, I will propone unto you certain pleaſant and neceſſary Queſtions of Arithmeticall Progreſſion, and to the performance of their workings, ſuch

ſuch neceſſary rules and documents, as are requiſite for the better underſtanding of them, or any ſuch like.

A queſtion of Velvet.

A certain Mercer *ſold* 20 *yards of Velvet to be paid in* 12 *weeks, by* Arithmeticall proportion; *that is to wit, to receive the firſt week* 6 ſhillings, *the ſecond week* 12 ſhillings, *the third week* 18, *and ſo forth, increaſing the* number *of weeks by* 6 ſhillings, *till the twelfth and laſt week were expired.* The Queſtion is how many pounds he had for 20 yards of Velvet.

6
12
18
24
30
36
42
48
54
60
66
72

To the performance of this Queſtion, and ſuch other the like, I ſet forth the 12 payments in ſuch ſort, as for example, here appeareth.

Then touching the adding together of theſe ſummes, without the aid of Addition, according to the Rules I taught you in Progreſſion Arithmeticall, I note the number of theſe places, which are 12, then adding the laſt number of the Progreſſion, which is 72, and the firſt number together make 78; and multiplying 78 by half the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 13 pounds 8 ſhillings. And ſo much hath the Mercer for his 20 yards of Velvet, which is nigh about 23 ſhillings 5 pence, a yard.

Scholar. I underſtand this work very well, but is there any proof for the juſtifying

hereof, as you have of other works?

Maſter. The work of it ſelf (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for juſtifying of the ſame: yet notwithſtanding, becauſe your requeſt is reaſonable, I will propone an example for the proof hereof,

The proof of the laſt queſtion.

A certain man is bound to pay for 20 *yards of Velvet, the* ſumme *of* 23 pound 8 ſhillings, *and it is to be paid weekly, in* 12 *weeks or terms by* Arithmeticall Progreſsion.

The Queſtion is therefore to know with what number the ſame Progreſsion is to be begun and continued in ſuch equall proportion Arithmeticall, that in 12 weeks the ſame may be juſtly accompliſhed.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings all into ſhillings, which maketh 468 ſhillings.

A generall rule.

Then adde 1 unto 12, the number of the terms, it maketh 13, which 13 you ſhall multiply by half the number of the terms, which is 6, it maketh 78; then divide 468 by 78, and you ſhall finde 6 in the Quotient, which is the true number that ſhall begin and continue the ſaid Progreſsion. That is to ſay, the firſt week 6 ſhillings, the ſecond 12 ſhillings, and the third week 6 ſhillings more, which is 18 ſhillings, and ſo every week as they riſe, 6 ſhillings

ſhillings more then the week before, as is manifeſt in the queſtion aforeſaid.

A Farm is to be ſold to be paid by the weeks in a year; the firſt week to pay 4 ſhillings, the ſecond week 8 ſhillings, the third week 12 ſhillings, and ſo forth, increaſing each number by 4, till the number of 52 (which are the number of weeks in a year expired.) The queſtion is, what the price of the Farm cometh to? A queſtion of a Farm.

Scholar. I doubt not, but by that you have already taught mée, to end this Queſtion very well; wherefore I ſet forth the Progreſſion with his exceſſe 52 times.

Maſter. May ſtay a while: And here for your further eaſe, (to abridge you of great labour that appeareth to fall out in this Queſtion, and ſo may doe in any other the like) If a Queſtion were propoſed of 100 or 200 places, or moe, and that this Queſtion, or any other the like can bée ended, unleſſe you may know abſolutely what the laſt number of the Progreſſion at the 52 place is, (or ought to bée) I will give you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, as well as if you had ordinarily proceéded by continuall Addition, till you had come to the laſt work, which is this.

Multiply the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall have your deſire. A generall rule.

Scholar.

Scholar. This Rule is well worth the noting: for if I underſtand you aright, I conſider that my exceſſe is 4, which I multiply by 51, which is one leſſe then the number of the places, and it maketh 204, whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208, which you ſay is, or ſhould bee the laſt Number of the Progreſſion.

Maſter. This is a moſt approved truth, if there were never ſo many places.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I doe not remember it. For by the benefit hereof, I have ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeem to paſſe a great many mens further ſtudy, and longer continuance.

Maſter. Many moe Conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to give you occaſion to think that *Rules* of Knowledge and *Arts*, are infinite capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written. But theſe things in my ſimple fantaſie, offer themſelves to be greatly beneficiall unto the aid of Progreſſion. Therefore now I will go forward with your Queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt number

numbers of the Progreſſion, which is 4, it maketh 212, which I multiply by half the number of the places, which is 26, and it amounteth to 5512 ſhillings. And ſo much is the totall ſumme or addition of this Progreſſion: which maketh 275 pounds 12 ſhillings, as appeareth here by my Tables.

Maſter. I like well your labour, and commend you for your diligence: I will here propone one example more, and therewithall for this time will end Progreſſion Arithmeticall.

A certain man bought 20 Ells *of* Holland, *to be paid in* 17 *weeks, or terms, by* Progreſſion Arithmeticall. *And the firſt week to pay* 1 ſhilling 8 pence, *the ſecond week* 3 ſhillings 4 pence, *the third week* 5 ſhillings, *the fourth week* 6 ſhillings 8 pence, *and ſo forth, each week ſucceeding* 20 pence *more then the week before.* A queſtion of Holland.

The Queſtion is, what the ſumme of his 20 Ells cometh to.

Scholar. Becauſe here is mention made both of Shillings and Pence, I fear there is ſome harder matter contained herein, then in the other before: therefore I pray you work it your ſelf, and I will diligently mark your labour.

Maſter. There is no more to be done in this, then in the other before; but becauſe your requeſt is ſo reaſonable, be attentive unto me.

Firſt, by the generall Rules, I ſeek to finde out the laſt Number of the 17 place, what this

this Progreſſion ought to be. Therefore here in my Tables multiplying the exceſſe 20 by 16, which is one leſſe then the number of the terms for places, and it cometh to be 320; and thereunto adding the firſt number of the Progreſſion, which is 20 pence, all is 340 pence, or 28 ſhillings 4 pence: for ſo much ought the laſt number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the firſt number of the Progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe half cannot be taken, I take the half of 360, which is 180, and multiplying 180 by 17, cometh to 3060 pence, which maketh as you ſee by Diviſion 12 pound 15 ſhillings. And ſo much is the Buyer to pay for his 20 Ells of holland. Which 3060 pence if you divide by 20, the number of Ells that was bought, you ſhall find 12 ſhillings 9 pence, and ſo much payed he for an Ell one with another.

The Proof.

A queſtion of debt. *A certain man doth owe* 12 pound 15 ſhillings, *to be paid in* 27 weeks *or* terms *by* Arithmeticall Progreſſion. *The queſtion is, to know with what* number *he ſhall begin and continue the* Progreſſion, *in ſuch equall proportion, as the ſame may be truly paid and ſatisfied in* 17 weeks.

The

The Anſwer.

First I reduce 12 pounds 15 ſhillings, all into pence, which as you ſée here in many Tables, make 3060 pence, that I let ſtand by a whyle.

Then I adde 1 to 17, the number of the places or terms, which maketh 18, which I ſhould multiply by half the number of the weeks or terms, which is $8\frac{1}{2}$ which $8\frac{1}{2}$ multiplied by 18 cannot well bée done, unleſſe you were acquainted with Fractions or broken numbers, therefore you ſhall let that paſſe and multiply 17 by the half of 18, which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$) and the multiplication of 9 into 17 maketh as you ſée 153, with which number you ſhall diuide the 3060 pence beforeſaid, and the Quotient bringeth forth 20 pence, which is the firſt number or payment to begin the Progreſſion withall: and ſo each wéek ſucceéding to riſe 20 pence more then the wéek before, and thereby in 17 wéeks ſhall 12 pound 15 ſhillings bée payed: as before was ſufficiently declared. Thus much for Progreſſion Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thanks for theſe benefits ſhewed mée, which me thinketh are ſo eaſie, delightfull, and pleaſant, that I count my ſelf happy to be in your company.

Maſter.

Maſter. I am glad you delight ſo well herein, which is an Art of wonderfull dexterity to all ſorts of men, of what degrée or profeſſion ſoever they be. And now will I proceed to Progreſſion Geometricall, wherein I will be more briefe, both becauſe I have been ſo long in this part of Arithmeticall Progreſſion, and alſo for that it would require the knowledge of Roots and ſurd numbers, (whereof ye have learned nothing) if I ſhould frame the like propoſitions in them as I have done in theſe. Therefore I will only teach you two practiſes about it, and ſo end the conſiderations and workes of theſe Progreſſions.

Progreſſion Geometricall.

PRogreſſion Geometricall, *is when the numbers increaſe by a like proportion, that is, if the ſecond number contain the firſt, 2, 3, or 4 times and ſo forth, then the third containeth the ſecond ſo many times, alſo: and ſo the fourth the third, and the fift the fourth, wherefore I ſet theſe examples.*

3	6	12	24	48
1	3	9	27	81
2	10	50	250	1250

Here in the firſt example you ſee, that every number containeth the other (that goeth next before him) two times; and in the ſecond example three times, and in the third example five times. Now if you will know how to finde eaſily the ſumme of any ſuch number, doe thus: Conſider by what numbers they be multiplied,

multiplied, whether by 2, 3, 4, 5, or any other, & by the ſame number multiply the laſt ſumme in the Progreſſion. To finde the totall ſumme in any Geometricall Progreſſion.

Scholar. I pray you work it by this Example, 2, 8, 32, 128, 512, 2048, which I have framed by proceeding from 2, and continuall multiply by 4.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192. Now muſt I abate from this ſumme the firſt number of the Progreſſion, which here is 2, then reſteth 8190, which ſumme I muſt divide by 1 leſſe then was the Number that I multiplied by. Seeing then I multiplied by 4. I muſt divide by 3, ſo dividing 8150 by 3, the Quotient will be 2730, which is the ſumme of all the Progreſſion. And now to prove whether you can doe the ſame, I give you theſe Numbers to adde by this Rule 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progreſſion doth increaſe.

Maſter. In any ſuch doubt doe thus: Divide the ſecond number by the firſt, and the Quotient will ſhew you the number that engendreth the Progreſſion.

Scholar. Then is that number in this Example 5, for ſo many times is 3 in 15.

Maſter. So is it. Now work as I taught.

Scholar. The laſt number is 46875, which I multiply by 5, and it yeldeth 323475, from

which I abate the laſt number of the Progreſsion, that is 3, and there reſteth 234372, which I divide by 4, for that is one leſſe then 5, and the Quotient is 58593, which is the whole ſumme of the Progreſsion.

Maſter. If you remember well this, you have learned the Art of Progreſsion both Arithmeticall and alſo Geometricall, which you may prove either by ſubtracting of each number alone from the ſumme, and ſo will there nothing remain: or elſe by adding together of all the parcels, for ſo will the ſame ſumme amount.

Aqueſtion of Satten. *A Mercer hath 12 yards of Satten, which he valueth at 16 ſhillings the yard, and ſelleth the ſame 12 yards to another man to be paid as followeth: That is to wit, for the firſt yard to have one ſhilling, for the ſecond yard two ſhillings, for the third yard four ſhillings, for the fourth yard eight ſhillings, &c. doubling each number following, till the twelfth and laſt yard.* The queſtion is, who hath made the better bargain of the Buyer or Seller?

Firſt you may ſet down 12, the number of the yards, as you ſee here in this Example. And againſt each number the number of ſhillings due to be paid as the order of Duplation or Multiplication by two teacheth.

Then reſorting to the adding up or ſumming of this Progreſsion, where I conſider that the increaſe of this ſumme proceeded by the Multiplication of 2, and therefore after I have drawn

a

0	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4095	

a line under the 12, I work and multiply the laſt ſumme by 2 alſo, & it yeelds 4096: from whence I abate the firſt number of the Progreſſion, which is 1, and then reſteth 4095: which I ſhould divide by one leſſe then I did multiply by, but ſeeing it is 1, I need not to divide it: for 1, (as I have ſaid before) doth neither multiply nor divide, therefore I take that ſumme 4095 for the whole ſumme of the ſhillings, which by Reduction amounteth to 304 pound, 15 ſhillings, and ſo much hath the Mercer for his twelve yards of Satten: which is 17 pound, 1 ſhilling, 3 pence a yard. But I think you will buy none ſo dear.

Scholar. No Sir, by the grace of God this yeer.

A queſtion of an horſe.

Maſter. *Then what ſay you to this* Queſtion? *If I ſold unto you an horſe having 4 ſhoes, and in every ſhoe 6 nails, with this condition, that you ſhall pay for the firſt nayl one* ob. *for the ſecond nayl two* ob. *for the third nayl four* ob. *and ſo forth, doubling untill the end of all the nails.*

Now I aſk you, how much would the price of the horſe come unto?

Scholar. Firſt, to know the number of the nails, I muſt multiply 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nails every one in order from 1 to 24, and againſt each number of the nails the ſumme of half pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next Figure following appeareth.

Then doe I reſort to the Rule of ſumming up the Progreſſion, where I conſider that the increaſe of this ſumme proceedeth by the Multiplication of 2, as the laſt Example did. And therefore multiplying the laſt ſumme by 2 alſo, and it yeeldeth 16777216, from which I abate the firſt Number which is 1, and then reſteth 16777215, which I ſhould divide by one leſſe then I

did

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2058	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24
16777216	

did multiply: but ſeeing that it is 1, I need not to divide it, for 1 (as you have before ſaid) doth neither multiply nor divide, therefore I do take the number 16777215 for the whole ſumme of the half pence, which by Reduction I finde to be 699050 ſhillings and 7 pence ob. that is 34952 pounds, 10 ſhillings, 7 pence, ob.

Maſter. That is well done, but I think you will buy no horſe of the price.

Scholar. No Sir, if I be wiſe.

Maſter. Well then, anſwer me to this Queſtion: *A Lord delivered to a Bricklayer a certain number of loads of Brick, whereof he willed him to make 12 Walls, of ſuch ſort, that the firſt Wall ſhould receive 2 thirdels of the whole number, and the ſecond 2 thirdels of that which was left; and ſo every other 2 thirdels of that that remained; and ſo did the Bricklayer: and when the 12 Walls were made, there remained one load of Brick.*

Queſtion of Bricks.

Now I aſk you, how many load went to each wall, & how many load was in the whole?

Scholar. Why Sir, it is impoſſible for me to tell.

Maſter. Nay it is very eaſie, if you mark it well. Mark well that I ſaid, that every wall ſhould receive two thirdels of the ſumme that was left. Now take away two thirdels from any ſumme, and you muſt néeds grant that that which remaineth, is one thirdell of the ſumme laſt before, Example of 9, from which if you take two thirdels, there will remain 3, which is one thirdle of 9. Likewiſe from 3 bate two thirdels, and there will remain 1.

Scholar. This is true, and now I perceive the leaſt wall had but two load of brick.

Maſter. And by the ſame reaſon may you know how many load every wall had, according as this Figure following doth ſhew. and likewiſe what the whole ſumme of bricks was, for if you make 12 ſummes, multiplying by 3, ſtill from the laſt remainer, as you may ſée here on the left ſide of the Table, there will appear all the remainers of the whole wall: and if you multiply the laſt of thoſe 12 ſummes by 2 alſo, then will that be the ſumme of the loads which was delivered to the Bricklayer.

	1	12	2
The remainer after every wall.	3	11	6 *Loads due to each wall.*
	9	10	18
	27	9	54
	81	8	162
	243	7	486
	729	6	1458
	2187	5	4374
	6561	4	13122
	19683	3	39366
	59049	2	118098
	177147	1	354294

Summe of the loads delivered — 531440

Again if you double every Remainer, as you may ſée at the right ſide of this Table, thoſe numbers will ſhew the ſumme of loads that went to each wall, whereby you may perceive that each wall was thrée times ſo great as the nert leſſe.

Scholar. Lo now it appeareth eaſſe enough. Now ſurely I ſée that Arithmetick is a right excellent Art.

Maſter. You will ſay ſo when you know more of the uſe of it: For this is nothing in compariſon to other points that may bée wrought by it.

Scholar. Then I beſéech you ceaſe not to inſtruct me further in this wonderfull cunning,

The Golden Rule, or Rule of Proportion direct, called the *Rule of Three.*

Maſter.

The Rule of Proportion.

BY order of the Science (*as men have taught it*) *there ſhould follow next* the Extraction of Roots of Number, *which becauſe it is ſomewhat hard for you yet, I will let it paſſe for a while, and will teach you the feat of* the Rule of Proportion, *which for his excellency is called* the Golden Rule. Whoſe uſe is, by three numbers known to finde out any other unknown; which you deſire to know, as thus.

The Golden Rule.

Queſtion of boardings.

If you pay for your board for three moneths ſixteen ſhillings, *how much ſhall you pay for* eight moneths?

To know this and all ſuch like Queſtions, you ſhall conſider which two of your numbers be of one Denomination, and ſet thoſe two the one over the other, ſo that the undermoſt be it that the Queſtion is of: as in my Queſtion 3 and 8, be both of one Denomination, for they both be moneths; and becauſe 8 is the number that the Queſtion is asked of, I ſet the one over the other, and 8 undermoſt thus, with ſuch a crooked draught of lines. Then doe I ſet

3 Z
8

the

the other number which is 16, against 3 at the right side of the line, thus.

3 Z 16
8

And now to know my Question, this must I do: I must multiply the lowermost on the left side, by that on the right side, & the summe that amounteth, I must divide by the highest on the left side: or in plainer words, thus, I shall multiply the number of which the Question is asked (which is called the third number) by the number of another Denomination (which is called the second) and the summe that amounteth, must I divide by the summe of like Denomination (which is called the first) Then for the knowledge of this Question, I multiply 1 into 16, and there amounteth 128, which I divide by 3, & it yeelds 42 shillings, and 2 shillings remaineth, which I turn into pence, and they bee 24 pence, of which third part is 8 pence, so the third part of 128 shillings, is 42 shillings, 8 pence, which summe I write at the right hand of the Figure against 8 thus.

Note.

The third number.
The second number.
The first number.

3 Z 16 s.
8 Z 42 s. 8 d.

Hereby I know that if 3 moneths boarding doe come to 16 shillings, that 8 moneths boarding will come to 42 shillings 8 pence, and likewise of any other like Question.

But here must you mark, that the first number and the third be of one Denomination, and also the second and the fourth, for which you

you ſéek: or elſe be of ſuch Denominations, that you in working may bring them into one; As if a man ſhould aſk me this Queſtion:

Queſtion of expence

Twelve weeks journeying coſt me 14 French Crowns at 6 ſhillings the peece, how many pounds is that in one year? Here you ſée no two numbers of one Denomination, but yet in working you may turn them into like Denomination: as thus, turn the one year into 52 weeks, and the fourth ſumme will bée French Crowns, by the order of the working. Then to know this Queſtion multiply the third ſumme 52, by the ſecond 14, and the ſumme will bée 728: that divide by your firſt number 12, and the Quotient will be 60 Crowns, and 8 Crowns remaining: which if you turn into ſhillings, they will bée 48 ſhillings, which if you divide by your firſt number 12, the Quotient will bé 4, which ſignifieth 4 ſhillings; put thoſe 60 French Crowns, which make 18 pounds with the 4 ſhillings for the ſumme that anſwereth to the Queſtion, and it is the juſt expences of a year: And the work will bée thus.

12	Z	14
52		6048.

A generall Rule.

And take this evermore for a Generall Rule touching this whole Art, That the doubtfull or unknown number that you would be reſolved of, ſhall alwaies be ſet in the third place. Note alſo the firſt number & the third, muſt ever be of one nature & denomination, or elſe muſt in working be brought

to

to like Denomination, and then of neceſſity muſt the other number be in the ſecond place.

Remember alſo that the place of the firſt number is higheſt on the left ſide, and the place of the ſecond, right againſt it on the right ſide; the place of the third number is under the firſt, as by thoſe Examples you have ſeen.

Scholar. This I truſt I can doe.

Maſter. But and if the Queſtion be asked thus: *In 8 weeks I ſpend 40 ſhillings, how long will 105 ſhillings ſerve me?* Here you ſée that 8 weeks anſwers himſelf, & ſaith 40 ſhillings. But how long time 105 ſhillings will ſerve you know not. Therefore you ſhall ſet 105 in the third place, according as I told you even now. And the firſt place muſt alwaies bée of the ſame nature or Denomination that the third is of, which here is 40. Then muſt 8 néeds bée that other: Now multiply 105 by 1, and it will bée 840, which if you divide by 40, it will yéeld 21, which is the fourth number, and ſheweth how many weeks 105 ſhillings will ſerve if you ſpend 40 ſhillings in 8 weeks.

The Figure of this Queſtion is this: as if you ſhould ſay: if 40 ſhillings ſerve for 8 weeks: 105 will ſerve for 21 weeks.

Shillings.	*weeks.*
40	8
105	21

Other diverſities there bée of working by this Rule, but I had rather that you would learn this one well, then at the beginning

to

to trouble your minde with many formes of working, ſith this way can doe as much as all the other, and hereafter you ſhall learn the other more conſtantly.

Note. ¶ And for your further aid and inſtruction, to make you better acquainted with this Golden Rule, I have here proponed ſix Queſtions, and their Anſwers, which I think moſt convenient and meet to preferre the deſirous to perfect underſtanding. The firſt four are all branches of one Queſtion ſprung out of the beſt tree (for a young Learner to taſte of) that groweth in this *Ground of Arts*: for that no manner of Queſtion in the *Rule of Three* whatſoever it be, can be proponed, but it muſt be comprehended under the reaſon or ſtile of one of theſe four.

The Queſtions.

If 15 *Ells of Cloth coſt* 7 pound 10 ſhillings, *what comes* 27 *Ells to at that rate?* Anſwer; 13 pounds 10 ſhillings.

If 27 *Ells coſt* 13 pound 10 ſhillings, *what are* 15 *Ells worth?* Anſwer; 7 pound 10 ſhill.

If 27 *Ells coſt* 13 pound 10 ſhillings: *how many Ells ſhall I have for* 7 pound 10 ſhillings? Anſwer; 15 *Ells.*

If I ſell 15 *Ells for* 7 pound 10 ſhillings: *how many Ells are to be delivered for* 13 pound 10 ſhillings? Anſwer; 27 *Ells.*

If 8 *pound of any thing coſt* 16 ſhillings 6 pence:

pence: *what money is to be received for* 49 li.
Answer: 5 li. 1 s. ob. $\frac{1}{4}$.

If 4 pound *of any thing cost* 7 d. *what money will* 8765 pound *of that commodity cost?*
Answer: 155 li. 4 s. 3 d. q.

Of all which Questions, I omit the work of purpose, that you shall whet your wit thereby at convenient leisure, to clime each branch, and gather the fruit of them, and doe minde now, before we make an end of this Rule, to give you some instructions of the backer Rule of three, whose order is quite contrary to this that you have learned.

Scholar. I thank you heartily for the six Questions, which I will (God willing) practise at convenient times: I pray you proceed therefore to the Backer or Reverse Rule.

The Golden Rule, or Rule of Proportion Backward, or Reverſe.

Maſter.

Note this well.

IN the former Rule evermore look how much the third number is greater then the firſt, ſo much the fourth number is greater then the ſecond. And contrariwiſe: look how much the firſt ſumme is greater then the third (if it doe chance ſo) ſo much is the ſecond ſumme greater then the fourth.

The Backer or Reverſe Rule of Three.

But in this Rule, there is a contrary order, as this: That the greater the third ſumme is above the firſt, the leſſer the fourth ſumme is beneath the ſecond: and this Rule therefore you may call the Backer or Reverſe Rule, as in example.

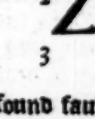
Queſtion of cloth.

If I have bought 30 yards of Cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards ſhould I need?

Scholar. Why, there is none ſo broad.

Maſter. I doe not care for that, I doe put this Example onely for your eaſie underſtanding: for if I ſhould put the Example in other meaſures, it would bee harder to underſtand. But now to the matter: If you would know this queſtion, ſet your numbers as you did

did before: but you ſhall multiply now the firſt number by the ſecond, and that ariſeth thereof, you ſhall divide by the third: which thing if you doe here, I mean if you multiply 30 by 2; it will bee 60: which ſumme if you divide by 3 there will appear 20, whereby I know, that if 30 yards of cloth of two yards broad, ſhould be lined with Canvas of three yards broad, 20 yards of Canvas would ſuffice, as this Figure ſheweth.



And now becauſe yée found fault with my Example, how ſay you, perceive you this?

Scholar. Yes Sir, I ſuppoſe.

Maſter. *Then anſwer me to this queſtion: How many Ells of Canvas of Ell breadth, will ſerve to line twenty Yards of Say, of three quarters broad?*

Scholar. In good faith Sir, I cannot tell; for I know not how to bring the ſummes to like Denominations.

Maſter. Then will I tell you, ſith there is mention here of quarters, and again every one of the meaſures both Ells & Yards may be parted into quarters, part them ſo both in the breadth and length, and then put forth the queſtion by quarters.

Scholar. Then ſhall I ſay thus: How many quarters of Canvas of 5 quarters broad will line 80 quarters of 3 quarters broad?

Maſter.

Maſter. Now anſwer to the Queſtion.

Scholar. Firſt, I will ſet them down in their forme thus: for 5 is ioyned with the Queſtion 6 is therefore the third number: then is 3 the number of the ſame Denomination, I mean becauſe they be both referred to breadth. Now I multiply 80 by 3, and it is 240, which I divide by 5, and it yeldeth 48. Then ſay I that 48 quarters of 5 quarters broad, will ſuffice to line 80 quarters of three quarters broad.

	Breadth.	Length.
	3	80
	5	

Maſter. Turn the quarters again into Ells and yards.

Scholar. Then I ſay, that 9 Ells and three quarters of a yard of ell-broad, will ſerve to line 20 yards of three quarters broad, as this Figure ſheweth.

	Breadth.	Length.
	3	80
	5	48

¶ Maſter. *Now what ſay you to this Queſtion? I lent my friend* 400 pound *for* 7 *moneths, how much money ought he to lend me again for* 12 *moneths to recompenſe my courteſie ſhewed him? can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2803, which I divide by 12, and it yeldeth

	Moneths.	Pounds.
	7	400
	12	

yeldeth 233 pound, and there is 4 pound remaining of my Diviſion, what ſhall I do therewith?

Maſter. Turn the ſame 4 pound into ſhillings, & then divide it by 12 as you did before.

Scholar. Well Sir, it ſhall be done: ſo have I 6 ſhillings for my quotient, and yet remaineth 8 ſhillings upon my Diviſion.

Maſter. You muſt alſo reduce that 8 ſhillings into pence, which maketh 96, and divide that alſo by your Diviſor.

Scholar. So have I done, and I finde 8 pence for my quotient, and nothing is left.

Maſter. This muſt you alwaies doe when any thing remaineth upon your Diviſion, whether it be money, weight, meaſure, or any kinde of thing whatſoever. This Rule is ſo profitable for all eſtates of man, that for this Rule only (if there were no more but it) all men were bound highly to eſteem Arithmetick.

By this rule may a Captain in warre, work many things, as Mr. Digges in his Stratiocos doth declare: Only now in this my ſimple addition for a taſte and incouragement, I will inlarge the Author with a queſtion or two more, wiſhing you and every my Countreymen or Gentlemen whatſoever, that by nature be any thing given to Military affairs, to be familiar & acquainted with this Excellent Art, the which he ſhall finde not only at the Sea, but alſo in the Camp and Field-ſervice, abounduntly to aid him, either in fortification, pay-

ing of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Inſtruments, whatſoever, as for example.

Queſtion of an Army. *If it ſhould chance a Captain which hath 40000 ſouldiers to be incloſed with his enemy, that he could have no freſh purveyance of victuals, and that the victuals which he had would ſerve that Army but only three moneths, how many men ſhould he diſmiſſe to make the victuall to ſuffice the reſidue 8 moneths.*

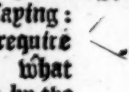
Scholar. As you taught me, I ſet the numbers thus, ſaying: If 3 moneths ſuffice 40000, to how many will 8 ſuffice?

To know this; I multiply the firſt number 3 into the ſecond 40000, and it yeldeth 120000, which ſumme I divide by 8, and there will be in the quotient 15000, which if I doe ſubtract from 40000, the remainder will declare that hee muſt diſmiſſe 15000 as this Figure ſheweth.

Aqueſtion of a Fort. Maſter. *Now anſwer me to this Queſtion: If 136 Maſons in a moneth be able to build a Fort to preſerve the Souldiers from the Enemie, and ſuch expedition requireth that I would have the ſame finiſhed in eight daies: how*

how many Work-men ſay you is there to be appointed?

Scholar. As you taught mée; I ſet the numbers thus, ſaying: If 28 daies requíre 136 Maſons, what number of men by the like proportion will 8 daies requíre?



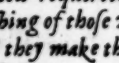
To know this, I multiply the firſt number 28 into 136, and it yéldeth mée 3808: which I divide by 8, and my Quotient is 476: which is the juſt number of Maſons that ſhall ſupply this work. And now me think theſe Queſtions are very eaſie.

Maſter. Truly if you tak: delectation herein, you ſhall finde this Art not only eaſie, but wonderfull pleaſant & profitable. Now therefor: one Queſtion mor: I will propoſe, and ſo leave off this Rule in whole numbers, untill we come to the uſe of it in broken numbers: for had you the underſtanding of broken numbers perfectly, not only in this Rule, but in all other, the Queſtion that in the ſight or appearance ſéemeth to bée 100 times more harder to abſolve, may thereby be wrought as ſoon, or ſooner then this.

Scholar. Your words doe greatly incourage me to be ſtudious to attain whole numbers: but might I once attain to bée a Practitioner in broken numbers, I ſhould think my ſelf happy.

Maſter. *What ſay you then to this Queſtion? If 48 Joyners in two daies make 200 light horſmen ſtaves (eſteeming they work but 12 hours a day) and ſuch need requireth that 384 Joyners are ſet to the finiſhing of thoſe 200 ſtaves; in what time ſay you, will they make them up?*

Scholar. I ſée here that I muſt turn my 2 daies into hours. And ſo doing, I ſet my numbers thus:



Saying, if 48 men are 24 hours, 384 men will make an end quickly. For it is grounded upon an old Proverb, Many hands make quick ſpeed.

I multiply 48 into 24, and it amounteth to 1159, which I divide by 384, and my quotient is 3 hours, which is my deſire.

Note. *I take this for a Note worthy the marking, either in the Rule of Three, forward or backward, when the two numbers are multiplied together, the Product is of the ſame nature and determination that the ſecond number is of.*

The double Rule of Proportion direct.

Maſter.

WEll, ſith you perceive now the uſe of this rule, I will ſhew other which inſue of the ſame, and firſt the *double Rule* which is ſo called, becauſe there is in it double working, by which thing only it differeth from this. The double Rule.

Scholar. Then by an example I ſhall underſtand it well enough.

Scholar. *So ſhall you, and let this be Example: If the carriage of* 100 *weight (that is* 112 *pound)* 30 *miles doe coſt* 12 *pence, how much will the carriage of* 500 *weight coſt being carried* 100 *miles?* Queſtion of carriage

Schol. I pray you ſhew me the working of it.

Maſter. You muſt make two workings of it: the firſt thus: If C. weight coſt 12 pence, how much will 500 weight coſt? Set your figures thus:

C. weight.	*Pence.*
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you divide by one, the Quotient will be ſtill 60, that is the price of 500 weight for 30 miles.

Then begin the ſecond work, ſaying: If

30 miles coſt 60 pence, how much will 100 miles coſt? Set your Figure thus.

Miles.	*Pence.*
30	60
100	

Then multiply 100 by 60 whereof amounteth 6000, which being divided by 30, will yeeld 200 pence. Then you may ſay, that ſo many pence ſhall coſt the carriage of 5000 pound weight 100 miles after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceive it alſo.

¶ *Maſter.* Theſe and ſuch other like Queſtions of the double Rule of Three, are to be anſwered much ſooner, at one onely working by the Rule of Proportion compoſed of five numbers, which anon I will ſhew you, and then when you have the uſe thereof, you may uſe it which way you think good.

Scholar. Sir, I thank you much for your courteſie: And I long now till this Rule be ended, that I may ſee how I may behave my ſelf with that new Rule of five numbers: for that I have ever ſince you taught me hitherto in the Golden Rule both forward and backward, wrought but with three numbers only.

Maſter. But yet a while we will go on forward with this Rule of Three, therefore anſwer to this Queſtion.

Queſtion of ſowing. Thirty buſhels of wheat ſowed, yeelded in one year 360, how many will 80 buſhels yeeld in ſeven years? I mean ſowing every year

year of thoſe ſeven, ſtill fourſcore buſhels?

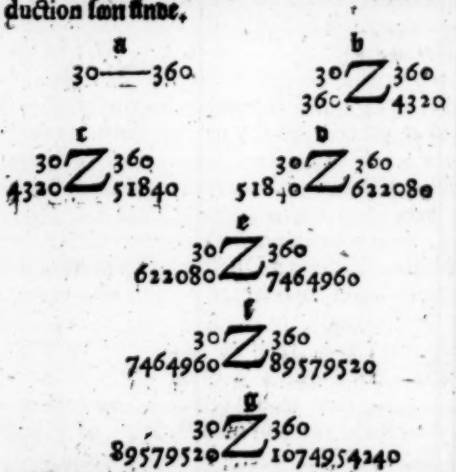
Scholar. First I ſay, that if 30 buſhels will yeeld 360 in one year, then 80 buſhels will yeeld 960 in one year. Then for the ſecond work, I ſay, If one year yeeld 960, then 7 years will yeeld 6720; as theſe two figures doe ſhew.

Seed.	*Encreaſe.*	*Year.*	*Encreaſe.*
30	360	1	960
80	960	7	6720

But now Sir, if I ſet forth 30 buſhels of Corn to another man for 7 years, agreeing ſo that he ſhall ſow every year the whole increaſe of the Corn, and I at the end of theſe 7 years to have the half of the whole increaſe: I would know how many buſhels will there amount to my part, ſuppoſing the increaſe to be after the rate of the laſt Queſtion, for 30 buſhels in one year to yeeld 360? Queſtion of Corn.

Maſter. In ſuch a Queſtion you muſt have ſo many ſeverall workings as there be years: As for Example, in the firſt year thirty buſhels yeeld 360. then to know the yeelding of the ſecond year, I muſt ſay, If thirty yeeld 360, how many yeeldeth 360? Work by your Rule, and you ſhall finde 4320. Then ſay for the third year. If thirty yeeld 360, How many will 4320 yeeld? you ſhall have 51840, and ſo every year multiplying the whole increaſe by 360, and dividing

it by 30, the increase of the next year will amount, as these 7 Figures following do orderly declare: where I have set 7 letters for the 7 years, of which the first is set without Art, because that is the increase which you doe presuppose: and the last number of each other doth shew the increase of that yeare that it standeth for, which the letters doe declare, so that the increase of the seventh yeare, is 1074954240 bushels: how many quarters that is, and also how many waies, you may by Reduction soon finde.



Question for knowing you: Now with one Question more I will prove you: *If 6 Mowers do mow 45 Acres in 5 daies, how many Mowers will mow 300 Acres in 6 daies?*

Scholar,

Scholar. If 45 Acres require 6 Mowers, then 300 Acres require 40. Now again, if 5 daies require 40 Mowers, then 6 daies need but 33 Mowers.

Maſter. Why dos you not make mention of the 2 that remaineth in the laſt Diviſion? for the laſt part of the Queſtion is wrought by the Backer Rule, where the firſt number 5 is multiplied into the ſecond, that is 40, whereof amounteth 200, which if you divide by the third number 6, the Quotient will be 33, as you ſaid; but then will there remain 2 which cannot well be divided into 6 parts: howbeit you may underſtand by the 6 part of 2, the third part of one mans work, which you muſt put to the 33; or elſe you muſt ſay that 33 Workmen will end all the 300 Acres in 6 daies ſave 2 mens work for one day, or two daies work for one man. But ſuch broken numbers called Fractions, you ſhall hereafter more better perceive, when I ſhall wholly inſtruct you of them.

Maſter. *Yet one Queſtion more of field matters I will propone, and ſo I will make an end of this double Rule of Three.*

Scholar. With all my heart. Sir I thank you, and I will diſpatch it as ſoon as I can, becauſe I would fain ſee the order of the next Rule of 5 numbers.

Maſter. *If a Captain over a band of men did ſet 300 Pioners a work, which in 8 hours did caſt a trench of 200 Rods: I demand how many labourers* Queſtion of entrenchings.

Labourers will be able with a like trench in three hours, to intrench a Camp of 3400 Rods.

Scholar. I think I am now in the Backhouse ditch: for I know not well which way to go about it. And besides that, truly I think I shall never come to preferment that way, my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and service into the favour of your Prince, for the avail of your Countrey.

Example for Navigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an adventure for the honour of his PRINCE and COUNTREY, as ever Englishman did.

Scholar. Sir, I thank you for your good incouragement. My minde, though I be little, is as desirous of knowledge, as any other: I have pondred now a little of it, and thus I set forth the work.

Rod.	*Men.*
200	300
3400	Z

Saying, if 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yeeldeth 1020000, which I divide by 200, and my quotient is 5100 men.

Then must I say for the second work, if in 8 hours 5100 men, be able to discharge it, how many shall perform the same in three houres? Now if I would worke by the Golden

Golden Rule of Proportion forward, I should finde a lesse number of men: because 3 hours is lesse then 8 hours: but because reason teacheth me, that the lesser the time is, wherein the trench must be made, the more Labourers I ought to have, thereupon I use now the Backer Rule, as in example. And I have in my Quotient 13600. So many Pioners must I have to intrench the Camp in 3 hours.

Master. You have answered the Question very artificially: And truly I commend you for your diligence and apt understanding: and now according to my promise, I will (in whole numbers) give you a little taste of the Rule of Proportion, compounded in 5 numbers.

The Rule of Proportion composed of 5 Numbers.

The first part of the Rule of Proportion compound, & direct.

THe Rule of Proportion compoſed, is diſtinct for moſt needfull queſtions, into ſeverall parts or workings: And there belongeth unto it alwaies five numbers, whereof in this Rule being the firſt part, the ſecond number and the fift, are alwaies of one nature and like denomination, which Rule is to be wrought thus: you muſt multiply the firſt number by the ſecond, and that ſhall be your diviſor: Then again, multiply the other three numbers, the one by the other, and their Product ſhall be your dividend.

And now according to my promiſe, we will firſt work the Queſtion of weight & carriage, which I delivered you in the double Rule of Three, to be abſolved by this Rule, which was this.

If the carriage of 1 *C. weight*, 30 *miles coſt* 12 *pence, what will the carriage of* 5 *C. weight ſtand me in, being carried* 100 *miles.*

C. weight. 1 — *Miles.* 30 — *Pence.* 12 — *C. weight.* 5 — *Miles.* 100

Now mark well how theſe five numbers ſtand: Then multiply the firſt number by the ſecond, as 30 by 1, which maketh but

30,

30. that number keep for your Divisor. Then multiply the other 3 numbers, the one into the other: that is to wit, 12 by 5, which maketh 60. Lastly 60 by 100, which as you see here in our Tables, ariseth to 6000, which 6000 you shall divide by the Product of the two first numbers, which here is 30. And you see there is found 200 pence. which is the duty that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the conclusion of the double Rule of Three.

Scholar. Sir I thank you, it is even so.

Master. *Yet note this in a generality in this Rule, look what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwaies your quotient.* Note this.

Scholar. Well now and if it please you, by your patience I will see how I can end the Question next following of 30 bushels of wheat sowed, which in 1 year yeelds 360, how many then will 80 bushels yeeld in 7 year following every year of those 7 still 80 bushels, and according to your reasons I set my numbers thus.

Bush. Year. Bush. Bush. Year.

30 — 1 — 360 — 80 — 7

80

28800

7

201600

When

Where I multiply 30 by 1, and it maketh 30 my Diviſor; then multiplying the other 3 numbers the one into the other, as here appeareth in my Tables, they make 201600, which I divide by 30: and my Quotient is 6720 buſhels, my deſire, for ſo much alſo it came to at two workings by the Rule of Three.

Maſter. Yet one Queſtion more I will propound unto you, and ſo leave this Rule, till it pleaſe God hereafter, that I may make you work it in broken numbers.

Queſtion of Intereſt

What comes the intereſt of 258 pound, *for* 5 *moneths to, after the rate of* 8 pound *taken in the* 100 pound *for* 12 *moneths?*

Scholar. Sir, this is yet within the compaſſe of ſome reaſonable uſance. Therefore to miniſter equity in this caſe, I will ſee how I can work the ſame, which I ſet down thus, praying you if I have not done well, to ſhew mee mine errour.

li. *moneths*, li. li. *men.*
100—12—8--258--5.

Maſter. Proceed, you have done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeldeth 1200, and the three other numbers multiplied together produce 10320, which I divide by 1200: and my Quotient is eight pounds. Then according as you have taught mee heretofore, I turn the 720 pound that I left, into ſhillings: and dividing it by the firſt number my Quotient is 12 ſhillings,

lings. So I answer, that the loan of 258 pounds for 5 moneths, after the rate of 8 pound in the 100 pound for a year, comes to 8 pound 12 shillings.

Master. You say true, I commend your diligence: now behold the manner of the second part of this Rule.

The

The backer Rule, or the ſecond part of the Rule of Proportion compound.

Maſter.

IN *the ſecond part of this* Rule of Proportion *compoſed, the third number is like unto the firſt. And the Rule is to be wrought thus: you ſhall now, contrary to the laſt Rule, multiply the third number and the fourth together, and that Product ſhall be your Diviſor. Then multiply the fift by the ſecond, and the Product thereof by the firſt: and that is the number that ſhall be divided.* For example, I propound this Queſtion, for a proof of my laſt Queſtion of Intereſt.

The proof of the laſt Queſtion. *A Merchant hath received* 8 pound 12 ſhillings, *for intereſt of certain money for* 5 *moneths term, which he received after the rate of Eight* pound *in the* 100. *for a year. The Queſtion is now, how much money was delivered to raiſe this intereſt?*

Behold therefore the manner, how the Queſtion is ſet forth.

li.	*moneths.*	li.	*moneths.*	li.	s.
100	12	8	5	8	12

Scholar. Sir I perceive it very well: and according

according to the doctrine which you prescribed for the working thereof: if please you now it is set down, I think I can follow the work.

Master. Nay, stay a while, and before you work, mark well how I deliver a reason for the perfect understanding of this Rule, which is thus:

If 8 *pound in* 12 *moneths doe yeeld me* 100 *pound, to take* 8 *pound* 12 *shillings for five months, must needs yeeld a great deal more.* Note.

So upon the knowledge that I have in this Art, The first part of this Rule is answerable to the Rule of Three forward: and this latter part accordeth to the Rule of Three backward.

Scholar. Sir, I yeeld you most hearty thanks for these your last instructions, they have given me great light into these two rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Master. You say well, go to now if you will, and trie your cunning in the Question: But this note take with you by the way, in as much as here is mention made of shillings: turn all your money as you work into shillings, for your more ease in working. Note.

Scholar. If it please you to behold me a little, I will quickly end it: for I have but my first, my second, and my last number to be multiplied together for my Dividend: And my third

into my fourth for my Diviſor.

li.	*Moneths.*	*Moneths.*	li.	s.
100	12	8 — 5	8	12
20		20	20	
2000		160	172	
12		5		
4000		800		
2000				
24000				
172				

48000 4128000 (
168000 8 00
24000
4128000

Which 4128000 I divide by 800, and my Quotient is 5160 ſhillings, which in pounds reſolveth 258, my deſire.

Maſter. I will here for this time in whole numbers end this Rule, and I will inſtruct you in the Rules of Fellowſhip. You may at your convenient leiſure for your exerciſe work the ſame by the Rule of Three at twice. And for your aid and incouragement therein, I ſet down here a proffer how to apply it.

A			B	
Moneths.	li. s.		*Pound.*	li.
5	8—12		8	100
12	412 $\frac{4}{5}$.		412 $\frac{4}{5}$	258 li.

The Rule of Fellowſhip.

The Rule of Fellowſhip without time.

BUt now will I ſhew you of the Rule of Fellowſhip or Company, which hath ſundry operations according to the divers number of the Company. This Rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſt I will ſpeak of that without difference of time, of which let this be an Example.

A queſtion of company.

Four Merchants of one Company made a bank of money diverſly: for the firſt laid in 30 pound, *the ſecond* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which ſtock they occupy ſo long, till it was increaſed to* 3000 pound. *Now I demand of you what ſhould each receive at the parting of this money.*

Scholar. I perceive that this Rule is like the other, but yet there is a difference which I perceive not.

Maſter. Then will I ſhew it to you: Firſt by Addition, you ſhall bring all the particular ſummes of the Merchants into one ſumme, which ſhall be the firſt ſumme in your working by the Golden Rule, and the whole ſumme of the gaines by that ſtock ſhall bee the ſecond ſumme. Now for the third ſumme you

ſhall

shall set the portion of each man one after another, and then work by the Golden Rule, and the fourth summe will shew you each mens gains: as in Example:

30
50
60
100

240

The parcels of the four Merchants make in one summe 240 pounds: set that in the first place, the gains in the second, and the first mans portion of stock in the third place, thus:

240 Z 3000
30

Now multiply the second by the third, and it will bee 90000, which you shall divide by 240, and there will appear 375 pounds, thus: And that is the gains for the first man.

240 Z 3000
30 375

Now for the second man, set the 50 pound that he brought, in the third place, and work as before: and his part will be 625 pounds, as this Figure sheweth.

Likewise for the third man, set his money which was 60 pounds, and his part of gains will be 750 pounds, as here appeareth.

240 Z 3000
50 624

And so for the fourth man; if you set his sum which is 100 pound, his gaines will bee 1250 pound, as the worke will declare.

240 - 3000
60 - 750

Scholar. This I perceive: but is there any way to examine whether I have well done or no?

240 - 3000
100 - 1250

Note this common proof.

Master. For the triall hereof, adde together all their four portions, and if their addition make the whole summe of their gains, then is the worke well done.

Scholar. That will I trie by and by; the four parcels are those, which added together make 3000, which is the just summe of money that they gained, whereby I know the worke is well done.

	375
	625
	750
	1250
	3000

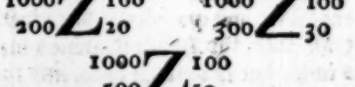
Master. Well, now another Example will I put to you, not of gains, but of losse: for one reason serveth for both.

A question of losse.

If three Merchants in one ship, and of one fellowship, had bought Merchandise so that the first had laid out 200 pound, *the second* 300 pound, *the third* 500 pound, *and it chanced by tempest that they did cast over board into the Sea Merchandise of the value of* 100 pound, *how much should each man bear in this losse?*

Scholar. If I shall doe in this as you did in the

the other Queſtion, then muſt I joyn their 3 portions together, 200,300,500, which maketh 1000. Then ſay I, If 1000 loſe 100 then ſhall 200 loſe 20, & 300 ſhall loſe 30, and 500 ſhall loſe 50, as by theſe three figures it doth appear plain.



Maſter. Well, ſith now you have done theſe, I will propound a Queſtion of more importance, which ſhall make you not only the abler to underſtand this Rule, but alſo it will greatly aid you in the next Rule of Fellowſhip with time, if ſuch need be that your money be of diverſ Denominations.

For this may not be forgotten in all ſuch Queſtions: If the number be of divers kindes, you muſt by reduction bring it into one kinde, that is to ſay, to the leaſt value that is named in the Queſtion. And likewiſe ſhall you do, if the time be of divers kindes, as ſome years, ſome moneths, weeks, & daies, you ſhall make all moneths, weeks, or daies, according as the leaſt name of time in the Queſtion is, as for Example.

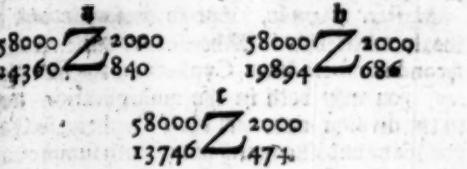
Firſt in diverſity of money: *Three companions bought* 2000 ſheep, *and paid for them* 241 pound 13 ſhillings 4 pence, *of which ſumme one paid* 101 pound, A Queſtion of ſheep.

pound 10 ſhillings. The ſecond 82 pound 17 ſhillings 10 pence, And the third paid 57 pound, 5 ſhillings 6 pence: How many ſheep muſt each of them have? Anſwer. *The firſt ſhall have 840. The ſecond 686. And the third 474. And that muſt you work thus:*

Solution. First, conſidering that your money is of diuers Denominations, you ſhall (by reduction) bring it all into the ſmalleſt Denomination which is in it, that is to ſay, pence, and ſo will the totall ſumme be 58000 pence.

Now if you turn each mans money into pence alſo, the firſt mans ſumme will be 24360 pence: The ſecond mans money will be 19894 pence: And the third mans money will bee 13746 pence.

Now to know how many ſheep euery man ſhall haue, let the whole ſumme of money, that is, 58000 pence be ſet in the firſt place, and in the ſecond place ſet the number of ſheep, and then orderly in the third place ſet each mans money, and then multiplying the third and the ſecond ſummes together, and diuiding that that amounteth by the firſt, there will appear the number of ſheep that each man ought to haue: as theſe 3 figures doe ſhew.



Scholar. Why doe you ſet the money in the firſt place, ſeeing in the Queſtion you ſay 2000 ſheep coſt 58000 pence, and not thus: 58000 coſt 2000 ſheep?

Maſter. You remember I taught you at the beginning of the Golden Rule, that the firſt and third numbers muſt be of one name, and of like things: and evermore the number that the Queſtion is asked of, muſt be ſet in the third place.

Now is the queſtion plainly this: If four men bought 2000 ſheep for 58000 pence, how many ſheep ſhall each man have?

But ſeeing in this Queſtion, there ought more reſpect to be had to the ſumme of money, then to the ſumme of the perſons; (for in the ſumme of money is their proportion toward the ſheep, and not in the number of perſons.)

If 58000 *pence bought* 2000 *ſheep, how many did* 24360 *buy? Again, how many did* 19894 *pence buy? And how many bought* 13746 *pence?*

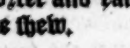
Scholar. I perceive it reaſonably, and ſo ſhall

shall I doe in all Questions.

Master. Even so. But for easinesse of the worke, mark this: Whensoever the first and second numbers have Cyphers in the first places, you may both in the multiplication and in the division leave out those Cyphers, so that you leave out like many out of both summes, as in this Question, the first number 58000 hath 3 Cyphers, and so hath the second, that is 2000: therefore cast away their Cyphers, and so will the first number be 58, and the second 2: set them in their places, and worke according to the Rule, and you shall perceive that will be all one, saving that this is the shorter and easier way, as these three figures doe shew.

Note.

a



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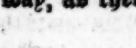


c

And this you see is both easier, and also the more certain way to know the Answer to this Question.

Scholar. Truly it is as you say: But Sir me seemeth I might ask a further Question here, not onely how many sheep each man should have, but also what every sheep cost.

Master. That Question doth not only be-

long

long to this Rule, but may also be discussed by Division, especially if the Questions number be one onely, as thus: Divide the totall summe 58000 pence by 2000 (or 58 by 2) omitting the cyphers, and the Quotient will be 29 pence, that is 2 shillings 5 pence. Howbeit, by this Rule you may doe it, and best when the number of the Question doth exceed 1; as if I should ask this Question, 2000 sheep cost 58000 pence, how much doe 20 cost? Then shall I set my figures as before.

2000 Z 58000
20

And doing after the Rule, there will amount 580 pence, that is, 2 li. 8 s. 4 d. the price of one score: but if you will use that easie way that I did teach you now, you may change the first and second number thus.

2 Z 58
20

Thus doe you perceive the use of the Rule without time.

Scholar. All this I understand very well: I pray you now instruct mee in the Rule of Fellowship with Time.

The Rule of Fellowſhip with Time.

Maſter.

The Rule of Fellowſhip with Time.

TO the intent you may as well perceive the ſame Rule with diverſity of time, I propoſe this example,

Four Merchants made a common ſtock, which at the yeares end was increaſed to 35145 pound. *Now to know what ſhall be each mans portion of gain, you muſt know each mans ſtock, and time of continuance.*

Queſtion of a Bank.

The firſt man of theſe four laid in 669 li. which he did take from the ſtock again at the end of 10 moneths. The ſecond man laid in 810 pound for 8 moneths. The third laid in 900 pound, for 7 moneths. And the fourth laid in 1040, for 12 moneths.

Note. A generall Rule.

This Queſtion ſhall you examine as you did the other before, ſaving that whereas in the third place of the Figure you did ſet each mans ſumme alone, here you ſhall ſet the ſame being multiplied by the number of their time: and likewiſe in the firſt place of the Figure you ſhall ſet the number which amounteth of their whole ſummes ſo multiplied by their time, and added into one whole ſumme, as thus.

The

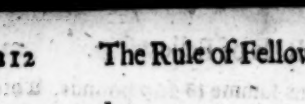
The first mans summe is 669 pounds, which I multiply by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 pound, multiplied by 8 (which was his time) maketh 6480. The third mans summe 900 pound, multiplied by 7 (for that was his time) yeldeth 6300. The fourth mans summe was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The four summes thus multiplied by their time, must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all four, which is 31950; and the gain must be in the second place, which is 35145. Now to end the Question, I say first, If 31950 did get 35145, what did 6690 get? Answer, 7359 pounds, as by this figure appeareth.

a

31950 Z 35145
6690 Z 7359

Likewise, the second man had to his part 7128 pound, the third must have 6930 pound, and the fourth man shall have for his part 13728 pound, as these figures doe partly declare.



Scholar. This I like very well: but what proof is there of this work?

Another proof.

Maſter. The ſame that I taught you for the other: howbeit, there is uſed both for this work and the other alſo, this manner of proof, to adde all the portions together, and if it may agree to the whole ſumme, then ſeemeth your work well done: But this is no ſure proof.

Scholar. Yet will I proove in this example: The foure parcels are theſe, which if I adde together, there will amount 35145, and that was the whole ſumme, whereby I perceive the work is well done.

	7359
	7128
	6930
	13728
	35145

Maſter. If it fall out otherwiſe, be ſure it is not well.

Scholar. Then doe I underſtand this work alſo very well: But what have I now to learn?

Maſter. There are many other excellent parts

parts behinde, of which I will not as now make mention, becauſe that without the knowledge of Fractions they cannot be duly taught, and much leſſe underſtood: Therefore will I propoſe to you two or three Queſtions more (that thereby you may better perceive the uſe of this Rule, & all other the like) and ſo make an end for this Time.

¶ *Three Partners by ſome ill adventure ſuſtained the loſſe of* 160 pound, *whereof the firſt laid into the common ſtock* 200 pound *for* 10 *months: the ſecond laid in* 350 pounds, *and the third* 100 pound, *but for how long the two latter, is unknown: But breaking off their Partnerſhip, the firſt found himſelf a loſer* 80 pound, *the ſecond* 56 pound, *and the third* 24 pound. *The Queſtion is, for how long time was the money of the two latter in company?* Aqueſtion of loſſe.

For the ſolution hereof, and of ſuch other like, you muſt alſo multiply the firſt mans 200 pound, that he put into the ſtock by his time of continuance, which was 10 moneths; and it maketh 2000: wherefore now I affirm, if his money that loſt 80 pound multiplied by his time make 2000: what ſhall his money make that loſt 56 pound, and his that loſt 24 pound? which two numbers I commit to the triall of the Rule of Three, at two workings, thus:

If 80 give 2000, what giveth 56? And again, if 80 give 2000, what giveth 24?

80 Z 2000 | 80 Z 2000
56 Z 1400 | 24 Z 600

To conclude, if you now divide 1400, the second mans portion, by 350, which was his stock that he laid into company, you shall find in your Quotient 4 moneths, and so for so long time did the second man put his money into the common stock.

Lastly, if you divide the third mans new laying in, which was 600 by 100, which was his stock that he put into the company, the Quotient declareth his time of continuance, which was six moneths. And thus is the Question resolved.

Scholar. Sir, I have attentively beheld your working, and the more we travell herein, the more me think I am in love with this excellent Art.

Master. Then what say you to this Question?

A question of Canons: *There is in a Cathedrall Church 20 Canons, and 30 Vicars, those may spend by year 2600 li. but every Canon must have to his part 5 times so much as every Vicar hath: how much is every mans portion say you?*

Scholar. I pray you make the answer your self also, so shall I perceive best the means to answer to such other like.

Master. In this Question you must doe as in those before-said, that have diversity of time,

time, for here is diverſity of portions. Therefore ſhall you multiply the number of the perſons by their difference of portions: (as you did in the other by time:) Then muſt you multiply the 20, (which is the number of Canons) by 5, (for that is the number of their portion) ſo will it be 100. Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put theſe two ſummes together, and they make 130. Then ſay thus; If 130 ſpend 2600 pounds, what may 100 ſpend? The Rule heweth 1000 pounds.

Again for Vicars: if 130 ſpend 2600 pound, what may 30 ſpend? Anſwer 600 pound, as theſe figures ſhew.

130 Z 2600	130 Z 3600
100 Z 2000	30 Z 600

But if every Canon ſhould have ſo often times 4 pound as the Vicar ſhould have 3 pound, then ſhould I multiply 20 by 4, (that were 80) and 30 by 3, (that were 90) and then both were 170. Then ſhould the figures be ſet as followeth.

li. s. d.	li. s. d.
170 Z 2600	270 Z 26000
80 Z 1223—10—7	90 Z 1376—9—5

But this ſort is too hard for you, by rea-

ſon of the fractions, therefore I will let it reſt to that place.

And by this Rule you ſée what the 20 Canons may ſpend; which ſumme if you divide by 20, you ſhall ſée each Canons proportion: and ſo of the Vicars, if you divide their ſumme by 30, the quotient will declare every Vicars portion.

The

The second Dialogue.

The accounting by Counters.

Master.

Now that you have learned the kinde of Arithmetick *with the Pen, you shall see the same Art in* Counters: *which feat doth not only serve for them that cannot write and read, but also for them that can doe both; but have not at some time their Pen or Tablet ready with them.*

This sort is in two forms commonly, the one by lines, and the other without lines. In that that hath lines, the lines doe stand for the order of places: and in that that hath no lines, there must be set in their stead so many Counters as shall need for each line one; and they shall supply the stead of lines.

Scholar. By examples I should better perceive your meaning.

Master. For example of the lines; loe here you see six lines, which stand for six places, so that the

Numeration by Counters.

nethermoſt ſtandeth for the firſt place, and the next above is for the ſecond, and ſo upward till you come to the higheſt, which is the ſixt line, and ſtandeth for the ſixt place.

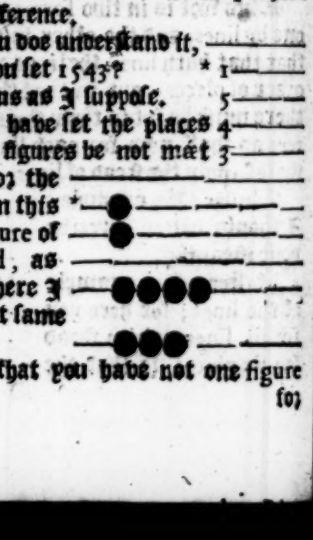
Now what is the value of every place or line, you may perceive by the Figure which I have ſet on them, which is according as you learned before in Numeration of figures by the Pen, for the firſt place is the place of Unites or ones, and every Counter ſet in that line, betokeneth but one: and the ſecond line is the place of 10, for every Counter there ſtandeth for 10: the third line the place of hundreds, the fourth of thouſands, and ſo forth.

Scholar. Sir, I doe perceive that the ſame order is here of lines, as was in the other figures by places, ſo that you ſhall not need longer to ſtand about Numeration, except there be any other difference.

Maſter. If you doe underſtand it, then how will you ſet 1543?

Scholar. Thus as I ſuppoſe.

Maſter. You have ſet the places truly, but your figures be not meet for this uſe: for the meeteſt figures in this behalf, is the figure of a Counter round, as you ſee here, where I have expreſſt that ſame ſumme.

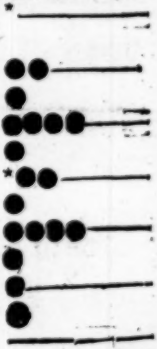


Scholar. So that you have not one figure for

for 2, nor 3, nor 4, and so forth, but as many digits as you have, so many counters you set in the lowest line, and for every 10 you set one in the second line, and so of other. But I know not by what reason you set that one Counter for 500 between two lines.

Master. You shall remember this, that whensoever you need to set down 5, 50, or 500, or 5000, or set forth any number whose Numerator is 5, you shall set one Counter for it in the next place above the line that it hath his denomination of: As in this example of that 500, because the Numerator is 5, it must be set in a void space, and because the denomination is a hundred, I know that the place is the void place next above hundreds, that is to say, above the third line.

And further you shall mark, that in all working by this sort, if you shall set down any summe between 4 and 10, for the first part of that number you shall set down 5, and then so many Counters more, as there rest numbers above 5. And this is true both of Digits and Articles. And for Example, I will set down this summe 297965, which summe if you mark well, you need none other examples for to learn the numeration of this form.



But this shall you mark, that as you did in other kinds of Arithmetick, set a prick in the places of thousands, in this work you shall set a Starre, as you see before.

Scholar. Then I perceive Numeration: But, I pray you, how shall I doe in this Art, to adde two summes or more together?

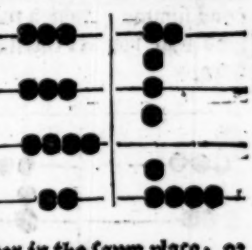
Addition.

Addition.

Maſter.

THe eaſieſt way in this is to adde but two ſummes at once together: Howbeit you may adde more, as I will tell you anon.

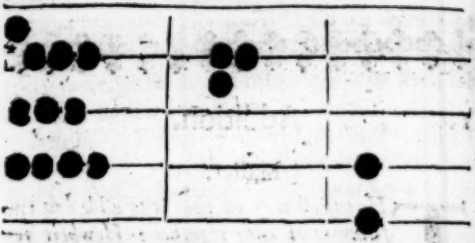
Therefore when you will adde two ſums, you ſhall firſt ſet down one of them, it forceth not which, and then by it draw a line croſſe the other lines. And afterward ſet down the other ſumme, ſo that the line may be between them: as if you would adde 2659 to 8342, you ſet your ſummes as you ſée here.



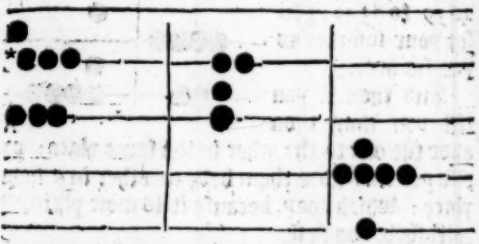
Addition of two ſummes.

And then if you liſt you may then adde the one to the other in the ſame place: or elſe you may adde them both together in a new place: which way, becauſe it is moſt plain, I will ſhew you firſt.

Therefore will I begin at the Unites, which in the firſt ſumme is but 2, and in the ſecond ſumme 9, that maketh 11: Thoſe doe I take up, and for them I ſet 11 in the new room, thus:



Then doe I take up all the Articles under a hundred, which in the first ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are four Articles of 10, and the ſecond ſumme 5, which maketh 9, but then take heed that you ſet them in their right lines, ſee here.



Where I have taken away 40 from the firſt ſumme, and 50, from the ſecond, and in their ſtead I have ſet 90 in the third room, which I have ſet plainly, that you might well

well perceiue: howbeit ſeeing that 90 with the 10 that was in the third room, already, doth make 100, I might better for thoſe 6 Counters, ſet 1 in the third line, thus:

For it is all in one ſumme, as you may ſee, but it is beſt never to ſet five Counters in any line, for that may be done with one Counter in a higher place.

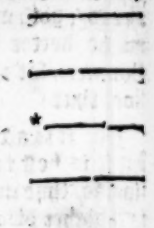
Scholar. I judge that good reaſon, for many are unredy̆ll where one will ſerue.

Maſter. Well, then will I adde forth of hundreds: I find 3 in the firſt ſumme, and 6 in the ſecond, which maketh 6000, them doe I take up and ſet in the third room, where is 100 already, to which I put 900, and it will be 1000: therefore I ſet one Counter in the fourth line for them all, as you ſee here.

Then adde I the Thouſands together, which in the firſt ſumme are 8000, and in the

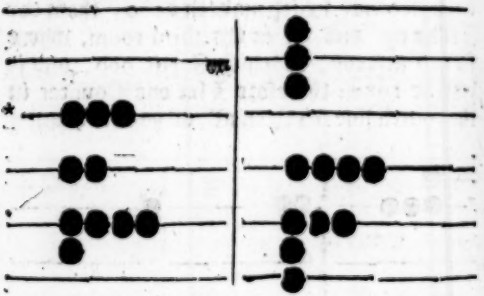
the ſecond 2000, that maketh 10000, them doe I take up for thoſe two places, and for them I ſet one Counter in the fifth line, and then it appeareth as you ſee to bée 11001, for ſo many doth amount of the Addition of 8342 to 2659.

To adde ſummes together.



Scholar. Sir, this I doe perceive: but how ſhall I ſet one ſumme to another, not changing them to a third place?

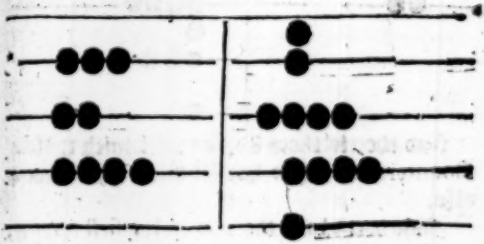
Maſter. Mark well how I doe it. I will adde together 65436 and 3245, which firſt I ſet down thus:



Then doe I begin with the ſmalleſt Denomination, which is 1 in the ſecond ſumme, and ſet it in his place: then doe I finde 5 in the firſt ſumme, and 5 in the ſecond, which put together, ſaving the two Counters, cannot

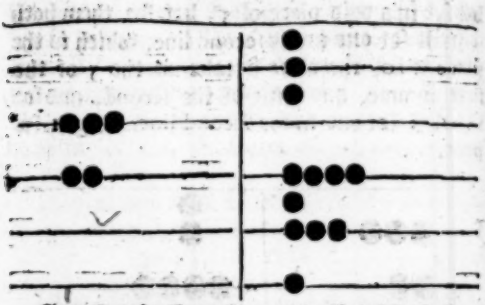
bée

be ſet in a void place of 5, but for them both I muſt ſet one in the ſecond line, which is the place of 10, therefore I take up the 5 of the firſt ſumme, and the 5 of the ſecond, and for them I ſet one in the ſecond line, as you ſee here.



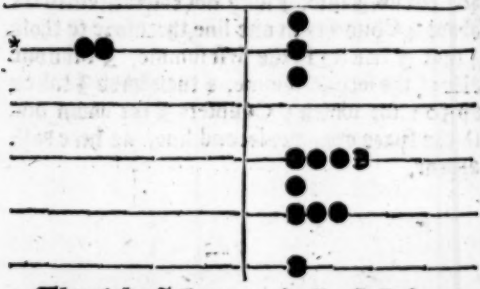
Then doe I likewiſe take the 4 Counters of the firſt ſumme and ſecond line, (which maketh 40) and adde them to the 4 Counters of the ſame line in the ſecond ſumme, and it maketh 80: but as I ſaid, I may not conveniently ſet above 4 Counters in one line, therefore to thoſe 4 that I took up in the firſt ſumme, I take one alſo of the ſecond ſumme, & then have I taken up 50: for which 5 Counters I ſet down one in the ſpace over the ſecond line, as here doth appear.

And



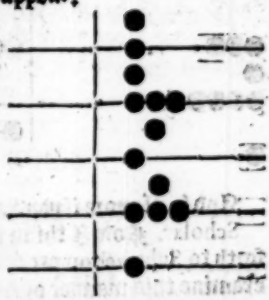
And then is there 80, as well with those 4 Counters, as if you had set down the other 4 also.

Now doe I take the 200 in the first summe, and adde them to the 400 in the second summe, and it maketh 600, therefore I take up the two Counters in the first summe, and three of them in the second summe, and for them 5, I set 1 in the space above, thus:



Then take I the 3000 in the first summe, unto

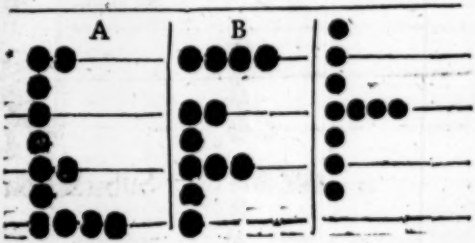
unto which there are none in the second summe agreeing, therefore I doe onely remove those three Counters from the first summe into the second, as here doth appear.



And you see the whole summe that amounteth of that Addition of 65436 with 3245, to bee 68681.

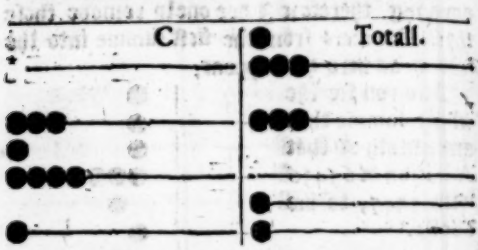
And if you have marked those two examples well, you need no further instruction in Addition of two only summes: but if you have more then two summes to adde, you may adde them thus:

First adde two of them, and then adde the third and fourth, or more, if there be so many: As if I would adde 2679, with 4286, & 1391. First I adde the two first summes thus:



And

And then I adde the third thereto thus:



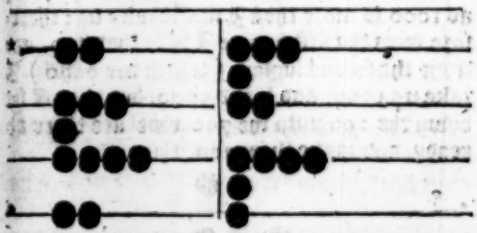
And so of more if you have them.

Scholar. Now I think it best that you passe forth to Subtraction except there be any way to examine this manner of Addition, then I think that were good to be known next.

Master. There is the same proof here that is in the other Addition by the Pen. I mean Subtraction; for that only is a sure way, but considering that Subtraction must be first known, I will first teach you the Art of Subtraction, and that by this Example.

Subtraction.

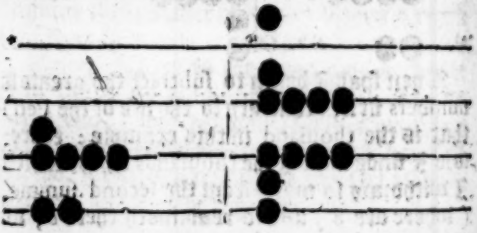
I *would ſubtract* 2892 *out of* 8746. *Theſe ſummes muſt I ſet down as I did in* Addition: *but here it is beſt to ſet the leſſer number firſt, thus:*



Then ſhall I begin to ſubtract the greateſt numbers firſt, (contrary to the uſe of the Pen) that is the thouſand in this example: therefore I finde among the thouſands 2, for which I withdraw ſo many from the ſecond ſumme, (where are 8) and ſo remaineth there 6, as this example ſheweth.

Then

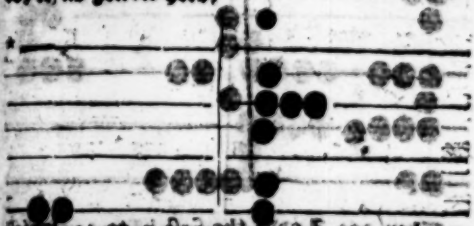
Then doe I likewise with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I cannot take 2, therefore this must I doe: I must lok how much my summe differeth from 10, which I finde here to be 2, then must I abate for my summe of 800, one thousand, and set down the excesse of hundreds, that is to say 2, for so much as 1000 is more then I should take up: therefore from the first summe I take that 800, and from the second summe (which are 6000) I take up 1000, and leave 5000, but then I set down the 200 unto the 700 that are there already, and make them 900, thus:



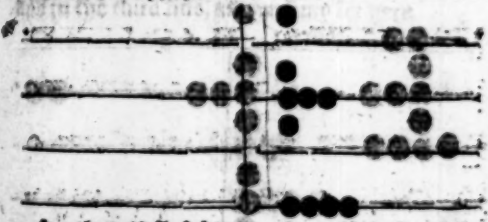
Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot bee abated from 40, I lok how much that 90 doth differ from the nert summe above it, that is, 100 (or else which is all to one effect) I lok how much 9 doth differ from 10, and I finde it to be 1 then in the stead of that 90, I doe take from the se-

cond

cond summe 100: but considering that is 10 to much, I set down on the next line beneath for it, as you se here.



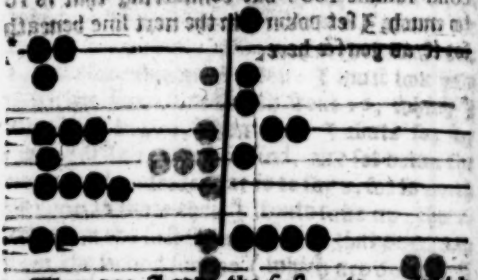
Giving that here I have set 1 Counter in the place in stead of 5 in the next line. And then I have subtracted all save 2, which I must abate from 6 in the second summe, and there will remain 4, thus:



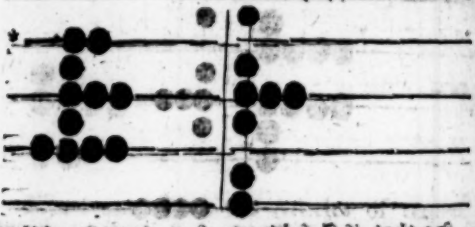
So that if I subtract 2892, from 8746 the remainer will be 5854.

And that this is truly wrought, you may prove by Addition: for if you adde to this remainer the same summe that you did subtract, then will the former sum 8746 amount again.

Scholar. That will I prove, and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus: A proof of Subtraction.

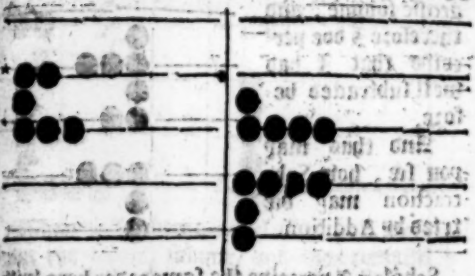


Then doe I adde the first 9. to 4. which maketh 6: so take I up 5 of those Counters, and in their stead I set 1 in the place, and one in the lowest line, as here appeareth.

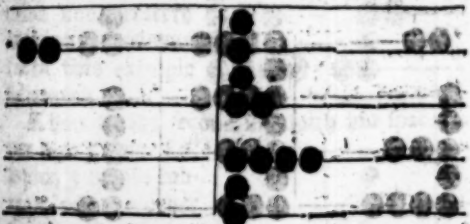


Then doe I adde the 90 next above to the 50 and it maketh 140. Wherefore I take up those 6. Counters, and for them I set 1. in the hundreds in the third line, and 4 in the second line thus:

Then



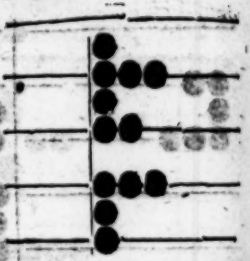
Then doe I come to the hundreds, of which I finde 8 in the first ſumme, & 8 in the ſecond, that maketh 1600, therefore I take up thoſe 8 Counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and in the third line, as you may ſee here.



Then is there left in the firſt ſumme but onely 200, and in the ſecond 1000, which is 7000, which I ſhall take up from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already; and there will the whole ſumme appear as you may well ſee, to bee 8746. which was the firſt

grosse summe, and therefore I doe perceive that I had well subtracted before.

And thus may you see, how Subtraction may bee tried by Addition.

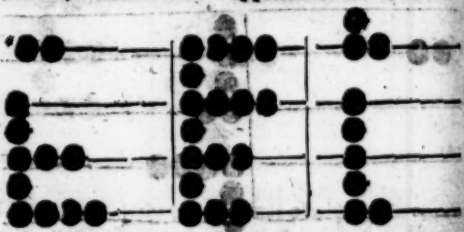


Scholar. I perceive the same order here with Counters, that I learned before in figures.

Master. Then let me see how you can trie Addition by Subtraction.

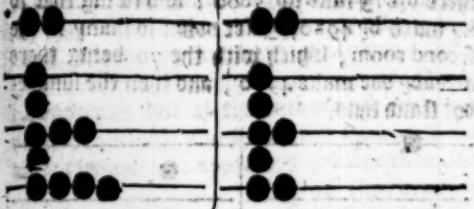
Scholar. First I will set forth this example of Addition, where I have added 2189 to 4988. And the whole summe appeareth to be 7177.

Proof of Addition by Subtraction.

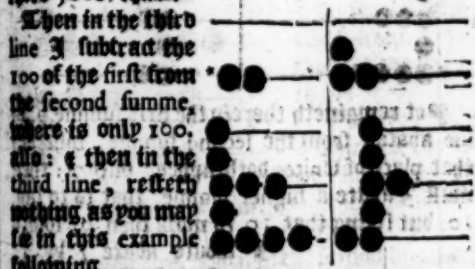


Now to trie whether that summe be well added or no. I will subtract one of the first two summes from the third. And if I have well done, the remainer will be like that other summe: as for example, I will subtract the first summe from the third, which I set thus in order.

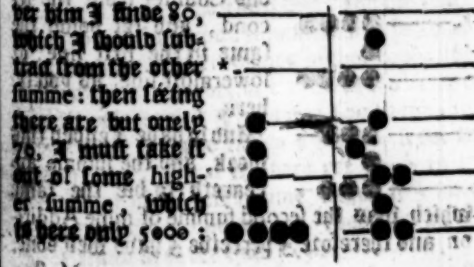
Then



Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, and then remaineth there 5000, thus:

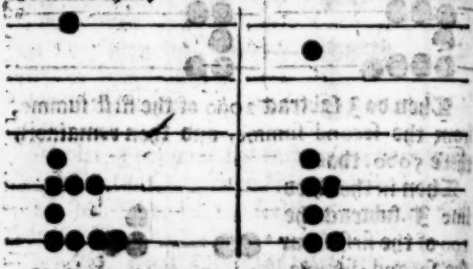


Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is only 100. alſo: & then in the third line, reſteth nothing, as you may ſe in this example following.



Then in the ſecond line with his ſpace over him I finde 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70. I muſt take it out of ſome higher ſumme, which is here only 5000:

therefore I take up 5000: and seeing that is too much by 4910, I set down so many in the second room, which with the 70 being there already doe make 4990, and then the summes doe stand thus.

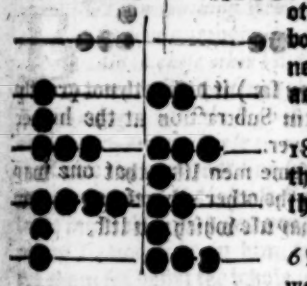


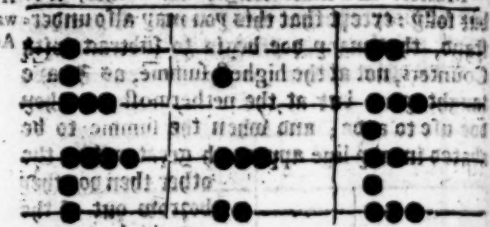
Yet remaineth therein the first summe 9, to be abated from the second summe, wherein that place of Unites doth appear only 7: then must I abate a higher summe, that is to say, 10, but seeing that 10 is more then 9 (which I should abate) by 1, therefore shall I take up one Counter from the second, and set down the same in the first line, or lowermost line, as you see here.

And so have I ended this worke, and the summe appeareth to be the same which was the second summe of mine Addition, and therefore I perceive I have well done.

Master.

Maſter. To ſtand longer about this, it is but folly: except that this you may alſo underſtand, that many doe begin to ſubtract with Counters, not at the higheſt ſumme, as I have taught you, but at the nethermoſt, as they doe uſe to adde; and when the ſumme to be abated in any line appeareth greater then the other, then doe they borrow out of the next higher room: as for example, I ſhould abate 1846 from 2378, they ſet the ſumms thus: First they take 6 which is the lower line, and his ſpace from 8 in the ſame rooms in the ſecond ſumme, and yet there remaineth two Counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the third line, and his ſpace, from 300 of the ſecond ſumme cannot be, therefore doe they abate it from a higher room, that is, from 1000. and becauſe 1000 is too much by 200 therefore muſt I ſet down 200 in the third line, after I have taken up 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdraw from the ſecond ſumme, then doe all Figures ſtand in order thus: 532. Another way of Addition.





So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Notwithstanding, as some men like that one way best, so some like the other: therefore you now knowing both, may use which you list.

Multi-

Multiplication.

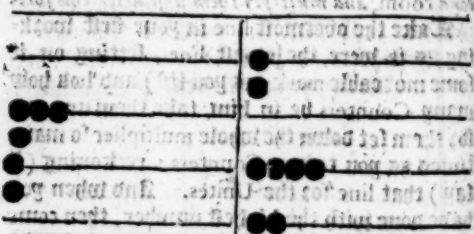
BUt now touching Multiplication: you shall set your numbers *into two* rooms *(as you did in those other kindes) but so that the multiplier be set in the first* room: *then shall you begin with the highest numbers of the second* room, *and multiply them first after this sort.*

Take the overmost line in your first working as it were the lowest line, setting on it some moveable mark (as you list) and look how many Counters be in him, take them up, and for them set down the whole multiplier so many times as you took up Counters: reckoning (I say) that line for the Unites. And when you have done with the highest number, then come to the next line beneath, and doe so even with it, and so with the next, till you have done all. And if there be any number in a space, then for it shall you take the multiplier five times, and then must you reckon that line for the Unites, which is next beneath that space. Or else after a shorter way yée shall take onely half the multiplier, but then shall you take the line next above the space for the line of Unites. But in each working, if by chance your multiplier bée an odde number, so that you

you cannot take the half of it justly, then must you take the greater half, and set down that, as if that it were the just half: and further, you shall set one Counter in the space between that line, which you reckon for the line of Unites, or else only remove forward the same that is to be multiplied.

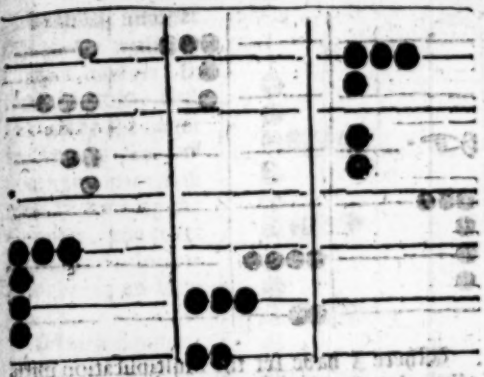
Scholar. If you set forth an example hereof, I think I shall perceive you.

Master. Take this example: I would multiply 1542 by 365, therefore I set my numbers thus.



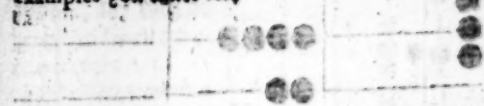
Then first I begin at the 1000 in the highest room, as if it were the first place, and I take it up setting down for it so often (that is once) the Multiplier which is 365, thus as you see here: where, for the one Counter taken up from the fourth line, I have set down other six which make the summe of the Multiplier, reckoning the fourth line, as if it were the first which thing I have marked by the Scarre set at the beginning.

Scholar.

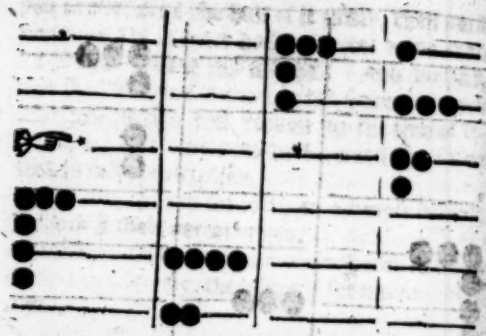


Scholar. I perceive well, for indeed this ſumme that you ſet down, is 365000. for ſo much doth amount of 1000, multiplied by 365.

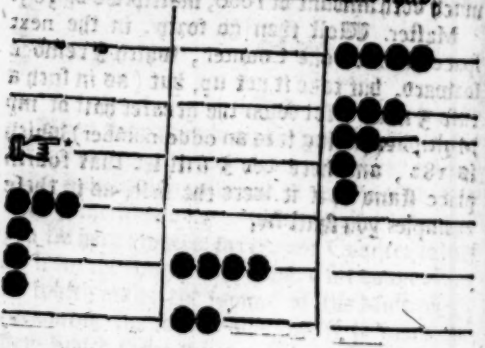
Maſter. Well then go forth, in the next ſpace I have one Counter, which I remove forward, but take it not up, but (as in ſuch a caſe I muſt) ſet down the greater half of my Multiplier (ſeeing it is an odde number) which is 182, and here dos I will let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſee.



Where



Where I have set the Multiplication with other, but for the ease of your understanding, I have set a little line between them. Now should they both in one summe stand thus,



How

Notwithstanding another forme to multiply such Counters in space, is this: first to remove the finger to the next line beneath the space, and then to take up the Counter, and to set down the Multiplier five times, as here you see.

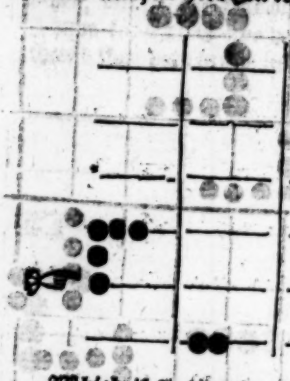
Another forme of Multiplication.

Which summes, if you doe adde together, into one somme, you shall perceive that it will be the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter, so this is plainer to reason, for such as have had small exercise in this Art.

Notwithstanding you may have them in your mind before you set them down: as in this example you might

might have said five times 300 is 1500, and five times 60 is 300, also five times five is 25, which all put together, doe make 1825, which you may at one time set down if you list.

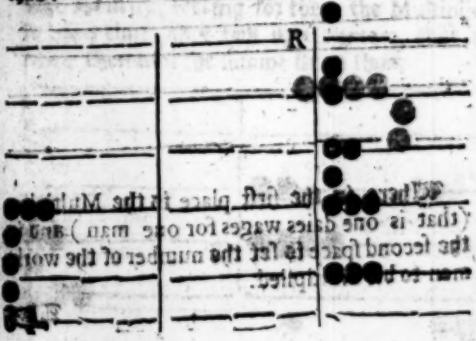
But now to go forth, I must remove the hand to the next Counters which are in the second line, and there must I take up those four Counters, setting down for them my multiplier four times severally, or else I may gather the whole summe in my minde first, and then set it down: as to say, four times 300 is 1200: four times 60 are 240: and four times 5 make 20, that is in all 1460: that shall I set down also, as here you see.



Which if I joyn in one summe with the former numbers, it will appear thus.

Then

Then to doo this Multiplication, I remoue the finger to the lowest line where are only 2, then doe I take up 9, and in their stead doe I set down twice 365, that is 730, for which I set one in the space aboue the third line for 700, and 30 more in the third line with that one that is there already, and the rest in their order, and so haue I well ended the whole summe thus:



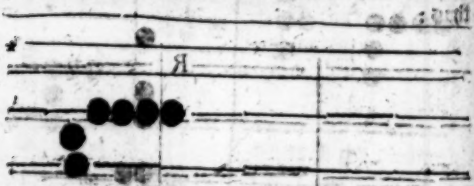
Whereby you ſee, that 1542 (which is the number of years ſince Chriſt his Incarnation) being multiplied by 365 (which is the number of the daies in 1 year) doth amount to 562830. which declareth the number of daies ſince Chriſts incarnation unto the end of 1542 years, beſides 385 daies, & 12 hours for leap years.

Example of wages.

Scholar. Now will I prove by another example, as this: 40 Laborers (after 6 pence the day for each man) have wrought 28 daies, I would know what their wages doth amount unto.

In this caſe muſt I work doubly: firſt I muſt multiply the number of the Laborers, by the wages of a man for one day, ſo will the charge of every day amount.

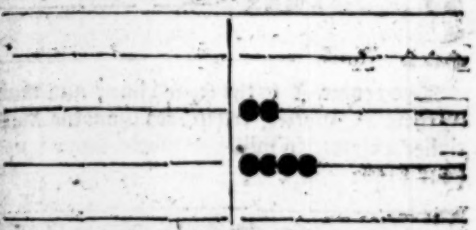
Then ſecondly, muſt I multiply the charge of one day by the whole number of daies, and ſo will the whole ſumme appear. Firſt therefore I ſhall ſet the ſummes thus:



Where in the firſt place is the Multiplier (that is one daies wages for one man) and in the ſecond ſpace is ſet the number of the workmen to be multiplied.

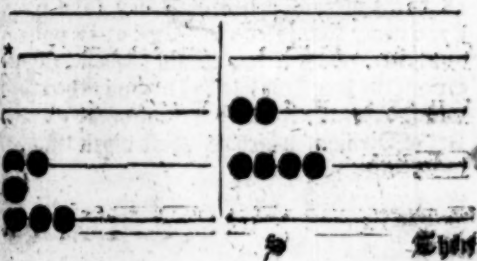
Then

Then ſay, If 6 times 4 (reckoning that ſecond line as the line of Unites) maketh 24, for which ſumme I ſhould ſet two Counters in the third line, and 4 in the ſecond, therefore doe I ſet 2 in the third line, and let the 4 ſtand ſtill in the ſecond line thus.

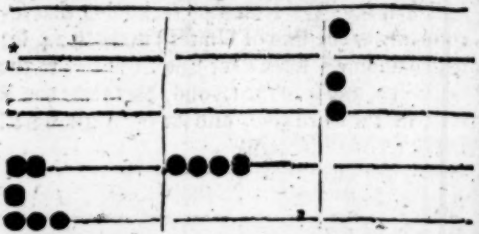


So appeareth the whole daies wages to be 240 pence, that is 20 ſhillings.

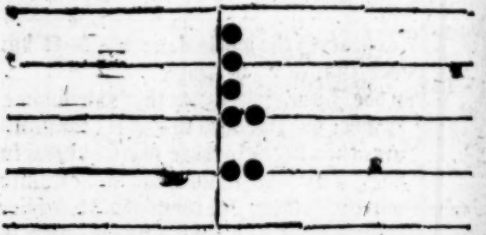
Then doe I multiply again the ſame ſumme by the number of daies, and firſt I ſet the numbers thus: then becauſe there are Counters in divers lines, I ſhall begin with the higheſt, and take them up, ſetting for them the Multiplier ſo many times as I took up Counters, that is twice, then will the ſumme ſtand thus.



So Then



Then come I to the second line, and take up those 4 Counters, setting for them the Multiplier 4 times, so will the whole summe appear thus:



So is the whole wages of 40 workmen for 28 dates after 6 pence each day for a man, 6720 pence, that is, 560 shillings, or 28 pound.

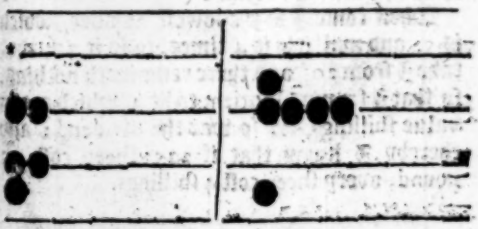
Master. Now if you would prove Multiplication, the surest way is by Division: therefore will I overpasse it till I have taught you the Art of Division, which you shall work thus.

Division.

Diviſion.

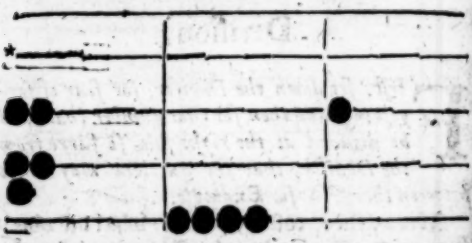
FIrſt, ſet down the Diviſor, for fear of forgetting, and then ſet that number that ſhall be divided at the right ſide, ſo farre from the Diviſor, that the quotient may be ſet between them: as for Example.

If 225 ſheep coſt 45 pound what vto every ſheep coſt? To know this, I would diuide the whole ſumme, that is 45 pound, by 225, but that cannot be: therefore muſt I firſt reduce that 45 pound, into a leſſer Denomination, as into ſhillings, then I multiply 44 by 20, and it is 900; that ſumme ſhall I diuide by the number of ſheep, which is 225, theſe two numbers therefore I ſet thus: An example of ſheep.



Then begin I at the higheſt line of the dividend, and ſeek how oft I may haue the diviſor therein, and that I may do four times,

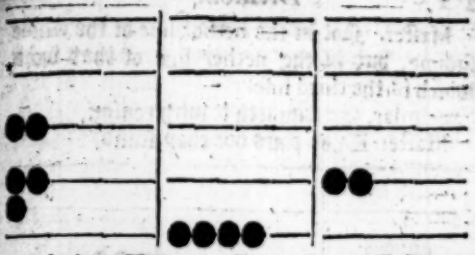
then ſay I, four times 2 are 8, which if I take from 9, there reſteth but but 1, thus:



And becauſe I found the Diviſor 4 times in the dividend, I have ſet as you ſee, 4 in the middle room, which is the place of the Quotient: but now muſt I take the reſt of the Diviſor as often out of the remainder, therefore come I to the ſecond line of the Diviſor, ſaying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

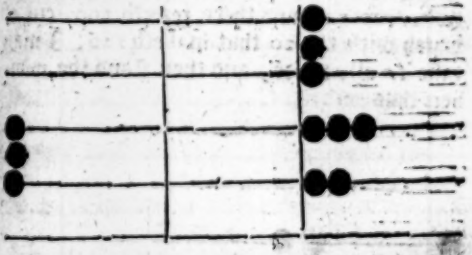
Then come I to the loweſt number, which is 5, and multiply it 4 times, ſo is it 20, that take I from 20, and there remaineth nothing, ſo that I ſee my Quotient to be 4, which are in value ſhillings, for ſo was the dividend: and thereby I know that if 225 ſheep coſt 45 pound, every ſheep coſt 4 ſhillings.

Scholar.



Scholar. This can I doe as you ſhall perceive by the Example: If 160 Souldiers doe ſpend every moneth 68 li. what ſpendeth each man? Example of Souldiers wages.

Firſt, becauſe I cannot divide the 68 by 160, therefore I will turn the pounds into pence by Multiplication, ſo ſhall there be 16320 pence: now muſt I divide the ſumme by the number of Souldiers, therefore I ſet them in order thus:

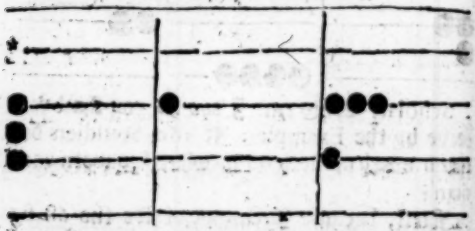


Then begin I at the higheſt place of the dividend, ſeeking my Diviſion there, which I finde once, therefore I ſet 1 in the nether line.

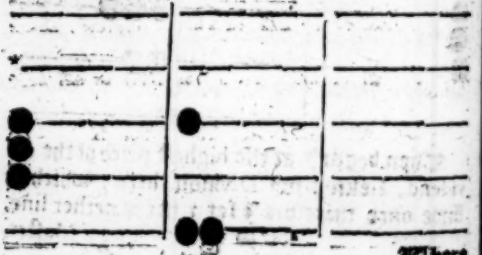
Maſter. Not in the nether line of the whole ſumme, but in the nether line of that work which is the third line.

Scholar. So ſtandeth it with reaſon.

Maſter. Then thus doe they ſtand.



Then ſeek I again the reſt. how often I may finde my diviſor: and I ſee that in 300 I might finde 100 three times: but then the 60 will not bee ſo often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300 and there reſteth 100, out of which with the 20 that maketh 120, I may take 60 alſo twice, and then ſtand the numbers thus:



Where

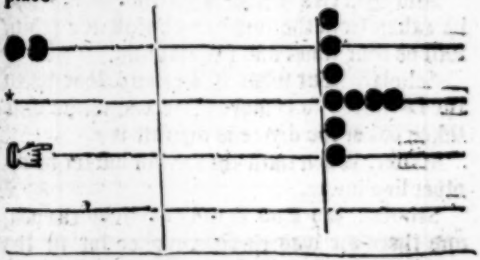
Where I have ſet the quotient 2 in the loweſt line: ſo is every Souldiers portion 102 pence that is 8 ſhillings 6 pence.

Maſter. But yet becauſe you may fully perceive the reaſon of diviſion, it ſhall be good that you ſet your diviſor ſtill againſt thoſe numbers from which you doe take it, as by this example I will declare.

If the purchaſe of 200 acres of ground did coſt 260 pound, what did one acre coſt?

First, will I turn the pounds into pence, ſo will there be 62400 pence. Then in ſetting down theſe numbers, I ſhall doe thus. An example of purchaſe.

First ſet the dividend on the right hand as it ought, and then the diviſor on the left hand againſt thoſe Numbers from which I intend to take him firſt. as here you ſee, where I have ſet the diviſor two lines higher then his own place.

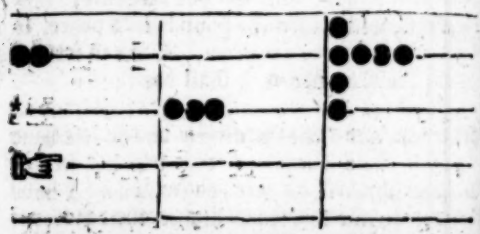


Scholar. This is like the order of Diviſion by the Pen.

Maſter. Truth you ſay, and now muſt I

ſet the quotient of this work in the third line, for that is the line of Unites in reſpect of the Diviſor in this work.

Then I ſée how often the Diviſor may bée found in the dividend, and that I finde 3 times, then ſet I 3 in the third line for the quotient, and take away that 60000 from the dividend: and further I ſet the Diviſor one line lower, as you ſée here.



And then ſéek I how often the Diviſor will be taken from the number againſt it, which will be four times and 1 remaining.

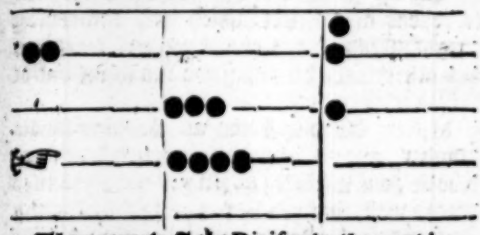
Scholar. But what if it chance that when the Diviſor is ſo removed, it cannot be once taken out of the dividend againſt it?

Maſter. Then muſt the Diviſor bée ſet in another line lower.

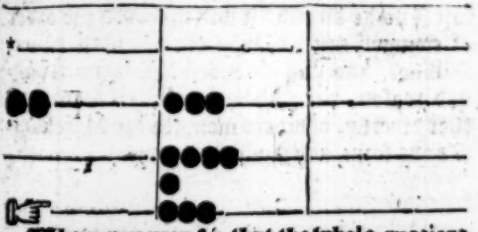
Scholar. So was it in Diviſion by the pen, and therefore was there a cypher ſet in the quotient: but how ſhall that be noted here?

Maſter. Here néedeth no token, for the lines doe repreſent the places, onely lok that you ſet your quotient in that place which ſtandeth

beth for Unites in reſpect of the Diviſor. But now to return to the example: I finde the Diviſor four times in the dividend, and 1 remaining: for 4 times 2 make 8, which I take from 9, and there reſteth 1, as this figure following ſheweth: and in the middle ſpace for the quotient, I ſet 4 in the ſecond line, which is in this worke the place of Unites.



Then remove I the Diviſor to the next lower line, and ſeek how often I may have it in the dividend, which I may do here 8 times juſt, and nothing remain, as in this form.



Where you may ſee that the whole quotient is 348 pence, that is 29 ſhillings, whereby I know

know that ſo much coſt the purchaſe of one Acre.

Scholar. Now reſteth the proofs of Multiplication, and alſo Diviſion.

Maſter. Their beſt proofs are each one by the other; for Multiplication is proved by Diviſion, and Diviſion by Multiplication, as in the work by the pen you learned.

Scholar. If that be all, you ſhall not need to repeat again that which was ſufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I ſuppoſe.

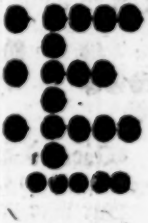
The reaſon of all the former rules.

Maſter. So will I doe as touching whole number, and as for broken number I will not trouble your wit with it, till you have practiſed this ſo well, that you be full perfect, ſo that you need not to doubt in any point that I have taught you, and then may I boldly inſtruct you in the Art of Fractions or broken Numbers: wherein I will alſo ſhew you the reaſons of all that you have now learned. But yet before I make an end, I will ſhew you the order of common caſting, wherein are both pence, ſhillings, and pounds, proceeding by no grounded reaſon, but only by a received form, and that diverſly, of divers men, for the Merchants uſe one form, and Auditors another.

Merchants

Merchants uſe.

Merchants Accompt.



But firſt for Merchants form, mark this example here, in which I have expreſſed this ſum, 198 pounds, 19 ſhillings, 11 pence. *So that you may ſee that the loweſt line ſerveth for pence, the next above for ſhillings, the third for pounds, and the fourth for ſcores of pounds.*

And further you may ſée that the ſpace betwéen pence and ſhillings, may receíve but one counter, (as all other ſpaces likewiſe doe) and that one ſtandeth in that place for 6 pence.

Likewiſe betwéen the ſhillings & the pounds one counter ſtands for 10 ſhillings.

And betwéen the pounds and 20 pounds, one counter ſtandeth for 10 pounds.

But beſide thoſe, you may ſée at the left ſide of ſhillings, that one number ſtandeth alone & betokeneth 5 ſhillings.

So againſt the pounds, that one counter ſtandeth for 5 pounds. And againſt the 20 pounds, the one counter ſtandeth for five ſcore pounds, that is, 100 pounds: ſo that every ſide counter is five times ſo much as one of them againſt which he ſtandeth.

Auditors

Auditors Accompt.

Auditors Accompt.

NOw for the Accompt of Auditors, take this Example.

Where I have expreſſed the ſame ſumme, 198 li. —19 s. —11 d.

But here you ſée the pence ſtand towards the right hand, and the other increaſing orderly towards the left hand.

Again you may ſée, that Auditors will make two lines (yea and more) for pence, ſhillings, and all other values, if their ſummes extend thereto. Alſo you ſée that they ſet one Counter at the right end of each row, which ſo ſet there, ſtandeth for 5 of that room, and on the left corner of the row, it ſtandeth for 10 of the ſame row.

But now if you would adde, or ſubtract after any of both theſe ſorts, if you mark the order of the other ſeat which I taught you, you may eaſily doe the ſame here without much teaching: for in Addition you muſt firſt ſet down one ſumme, and to the ſame ſet the other orderly, and in like manner, if you have many; but in Subtraction, you muſt ſet down firſt the

the greateſt ſumme, and from it muſt you abate the other, every Denomination from his due place.

Scholar. I doe not doubt but with a little practiſe I ſhall attain theſe both: but how ſhall I multiply and divide after theſe formes?

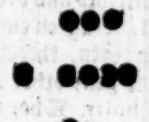
Maſter. You cannot duely doe any of both by theſe forts, therefore in ſuch caſe you muſt reſort to your other Arts.

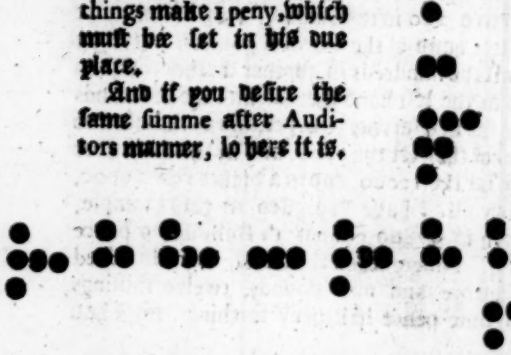
Scholar. They that uſe ſuch Accounts that it exceed 200 in the ſumme, they ſet not 5 at the left hand of the ſcores of pounds, but they ſet all the hundreds in another farther row, and 500 at the left hand thereof, and the thouſands they ſet in a farther row yet, and at the left ſide thereof they ſet the 5000, and in the ſpace over they ſet the 10000, and in a higher row 20000, which all I have expreſſed in this example, which is 97869 pounds 12 ſhillings, 9 pence ob. q. Ninety ſeven thouſand, eight hundred threeſcore and nine pounds, twelve ſhillings and nine pence half-peny farthing, for I had

not

not told you before, where neither how you should set down farthings, which (as you see here) must be set in a void place standing beneath the pence, for a farthing, one counter, ob. 2 counters, for ob. farthing 3 counters, and more there cannot be: for 4 farthings make 1 peny, which must bee set in his due place.

And if you desire the same summe after Auditors manner, lo here it is.





But in this thing you shall take this for sufficient, and the rest you shall observe as you may see by the working of each sort, for the divers wits of men have invented divers and sundry wates, almost innumerable.

THE

THE SECOND PART OF ARITHMETICK; Touching Fractions, briefly set forth.

Scholar.

Albeit I perceive your manifold busines doth so occupie, or rather oppresse you, that you cannot as yet compleatly end the Treatise of Fractions Arithmeticall, *which you have prepared, wherein not onely sundry works of Geometry, Musick, and Astronomy be largely set forth, but also divers conclusions and naturall*

Arithmeticall fractions.

naturall works, touching mixtures of Metall, and compositions of Medicines, with other ſtrange examples. Yet in the mean ſeaſon, I cannot ſtay my moſt earneſt deſire, but importunely crave of you ſome brief preparation toward the uſe of Fractions, whereby at the leaſt I may be able perfectly to underſtand the common works of them, and the vulgar uſe of thoſe rules, which without them cannot well be wrought.

Maſter. If my leaſure were as great as my will is good, you ſhould not need to uſe any importunate craving, for the attaining of that thing, whereby I may bee perſwaded that I ſhall any way profit the Common-wealth, or help the honeſt ſtudies of any good member in the ſame: wherefore while mine attendance will permit mee to walk and talk, I am well willing to help you as I may.

Therefore, firſt to begin with the explication of this name Fraction, what take you it to bee?

What a Fraction is.

Scholar. Marry Sir, I think a Fraction (as I have heard it often named) to bee a broken number, that is to ſay, to be no whole number, but part of a number.

Maſter. A Fraction indeed is a broken number, and ſo conſequently the part of another number, but that muſt be underſtood of ſuch another number as cannot be divided into any other parts then Fractions: for although I may take the third part of 60, or the fourth

part

part of it, and ſo of other parts diverſly, yet thoſe parts bee not properly, nor ought to bee called Fractions, becauſe they may be expreſſed by whole numbers, for the third part of it is 20, the fourth part is 15. the twelfth part is 5, and ſo forth of other parts, all which bee whole numbers.

Wherefore properly a Fraction expreſſeth the parts or part onely of a unite, that is to ſay, that the number which is the whole or entire ſumme of any Fraction, may not bee greater then one: and therefore it followeth, that no one Fraction alone can be ſo great, that it ſhall make 1, as by example I will declare, aſſoon as I have taught you to know the forme how a Fraction is expreſſed or repreſented in writing. What a Fraction is properly.

Numeration.

The expressing of Fractions.

But first to begin with expressing of a Fraction, which is the Numeration of it: you must understand that a Fraction is represented by two numbers set one over the other, and a line drawn between them, as thus, $\frac{1}{3}\ \frac{1}{4}\ \frac{2}{5}\ \frac{10}{17}$, *which four Fractions you must pronounce thus,* $\frac{1}{3}$ *one third part,* $\frac{1}{4}$ *three quarters,* $\frac{2}{5}$ *two fift parts,* $\frac{10}{17}$ *ten seventeen parts.*

Scholar. I understand this form of their expression and pronunciation, but their meaning or valuation seemeth more obscure. Yet I think that by the two first Fractions, I understand the valuation of the two latter Fractions, and consequently of other.

Master. Value them then, that I may perceive your taking of them.

Scholar. $\frac{2}{5}$ betokeneth two fift parts, that is to say, if one be divided into 5 parts, that Fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing be divided into three parts, I must take out one of them: so $\frac{1}{4}$, that is three quarters, doth declare that one being divided into four quarters, I must take (for this Fraction) three of

of those quarters.

If there be no more difficulty in the Numeration, then I pray you go forward to their Addition and Subtraction, and so to the other kinds of works. For I understand that the same kindes of works be in fractions, that be in whole numbers.

Master. There are the same kindes of works in both, albeit the order of them is divers, as I will anone declare: but yet more in Numeration before we leave it. You must understand, that those two numbers which expresse a Fraction, have severall names, the overmost, which is above the line, is called the Numerator, and the other beneath the line, is called the Denominator. Numerator, Denominator,

Scholar. And what is the reason of their divers names? For (in mine opinion) both be Numerators, seeing both they doe expresse the numeration of the fraction.

Master. You are deceived; for one onely (which is the overmost) doth expresse the Numeration, and the Denominator doth declare the number of parts, into which the unite is divided, as in this Example; when I say: divide a pound weight of Gold between four men, so that the first man shall have $\frac{3}{15}$, the second $\frac{4}{15}$, the third $\frac{3}{15}$, and the fourth $\frac{5}{15}$.

Now doe you perceive that by the Denominator (which is one in all four fractions) it is intended that the pound weight should be divided into so many parts, I mean 15.

& by the 4 ſeverall Numerators, is limited the divers portion that each man ſhould have, that is, that when the whole is parted into 15, the firſt man ſhall have two of thoſe 15 parts: the ſecond man three of them: the third man four: and the fourth man ſix. And ſo may you ſee the ſeverall offices (as it were) of thoſe two numbers, I mean of the Numerator and the Denominator.

And hereby you perceive that a man can have no more parts of any thing then it was divided into, neither yet aptly ſo many: ſo that it were unaptly ſaid: You ſhall have $\frac{15}{15}$, that is 15, fifteen parts of any thing, ſeeing it were better ſaid, you ſhall have the whole thing.

Scholar. So doth it appear reaſonably, for the labour is vain to divide any thing, and then to apply the Diviſion to no uſe. And much leſſe reaſonable were it to ſay $\frac{16}{15}$: for if the whole be divided into 15 parts only it is not poſſible to take 16 of them, that is to ſay, more then all together.

Maſter. This is true touching the proper and apt uſe of the name of a Fraction; yet improperly (and after a vulgar acceptation for eaſineſſe in work) both theſe forms be called Fractions, becauſe they be written like Fractions, although they be none indeed: for $\frac{15}{15}$ and generally in ſuch other, where the Numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts: And ſo $\frac{15}{13}$ is not to be called a Fraction,

tion, but a mixt number, of a whole number and a Fraction, for it is as $1\frac{3}{12}$, that is one whole and $\frac{3}{12}$ parts; as shall be declared in Reduction. Therefore they doe abuse the names that call them Fractions, where the Numerator is either equall, or greater then the Denominator. An improper Fraction of a mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in work write some numbers after that sort like Fractions: but they needed not to call them Fractions, but (as they bee) whole numbers, or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions, or as improper Fractions.

Now must you understand, that as no Fraction properly can bee greater then one, so in smalnesse under one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increase above one infinitely, so that not onely one may bee divided into infinite Fractions or parts, but also every Fraction may bee divided into infinite Fractions or parts, which commonly bee called Fractions of Fractions: and they bee expressed diverslly: As for examples $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, three quarters of two third parts of one half part. Whereby is signified, that if one bee divided into two halfs, and the one halfe into three parts, and two of those Fractions of Fractions.

three parts be divided jointly into four quarters, this Fraction of Fractions doth represent three of those quarters.

Scholar. I pray you let me prove by an Example in common money, whether I doe rightly understand you or no. One Crown which I take for an unite, doth contain 60 pence; therefore the half of it is 30 pence, $\frac{2}{3}$ of that half is 20 pence, whereof $\frac{1}{4}$ is fifteen pence, so then fifteeen pence is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crown: and so is three pence $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceive this well enough: yet this note I give you by the way, that the form of expressing the Fractions is voluntary, and hath no other reason then the will of the Divisor, which form many follow: for some expresse them thus, $\frac{1}{4}\frac{2}{3}\frac{1}{2}$ without any Figure of distinction between them, which form also may follow. Some other doe make lines between every fraction, and adde words of distinction, after this sort, $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, which form is best.

Some other expresse them thus in slope form, to distinct them from Fractions of one whole numbers, for if they were in one right line thus, $\frac{1}{4}\frac{2}{3}\frac{1}{2}$ then ought it to be pronounced, three quarters, and two third parts, and a half, which maketh almost two whole unites, lacking but one twelfth part. And so is it nothing agreeable

$\frac{1}{4}$ $\frac{2}{3}$ $\frac{1}{2}$

with

with the other Fraction of Fractions: wherefore it is a great oversight in certain learned men, which doe expresse them so confusedly with such severall Fractions, that a man cannot know the one from the other.

Therefore some men (*as* Stifelius) *doe expresse without a line, numbers of proportion, being applied to* Addition *or* Subtraction, *because they must be taken as two, where the line in Fractions maketh them to be taken for one: for of the* Numerator *and* Denominator *is made one number.*

Scholar. Then I perceive there be three severall varieties in Fractions: First, when one only Fraction is set for one number, as $\frac{4}{5}$, that is, four fift parts. The second is, when there be set two or more severall Fractions of one number, as $\frac{4}{9}$, $\frac{4}{5}$, that is four ninth parts, and two fift parts. The third sort is Fractions of Fractions, as $\frac{4}{9}$, of $\frac{4}{5}$, that is, four ninth parts of two fift parts. Three severall varieties.

Master. You have said well, if you understand well your own words.

Scholar. If it shall please, I will by an example in the parts of an old English Angel, expresse my meaning.

Master. Let me hear you.

Scholar. The old English Angel did contain 7 shillings 6 pence, that is, 90 pence: Now $\frac{4}{5}$ of it is 79 pence: And of the same 90 pence; if I take $\frac{4}{9}$ and $\frac{4}{5}$, that is four ninth parts, and two fift parts, $\frac{4}{9}$ is 40, and $\frac{4}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{4}{5}$, that

is four nine parts of two fift parts, ſeing $\frac{2}{3}$ is but 36, then $\frac{4}{9}$ of 36 will yeld but 16, ſo $\frac{1}{9}$ of 36 is but 4, and that taken four times maketh 16.

Maſter. This is plainly expreſſed and truly, and hereby (I doubt not) but you dee perceite, that as great a difference, as is betwéen 16 & 76, ſo much difference is betwéen thoſe two Fractions $\frac{4}{9}$ and $\frac{4}{5}$: and $\frac{4}{9}$ of $\frac{2}{3}$.

And now that yée underſtand theſe varieties, I will proceéd to the reſt of the works: First, admoniſhing you, that there is another order to bée followed in Fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diviſion, and Reduction: but in Fractions, (to follow the ſame aptneſſe in proceéding from the eaſieſt works to the harder) wée muſt uſe this order of works, Numeration, Reduction, Addition, Subtraction, Multiplication, and Diviſion.

The order of works in Fractions.

Scholar. That Addition and Subtraction ſhould go together, and Diviſion to follow Multiplication, naturall order doth perſwade: but why Reduction ſhould bée firſt in order here, next to Numeration, and Addition and Subtraction in the middle, I deſire to underſtand the reaſon.

Maſter. As in the Art of whole numbers, Order would reaſonably begin with the eaſieſt, and ſo goé forward by degrées to the hardeſt:

hardest: even reason teacheth in Fraction the like order. And consider that Addition or Subtraction of Fractions, can very seldome bee wrought without Reduction: and contrariwise, Reduction may be wrought without this form of Addition or Subtraction: therefore was it orderly required, that Reduction should goe before Addition and Subtraction, and this reason serveth for the placing of Reduction before the other.

Scholar. Then, if Reduction bee the easiest, I pray you declare the form of it, first by rule, and then by example.

Master. Your request is good.

Reduction

Reduction of Fractions.

Of Reduction of Fractions, there are 5 varieties.

THerefore will I now declare the diverſities of Reduction of Fractions, which commonly have five varieties, or forms.

1. *Firſt, when there be ſundry Fractions of one intire unite, they muſt be reduced to one Denomination, and alſo into one Fraction.*

2. *Secondly, when there be proponed Fractions of Fractions, they muſt be reduced likewiſe into one Fraction: for otherwiſe they cannot be brought into one Denomination.*

3. *Thirdly, when an improper Fraction is proponed, that is to ſay, a Fraction in form, which indeed is greater then an Unite: it muſt be reduced into apt form, expreſſing the Unite or Unites of it, and the proper Fraction diſtinctly. And ſometimes alſo it ſhall be needfull to convert ſuch a mixt number of Unites with Fractions, into the form of a Fraction, that is, into an improper Fraction: which two forms I eſteem but as one, becauſe they work one kinde of number.*

4. *Fourthly, there happeneth ſometimes Fractions to be written in great numbers, which might be written in leſſer numbers: therefore is there a mean to reduce ſuch great numbers into their ſmalleſt terms.*

5. *Fiftly, when any Fraction betokeneth the parts of a whole thing, which hath by common partition certain*

certain parts, but none of like denomination *with that* fraction, *then may you reduce the said* fractions *into another, whose* denomination *shall expresse the common parts of that whole thing.*

Scholar. This distinction in Doctrine delighteth mee much, but more with hope then present fruit: for as yet I doe not understand scarcely the varieties, and much lesse the practise and use of their workes.

Master. Reduction is an orderly alteration of Numbers out of one form into another, which is never done orderly but for some needfull use, as in every of the said five severall forms I will distinctly declare.

First therefore, when two, or more severall Fractions of any Unite be proponed: As for Example $\frac{1}{16}$ and $\frac{4}{6}$; because it is hard to tell what proportion of the intire number those two fractions doe expresse, therefore was Reduction devised to be a mean whereby these severall fractions might be brought into one Denomination and fraction. The first form of Reduction.

And in these fractions, this is the Art for bringing them to one Denomination.

Multiply first the denominators *together, and the totall thereof you shall set twice down under two severall lines for two new* denominators, *or rather for one common* denominator. *Then multiply the* numerator *of the first fraction, by the* denominator *of the second, and set the totall thereof for the* numerator *over the first line. Likewise multiply the* numerator *of the second Fraction* How to reduce fractions of divers denominations into one denomination.

by

by the Denominator of the first, and set that totall over the second line for the Numerator of that fraction: and so are these two first Fractions of severall Denominations, brought to one Denomination.

Scholar. If I understand you, as I think I doe, my Example shall declare the same. The Fractions, which you proponed were these, $\frac{1}{16}$, and $\frac{2}{6}$, whose Denominators (being 16 and 6) I multiply together, and there amounteth 26, which I set under two lines, thus: $\frac{\quad}{96}$ $\frac{\quad}{96}$.

Then I multiply the Numerator of the first Fraction by the Denominator of the second, saying, 3 into 6 maketh 18, that I set over the first line for a new Numerator, and it will be thus, $\frac{18}{96}$.

Likewise I multiply the Numerator of the second Fraction, by the Denominator of the first, saying, 4 times 16 maketh 64, that I set for the second Numerator, and the Fraction will appear thus, $\frac{64}{96}$.

So that both Fractions brought to one Denomination, must stand thus, $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar. I beseech you let me examine it after my accustomed form, by common parts of coyn or other measure.

Master. Go to.

Scholar. I have a peece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is the sixteenth part, is 6 pence, and $\frac{1}{16}$ is 18 pence, that

is

is $\frac{15}{96}$. Again $\frac{1}{2}$ of the same peece of gold is 16 pence, so that $\frac{2}{3}$ parts maketh 64 pence, that is $\frac{48}{96}$. And so I finde the summes to agree with the other before.

Master. *So have you now the Art to bring two such Fractions into one Denomination: And if there be more then two, then must you multiply all the Denominators together, and set the totall thereof so many times down as there be Fractions; and then to get for each one a new Numerator, multiply the Numerator of the* first, *by the Denominator of the* second, *and the totall thereof, multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the* third *into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth if there were moe. So these three Fractions* $\frac{2}{5}\frac{1}{4}\frac{1}{3}$ *doe make by Reduction these other three Fractions of Denomination.* $\frac{24}{60}\frac{15}{60}\frac{20}{60}$. *All which you may bring into one Fraction by adding the Numerators together, and putting the totall for the totall Numerator, reserving still that same common Denominator. And those three Fractions make one improper Fraction, thus:* $\frac{109}{60}$.

Note the Reduction of three Fractions, or more, into one.

Scholar. All this I perceive, and also that this last fraction is more then an unite, and therefore you did call it an improper fraction.

Master.

Master. There bee certain other formes of working in this Reduction, which I will briefly touch also, to give you an occasion to exercise your wit therein.

The first variety of Reduction

The first variety is this: When you have made and written down your common Denominator *(as I have taught before) then to get a* Numerator *for the first, doe thus: Divide the* common Denominator *by the* Denominator *of the first Fraction, and the quotient multiplied by the* Numerator *of the same, yeeldeth a new* Numerator *for the first new Fraction. So likewise doe with the second and the third, and with all the residue, if there be more.*

Scholar. That will I prove in your last Example of these three fractions, $\frac{2}{5}\frac{1}{4}\frac{2}{3}$. When the Denominators be multiplied they make 60, for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60, that I set down three times thus: $\frac{\ }{60}\frac{\ }{60}\frac{\ }{60}$ then to have a Numerator, for the first. I must divide 60 by 5 (the Denominator of the first) and the quotient is 12, which I must multiply by 2 (the Numerator first, and that maketh 24, and so have I for the first Fraction, $\frac{24}{60}$.

The second variety.

Likewise for the second Fraction: I divide 60 by 4, and there cometh 15, which I multiply by 3, and so have I 45, for the second Fraction $\frac{45}{90}$. Then for the third in like sort will come $\frac{40}{60}$.

Master. *Another way is this: If it happen so, that the lesser* denominator, *can by any* multiplication *make the greater, then note the* multiplier, *& by it multiply the* numerator *over that lesser* denomina-

tor,

tor, *and for the leſſer* Denominator *put the greater, as thus in theſe two fractions* $\frac{1}{12}$ *and* $\frac{2}{3}$, *three being the leſſer* Denominator *multiplied by* 4, *will make* 12, *which is the greater* Denominator: *therefore by the ſame* 4 *I doe multiply* 2 *which is the* Numerator *over* 3, *and that maketh* 8: *under which I do put* 12, *being the greater* Denominator *which is alſo made by multiplication of* 4 *into* 3, *and ſo have I theſe two fractions* $\frac{1}{12}$, $\frac{8}{12}$, *thus ſhortly reduced, without altering the one* Fraction.

Scholar. This I underſtand.

Maſter. *Then mark this third way: If the* Denominators *doe not happen ſo, that one by* Multiplication *may make the other, then look whether they both may be parts of any other one* number, *as in* $\frac{1}{12}$, *and* $\frac{1}{18}$, *although the leſſer taken but twice, be too much to make* 18, *yet they both may be parts unto* 36, *therefore look how many times twelve is in* 36, *and that quotient being multiplied by the* Numerator *over* 12, *the total ſhall be put in ſtead of the* Numerator *over* 12, *and for* 5 *put* 15, *thus,* $\frac{15}{36}$. *So likewiſe look how often is* 18 *in* 36, *becauſe it is twice, therefore by* 2 multiply 7, *which is over* 18, *and it will be* 14: *ſet that for the* Numerator, *and in ſtead of* 18 *put* 36; *and then your* Fractions *reduced ſtand thus,* $\frac{15}{36}$ $\frac{14}{36}$, *in ſtead of* $\frac{1}{12}$, *and* $\frac{1}{18}$. The third variety.

And if you will prove whether you have wrought well or no, that may be proved by Reduction of them again to their former Denominations, which Art ſhall be taught in the fourth kinde of Reduction, where greater terms of Fractions be reduced into ſmaller in Proof.

number

☞ number. but no ſmaller in proportion. And if in ſuch Reduction the ſame terms or numbers come again that were before, then is the work good, elſe not.

Scholar. Sir, I hear your words, but I doe not underſtand many of them: which if it pleaſe you, declare.

Maſter. With a good will when convenient place ſerveth, but that muſt be in the ſaid fourth kinde of Reduction, which teacheth how to reduce Fractions of Fractions into one Fraction, and ſo to one Denomination.

The ſecond form of Reduction of Fractions of Fractions into one Fraction and Denomination.

When Fractions of Fractions be propoſed, you ſhall multiply the Numerators of each into other, and ſet the totall for the new Numerator, and then multiply all the Denominators likewiſe, and take their totall for the new Denominator, and ſo are they ſpeedily reduced.

Scholar. If that bee all, then I underſtand it already, as by this example I will declare. Theſe be the Fractions $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ which I would reduce to one Denomination, and proper ſimple Fraction.

Therefore begin I with the Numerators, and multiply them together, ſaying, 3 by 2, maketh 6: and 6 by 6, maketh 36, which multiplied by 7, yeeldeth 252: that I ſet over a line for the Numerator thus:

252

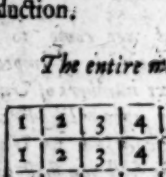
Then I multiply the Denominator, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeeldeth 756, the new

new Denominator. And so the whole reduced Fraction is this, 252/756 which is too hard a Fraction for me to understand yet.

Master. You think so, and no marvell, but anon you shall learn to judge it easily, for this Fraction is no more indeed then 1/3, although it be in greater terms, and therefore more strange and more obscure.

And this sufficeth for this Reduction, save that I will shew you by a Figure of measure the just rate and reason of this kinde of Fractions, and also the due understanding of their Reduction.

The entire measure parted into 9.



Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 divisions, whereof 7 are severed by the second measure: and thereof again are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being divided into four parts, the lowest

meaſure doth contain $\frac{1}{4}$, ſo that the ſame $\frac{1}{2}$ muſt be named, not $\frac{1}{2}$ of the whole meaſure, but indeed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{7}{9}$.

Scholar. This example is ſo ſenſible, that I cannot chuſe but ſée it. And furthermoꝛe ſée alſo, that the ſame Fraction is equall to $\frac{1}{2}$ of the entire meaſure, as the lines which run up and down doe expreſſe ſet foꝛth. Alſo I ſée here that $\frac{2}{7}$ of $\frac{4}{7}$ is equall to $\frac{2}{7}$. And further yet that $\frac{2}{7}$ of $\frac{1}{2}$ is equall to $\frac{4}{9}$ oꝛ $\frac{2}{3}$.

Maſter. I am glad that you ſée it ſo well, not doubting but you will gather greater light of knowledge hereby.

The third forme of Reduction of improper Fractions.

But now it is time that wee come to the form of Reduction, which teacheth of improper Fractions, that is to ſay, mixt numbers of Unites and Fractions although they appear like Fractions, as thus $2\frac{1}{3}$, *which doth conclude* 25 *Unites wholly, and* $\frac{1}{3}$ *over. Wherefore firſt you ſhall know them, by that the Numerator is greater then the Denominator.*

Scholar. Indéed Sir, that appeareth reaſonable, that if the Numerator doe expreſſe moꝛe parts to bée taken of any Unite, then the Denominator doth ſignifie that the Unite to be diuided into, it muſt néeds follow, that ſuch a Fraction importeth moꝛe then the whole, that is to ſay, the whole with certain parts oꝛer: but what Reduction is there in it?

Two ſeuerall waies in this Reduction.

Maſter. There be two ſeuerall kindes of Reduction, concerning ſuch Fractions. Some-

times

times it ſhall bee needfull to convert theſe Fractions into Unites, and the proper Fraction, that will remain. And ſometimes contrariwiſe, it ſhall be meet to reduce mixt numbers, that is Unites written with Fractions into the form of one ſimple Fraction, and ſo be there two waies.

Scholar. What is the meane of the firſt way to turn improper Fractions into Unites with their proper Fractions?

The firſt way. Reduction of improper Fractions into Unites, with their proper Fractions.

Maſter. *That is thus; Your Numerator being greater then the Denominator, muſt be divided by the ſame denomination, and the Quotient thereof expreſſeth the Unites: the remainer ſhall be put for the Numerator of the Fraction that reſteth, and the Denominator muſt be the ſame that was before.*

Scholar. For Example, I take $\frac{17}{5}$, and diviſing 17 by 5, the Quotient will be 3, and there will remain 2.

Maſter. That you muſt write thus, $3\frac{2}{5}$, where (you ſee) I have written 3 without any line, as entire numbers ought to be written, and the 2 that remained, I have ſet over the former Denominator, with a line, as a proper Fraction. And this number doth ſignifie now three Unites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by Unites here underſtand Crowns, ſo it were 3 Crowns, and $\frac{2}{5}$, that is 2 s.

Maſter. Even ſo, and therefore $\frac{17}{5}$ did ſignifie the ſame: But this happeneth ſometimes

that when the Reduction is ſo wrought, there remaineth nothing: And then it is not a mixt number, but a ſimple intire number, repreſented like a Fraction.

The ſecond way.

Scholar. As $\frac{14}{3}$ will make 3 juſt, and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the ſecond form of Reduction that you ſpeak of for theſe ſorts of Fractions?

Reduction of whole numbers either alone, or joyned with Fractions into improper Fractions.

Maſter. *Whenſoever you have any of theſe two ſorts of Numbers, that is to ſay whole Numbers without Fractions, or whole Numbers with Fractions, and you would turn them into the form of a Fraction, you muſt multiply the whole Number by that Denominator which you will have to remain ſtill, and to the totall thereof adde the Numerator, which you have already, and all that ſhall you ſet for the new Numerator, keeping ſtill the former Denominator: As if you have 6 $\frac{1}{4}$ which you would convert into an improper Fraction, you muſt multiply 6 by 4, whereof cometh 24, & thereto add the Numerator, which is 3, and ſo have you 27 for the Numerator, and 4 ſtill for the Denominator.*

Scholar. Then is $\frac{27}{4}$ equall to 6 $\frac{1}{4}$.

Note.

Maſter. Even juſt, and ſo backward (as appeareth by the former Reduction) 6 $\frac{1}{4}$ maketh $\frac{27}{4}$. And thus one of their Reductions may be the proof of the other work.

Scholar. This I perceive: But now if you would turn whole numbers without Fractions into any Fractions, I ſee not how that may be done, becauſe there is no Denominator to make the multiplication by.

Maſter.

Master. That is well marked: but this you know, that no man intendeth to turne any whole number into a Fraction, but he hath in his minde that Denominator by which the multiplication must bée made: for the ꝓof whereof I set down 7, which is a whole number. And if you will have this number converted into any certain Fraction, will mée to doe it.

Scholar. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some Fraction.

Scholar. No, I passe not for the sort of the Fraction.

Master. Then how can you think that you require me to doe any thing certain, when you leave me to doe as I list? And seeing you stand at that stay, whether think you that I must first intend in minde what Fraction I will make of it before I can doe it indéed?

Scholar. Else you should doe ignorantly.

Master. Then will I limit my self (seeing you will not) to turn it into quarters. And therefore I multiply 7 by 4 (which is the denomination by quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the Denominator, and the Fraction will be thus $\frac{28}{4}$.

Scholar. Indéed I perceive this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then

if I would turn 8 into 5 parts, it will make $\frac{40}{5}$, which is all one with 8: for 8 Crowns turned into 5 parts, (that is into shillings) will make 40 shillings, that is, $\frac{40}{5}$ of a Crown.

The fourth form of Reduction

Master. *Seeing you understand now these three kinds of Reduction, I will declare unto you the fourth kinde, that is, when Fractions be written in greater terms then they need, how they may be brought to lesser terms.*

Scholar. To write any thing in greater terms then needeth, seemeth to be a fault, and so this Rule seemeth to amend that fault.

Master. It were a fault to doe any thing without need, which after must be redressed: but in this case it is not so, neither did I say absolutely (as you doe) that it needeth not to expresse those Fractions in so great terms, but that the Fractions doe not need, I mean for their Value to be understood: but yet it may be needfull for the ease of these workes wherein they be applied; as for Example, In the first kind of Reduction this was your own example, $\frac{1}{2}$ and $\frac{2}{3}$ which when you would reduce, you were faine to turn them first into one denomination, and so appeared they thus $\frac{3}{6}$ and $\frac{4}{6}$, where the Fractions (for their own understanding) needed not to be turned out of smaller terms into greater, but yet the easinesse of working needeth it.

Scholar. Sir, I understand now, not only the difference of this need (for the Fractions might better be understood as Fractions

several,

severall, each in his value, when they were in lesser terms, although they could not so well be reduced) but also I understand what you mean by greater terms and lesser terms, where- of before I was in doubt: for I see you call the Numerator and Denominator, the terms of the Fraction. Terms of Fractions.

Master. *I am glad you understand it so well: now when then you would value any Fractions (because they may best be done when the terms are smallest) you shall reduce them to the smallest that you can, which thing you may do thus: Divide the greatest of any such two terms by the lesser, and if any thing remain, by that remainer divide the last Divisor: and if any thing remain now, by that divide the first Divisor (which was before the remainer of the last Division) and so continue still, till nothing do remain in the Division: and then mark your last Divisor, for it is the Number that will easily reduce your Fractions, if you divide both the Numerator and the Denominator by the same Number, and put for the Numerator the Quotient of his Division, and for the Denominator also his Quotient, that riseth by his Division.*

Scholar. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I divide it by 18, & the Quotient is 5, and there resteth 6, what shall I doe with this Quotient?

Master. Nothing in this work, but now seeing there remaineth somewhat by that remainer must you divide the last Divisor.

Scholar. If I shall divide 18, (which was

the laſt Diviſor) by 6, that was the remainer, ſo is the Quotient 3, and nothing reſteth.

Maſter. As for the Quotient, I omit him yet: but becauſe there doth remain nothing, therefore is 6 (which was your laſt Diviſor) that number by which you may reduce the Fraction proponed.

Scholar. Then as you taught me, I muſt divide the Numerator 18 by 6, and the Quotient is 3, which I muſt put for the Numerator over a line, thus: And then by the ſaid 6 muſt I divide alſo the Denominator 96, and the Quotient will be 16, which I muſt take for the Denominator, and ſo is the Fraction $\frac{3}{16}$. And ſome thinketh this Rule doth prove the work of the firſt Reduction.

3
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16

Maſter. That is true, if the firſt Reduction were made of Fractions into their leaſt terms, and elſe not, without ſome help, as the ſecond number in that place will declare.

Scholar. The ſecond number was $\frac{2}{3}$, which was turned into $\frac{64}{96}$ by that Rule. Now if I ſhall by this Rule reduce it again into the leaſt terms, I muſt divide 96 by 64, and there remaineth 32, wherefore I muſt take that 32 for the Diviſor, to reduce the ſaid Fractions. Then doe you divide 64 by 32, and the Quotient is 2, which I ſet for my Numerator. Again, I divide 96 by 32, and the Quotient will be 3; and ſo I have but $\frac{2}{3}$.

Maſter. Muſe not at the matter, for you have

have done well enough: but you think you have not the Fraction that you looked for, that is, $\frac{2}{3}$, yet have you one equall to it, as by the parts of a shilling you may prove.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{4}{12}$ and $\frac{8}{24}$ and $\frac{16}{48}$ be all three equall. And now I perceive that because it was not written in the least terms that it might be, therefore this Reduction brought forth not it, but that other which is written in the least terms. Now understand I this Rule well. But is there any other way to work this Reduction?

Master. Yes, but first note this, that if you finde no such Divisor, to reduce the Fraction till you come to 1, because one doth make no division, therefore that fraction is already in his least terms, as by $\frac{21}{100}$ you may prove, and so $\frac{21}{98}$, and many other like. Another way to work this Reduction.

But now for your better aid to finde the due proportion in least terms, with more ease for a young learner, you shall mediate or take the half of the Numerator, and also of the Denominator as long as you may upon a line, alwaies parting them with a right down dash of your pen as you work, which may easily be done, if the numbers be even: as 2, 468, or 10, but if they be odde (though it be but one of them) then must you abbreviate them by 3, 5, 7, or 9, &c. And because examples do most instruct, I have here set down the manner of two or three, whose last number at the end of the line shew- Note that to mediate any number is to divide by two.

sheweth the least term of valuation of that Fraction. As for Example: I would reduce $\frac{298}{576}$ into his least term or value, whereupon I set forth $\frac{298}{576}$ with a long line drawn from it, thus:

298	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator & the Denominator end in even numbers, I see this may be abbreviated by 2, or 4, or 6, &c. Therefore on the other side of the right down path toward the right hand, I first take the half of the Numerator: saying, the half of 2 is 1, the half of 8 is 4: and again, the half of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right down path as before.

Then doe I also take the half of 576, in saying, the half of 5 is 2, and the half of 17 is 8, and the half of 16 is 8, and so have I 288 for a new Denominator.

Then beginning again; saying, the half of 144 is 72, and the half of 288 is 144: thus continuing the mediation or division by 2, untill you come to the last work, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$ which is equall $\frac{298}{576}$.

So the second example $\frac{28}{112}$ first abbreviated by 2, and again by 2, and last by 7, is reduced to $\frac{1}{4}$ which is equall to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Again,

Again, $\frac{1465}{4395}$ abbreviated firſt by 5, then by 293.

1465	293	1
4395	879	3

Scholar. Sir, I thank you much, this is very eaſie and good for a young learner.

Maſter. So it is, but yet notwithſtanding, if you can without that diviſion by memory, eſpy the greateſt number that may divide exactly both termes of your Fractions proponed, then need you not to uſe that diviſion, as in this Fraction $\frac{22}{91}$, I ſee that 11 is the greateſt number that can divide them both: and therefore without any work, by memory only, I turn that into $\frac{2}{8}$; but this ability in knowledge is got by exercise.

Yet one other way of eaſie Reduction in this kinde there is, when your Fraction hath any Cyphers in the firſt places of both termes, then may you by caſting away the Cyphers, make a briefe Reduction, as thus: $\frac{100}{400}$ here take away the Cyphers, and it will be $\frac{1}{4}$, which is the ſame in value with $\frac{100}{400}$.

Scholar. And ſo if I have $\frac{600}{7500}$, it will be $\frac{6}{75}$.

Maſter. You are deceived, for you take away more Cyphers from the Numerator then you doe from the Denominator, which you may not doe.

Scholar. I confeſſe my fault, which came of too much haſte, I was more gladder of the Rule

Rule then wise in using it: but now I understand it I trust.

Master. Then may I goe in hand with the fift or last kinde of Reduction, which teacheth how to turn any Fraction proponed into any other Denomination that you list, or into any part of common coyns, weights, or measures, or such like.

The fift kinde of Reduction.

For declaration whereof, first you shall mark whether your Fraction be a simple Fraction, either else a Fraction of sundry parts, I mean of more terms then two. And if your Fraction be a Fraction of Fractions, or otherwise compound, you must reduce it to one simple Fraction: And then mark well the Denomination of that other Fraction, into which you would turn this: for by that Denominator you must multiply the Numerator of your first Fraction, and the totall Product thereof shall you divide by the Denominator of your first Fraction, and that Quotient shall be the Numerator of the Denominator proponed: As for Example, *I have this Fraction* $\frac{1}{2}$, *which I would turn into ten parts: therefore I multiply this* 10 *by* 3, *that is the Numerator of my Fraction, and there ariseth* 30, *which I divide by* 5, *and the Quotient is* 6, *which must be the Numerator to* 10, *and so* $\frac{1}{2}$ *will be* $\frac{6}{10}$.

To reduce Fractions to a Denomination appointed.

Scholar. This is easie enough to doe.

Master. Then shall you see another Example of the same Fraction that is not so easie: as if I would turn $\frac{1}{2}$ into 8 parts, prove you that work.

Scholar. I must multiply 8 by 3, and there amount:

amounteth 24, which I divide by 5, and the Quotient is 4, then is the new Fraction $\frac{4}{5}$.

Maſter. And ſee you nothing doubtfull in this worke?

Scholar. I ſee that when 24 was divided by 5, there remained 4, which I did not paſſe of, becauſe yée ſpake nothing of any remainer, but only of the Quotient.

Maſter. By likelihood you remember what I ſaid to you in Diviſion of whole numbers, that you ſhould not paſſe of the remainer there but only note it as a ſumme that could not be divided without knowledge of Fractions.

Wherefore now mark this, that in all diviſions of whole numbers, when there is any remainer, you ſhall ſet it over a line as a Numerator, and ſet the Diviſor for the Denominator, and that Fraction doth make the Diviſion compleat, and is part of the Quotient: As if I would divide 48 by 5, the Quotient will bee $9\frac{3}{5}$: ſo in your former worke when 24 was divided by 5, the Quotient ſhould bee $4\frac{4}{5}$, and ſo the new Fraction ſhould be thus, $\frac{4}{5}$ and $\frac{4}{5}$ of $\frac{1}{5}$, that is, $\frac{4}{5}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{5}$ part of any thing, which you may proue by Example of ſome Coyn.

Scholar. When I take a Crowne, whoſe $\frac{1}{5}$ is 3 s. Now if I would proue whether the 3 s. bée $\frac{4}{5}$ adde $\frac{4}{5}$ of $\frac{1}{5}$, I ſhall haue a cumbrous worke to doe.

Maſter. Indéed for whole pence, your Example is a little troubleſome: yet turning the crowne

crown into half pence, it is easie enough.

Scholar. That will I trie.

¶ First, I see that $\frac{4}{5}$ of a Crown is 3 shillings, which is 36 pence, or 72 half pence. Now if I can finde that this Fraction $\frac{3}{8}$ and $\frac{1}{2}$ of $\frac{1}{2}$ be equall unto 3 shillings, then am I fully answered.

Because I cannot take $\frac{3}{8}$ of a Crown, I turn the Crown into half pence, as you will'd me, which makes 120 which I divide by 8, my Quotient is 15, which taken four times, make 60 ob. Now resteth me to have $\frac{3}{5}$ of the $\frac{1}{2}$ part of a Crown, whereof $\frac{1}{2}$ part is 15 ob. the 15 being parted into 5 parts, the Quotient is 3, which taken 4 times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{4}{5}$, my desire.

Master. I commend you for your diligence, you might have wrought it thus: either $\frac{3}{8}$ being abbreviated as before I taught, is $\frac{1}{2}$. Now half a Crown is 2 shillings 6 pence. Now $\frac{4}{5}$ of $\frac{1}{2}$ is a Fraction of Fractions, which if you doe reduce into one entire Fraction, as before you have learned in saying, 5 times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abbreviated also make $\frac{1}{10}$. Now the 10 part of a Crown is 6 pence, which put to 2 shillings 6 pence, make also 3 shillings your desire.

But now one Example more for this Rule, and then we shall end it. If I have $\frac{2}{15}$ of a Sovereign (accounting the Sovereign 20 shillings)

lings) how many ſhillings is that $\frac{7}{11}$?

Scholar. I muſt multiply 7 by 20 and that maketh 140, which I ſhall divide by 15, and the Quotient will be $9\frac{1}{11}$, or in leſſer terms $\frac{4}{3}$.

Maſter. That is 19 ſhillings and one third part of a ſhilling, that is 4 pence, as by the ſame Rule you may prove. And this for this time ſhall ſuffice for Reduction. And now I will proceed to Addition.

Addition.

Addition of Fractions of one Denomination.

WHenſoever you have any Fractions to be added, you muſt conſider whether they be of one Denomination or not, and if they be of one Denomination, then adde the Numerators together, and ſet that that amounteth for the Numerator over the common Denominator, and ſo have you done: The reaſon is, becauſe that ſuch differ little in Addition or Subtraction from the work of vulgar Denominations, where the Denominators be of the Number, as 3 d. *and* 5 d. *make* 8 d. *where the Denomination is not altered. But if the Fractions be not of one Denomination, or any of them be mixt of whole Numbers and Fractions, then muſt you firſt reduce them to one Denomination, and after adde them. And if they be many, then adde firſt two of them, and ſo the Summe that doth amount of the Addition, and the third, and then the fourth, &c. if you have ſo many.*

Scholar.

Scholar. This seemeth easie enough, now that I have already learned to reduce, without which I could never have wrought this: And therefore now I see good reason why you do place Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your understanding of it by an Example.

Scholar. Then would I adde first $\frac{5}{18}$ with $\frac{7}{18}$ and because the Denominators are like (and so needeth no Reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreviated $\frac{2}{3}$.

To adde Fractions of divers Denominations.

And if you have many numbers to be added, as here $\frac{1}{8}$ $\frac{4}{5}$ $\frac{2}{10}$, first I must reduce them (because they have divers Denominators) into one denomination, and then they will be thus, $\frac{112}{400}$ $\frac{128}{400}$ $\frac{160}{400}$, or in lesser terms, $\frac{11}{40}$ $\frac{12}{40}$ $\frac{16}{40}$, which by Addition doe make $\frac{39}{40}$, that is $2\frac{1}{40}$.

Master. Now may we go to Subtraction.

Subtraction.

Subtraction of Fractions.

Subtraction of Fractions.

Subtraction *hath the same precepts that* Addition *had, for if the* denominators *be like, then must you subtract the one* numerator *from the other, and the rest is to be set over the common* denominator, *and so your* Subtraction *is ended: but and if you have many* fractions *to be subtracted out of many, then must you reduce them to one* denomination, *and into two severall* fractions, *that is, all that must be subtracted into one* fraction, *and the residue into another* fraction, *and then work as I said before.*

Scholar. For the first example I take $\frac{11}{12}$ to be subtracted out of $\frac{12}{12}$, and the rest will bee $\frac{1}{12}$ or $\frac{1}{0}$.

For another example I take $\frac{1}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{8}{32}$ and $\frac{28}{32}$.

Then do I subtract 24 out of 28, and there resteth 4, which I set over the common denominator for a Remainder, thus $\frac{4}{32}$: that is $\frac{1}{8}$.

Now for the third example, I take $\frac{1}{4}$ and $\frac{5}{8}$ to be subtracted from $\frac{7}{8}$ and $\frac{7}{10}$: and because their Denominators be divers, I do reduce them into one Denomination thus $\frac{1232}{192}$ $\frac{1568}{1920}$ $\frac{1728}{1920}$.

Then do I adde the two first, and they make $\frac{3162}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$. And thus have I done with Subtraction, except you have any more to teach me.

Master. Prove one example or more out of Fractions of diverse Denominations.

Scholar. I take the two Fractions $\frac{7}{12}$ to be subtracted from $\frac{2}{24}$ which being reduced, will stand thus $\frac{168}{192}$ and $\frac{72}{192}$ Now would I subtract 168 out of 72, but I cannot.

	168	72
	$\frac{7}{8}$	$\frac{3}{8}$
	192	

Master. Then may you perceive that you mistook the Fractions: for you can never subtract the greater out of the lesser, although you may adde, multiply or divide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his terms lesser then $\frac{2}{24}$ yet is $\frac{2}{24}$ the lesser Fraction: for generally if you multiply the Numerator and the Denominators of two Fractions crosse-wayes, that Fraction is the greatest of whose Numerator commeth the greatest summe, as in this example: 7 multiplied by 24 maketh 168, and 9 being multiplied by 8 yeeldeth but 72, therefore is the first Fraction 7 the greatest of these two, so can you not subtract it out of the lesser Fraction.

The greatest of two Fractions.

But if you should subtract a Fraction out of a whole number, what should you doe?

Schola.

Scholar. Marry I would reduce the whole number into a Fraction of the same Denomination that my Fraction is; and then work by Subtraction.

Master. So may you doe, but it is much easier, if your Fraction be a proper Fraction, that is to say, lesse then an unite, to take an unite from the whole number, and then turne it into an improper Fraction, and so work your Subtraction. As if I would subtract $3\frac{2}{3}$ from 4, I may take one from 4, and turn it into $\frac{3}{3}$, from which I take $3\frac{2}{3}$, there will remain $\frac{1}{3}$. And if the first Fraction be an improper Fraction, then may I take so many unites from the whole number, that they may make an improper Fraction, greater then that first, and then worke by Subtraction. As if there be proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more then 3, and not so much as 4, I must take 4 from 6, and turn them into thirds thus $\frac{12}{3}$, then abate $\frac{10}{3}$ and from $\frac{12}{3}$, there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$, which is $\frac{10}{3}$, from 6 whole: then set 1 under 6, as thus $\frac{6}{1}$: And then to reduce those two Fractions into one Denomination, as here appeareth $\frac{10}{3}$ from $\frac{6}{1}$: Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$, which maketh $2\frac{2}{3}$, your desire. And thus will I make an end of the worke of subtraction of Fractions, and proceed to Multiplication.

$$\frac{8}{3} \times \frac{18}{3}$$

$$3$$

Multiplication of Fractions.

THerefore when any two fractions be proponed to be multiplied together, the numerator of the one must be multiplied by the numerator of the other: and the sum that amounteth thereof must be set for a new numerator: likewise the Denominator of the one must be multiplied by the Denominator of the other, and that that amounteth shall be set for the Denominator, and this new third Fraction expresseth the Product of the Multiplication of the two first Fractions proponed, whereof take this Example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$ doth make $\frac{15}{60}$.

$$\frac{3}{5} \times \frac{5}{12} = \frac{15}{60} = \frac{1}{4}$$

Schol. I perceive then that 3 being the Numerator of the first Fraction, is multiplyed by 5 being the Numerator of the second Fraction, whereof amounteth 15, the Numerator of the third Fraction. And so likewise 5 being the Denominator of the first Fraction, is multiplied by 12 the Denominator of the second Fraction, whereof amounteth 60 the new Denominator, so that I perceive how the work is done, but I do not perceive how $\frac{15}{60}$ is greater then $\frac{1}{4}$, for if I shall use my former manner of examination by the parts of some coin, I see that $\frac{1}{4}$ of a Crown is 36 pence, and $\frac{5}{12}$ of a Crown is 25 pence, whereof the one multiplied by the other, doth make 900 pence, which is 15 Crowns, but by your multiplication

tiplication there amounteth $\frac{11}{20}$, which is but 15 pence, and that is much leſſe then any other of both the firſt Fractions.

Maſter. That difference is between Multiplication in whole numbers, and Multiplication in broken numbers, that in whole numbers, the ſumme that amounteth is greater then both the other whereof it came: but in Fractions it is contrariwiſe: for the ſumme that amounteth is leſſer then any of the other two Fractions whereof it is produced.

Scholar. I deſire much to underſtand the reaſon thereof.

Maſter. Although I purpoſed to reſerve the reaſons of workes Arithmeticall for the perfect Book of Arithmetick, yet I will ſhew you this, becauſe of the ſtrangeneſſe of the work.

You ſee in whole numbers, that of two numbers being multiplyed together, is made the third number, which third number doth beare the ſame proportion to the number multiplyed, that the multiplyer doth beare to an unite. And ſo in Fractions the third number which amounteth of Multiplication, beareth the ſame proportion to each of the two firſt Fractions, that the other of thoſe two Fractions doth beare to an unite.

Scholar. Sir, I underſtand your words thus: when 40 is multiplyed by 12, there doth amount 480, which 480 doth contain 40 ſo many times in it, as 12 doth containe Unites, that is to ſay, twelve times. And ſo it

appeareth that 480 doth contain twelve so many times also as 40 doth contain unites, that is 40 times. But now I see not how the third number in this example of Fractions can contain any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you cannot see that thing which is not possible to be seen of any man, how the third number in Multiplication of Fractions should be greater then any of the two former Fractions: but yet this may you see (which I said) that the third number in Fractions so multiplyed, doth bear the same proportion to any of the two former Fractions that the other of those two Fractions doth bear to an unite, as in your example, $\frac{4}{5}$ being multiplyed by $\frac{1}{10}$, doth make $\frac{11}{90}$. Now I say that $\frac{11}{90}$ doth bear the same proportion to $\frac{1}{5}$ that $\frac{4}{72}$ doth bear to an unite, as you may in your own some of examination by Coine, try it: for in an old angell (which in times past was currant for 7 shilling 6 pence, are 180 half pence) which I set for the entire unite, whose parts (according to the Fractions aforesaid) are these, for $\frac{1}{72}$ set 45 half pence, for $\frac{1}{5}$ take 108 half pence, and for $\frac{4}{72}$ put 75 half pence. Now doth 45 bear the same proportion to 108, that 75 doth bear to 180, for 45 is $\frac{1}{72}$ of 108, and so is 75 also $\frac{1}{72}$ of 180.

But these reasons may be better reserved till another time, when the knowledge of

pro-

proportions in due order shall be taught: yet in the mean season I will shew you how it commeth to passe, that in Fraction the third summe must needs be lesser then any of the other two.

Consider this, that when a Fraction is proponed, as in the former example $\frac{1}{5}$ if it be multiplied by more then 1, it will make more then one entire number. As if I multiply $\frac{1}{5}$ by 5, that is to say, if I take it 5 times, it will make three entire unites. Example: in a Crown $\frac{1}{5}$ of it maketh 3 shillings, which if I take five times, it will amount to 15 shillings, that is, three entire Crowns; so if I take the same $\frac{1}{5}$ but twice, it will yeeld 6 shillings, that is, one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more then it was before, that is 5 shillings. And if I take it lesse then once, it cannot be so much as it was before, Then seeing that a Fraction is lesse then one, if I multiply a Fraction by another Fraction, it followeth that I do take the first Fraction lesse then once, and therefore the summe that amounteth, must needs be lesse then the first Fraction. Note. ☞

Scholar. Sir, I thank you much for this reason. And I trust I doe perceive the thing; as by example of this same Fraction $\frac{1}{5}$ I will expresse. If I take $\frac{1}{5}$ of a Crown once, that is to say, if I multiply $\frac{1}{5}$ by 1, it will be as it was before, but 3 shillings: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then will

will it be but half so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the first Fraction.

And so to make an end: if I take but the twelfth part of one, that is, if I do multiply it by $\frac{1}{12}$ it will yeeld but the twelfth part of the first Fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make 3 pence, then $\frac{1}{12}$ must needs make five times so much, that is, 15 pence, which was the summe that hath given the occasion of all this doubt.

Master. Then I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will proceed to the rest.

To multiply a whole number into a fraction.

In Multiplication *it happeneth sometime, that there be whole numbers to be multiplyed with Fractions, and may be in two sorts: for either the whole number is severall from the Fraction, and is the* Multiplier, *or else the whole number is joyned with one, or both of the Fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no* Reduction, *but onely multiply the* numerator *of the Fraction by that whole number and the totall thereof set for the new* numerator.

Scholar. I understand you thus. If I have $\frac{6}{24}$ to be multiplyed by 16, then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator keeping still 23

for

for the Denominator, and so the Fraction will be $\frac{26}{33}$ that is $4\frac{6}{34}$.

Master. And in this sort of work you may abridge the labour thus. If it happen the Denominator to be such a number as may evenly be divided by the said whole number proposed, then divide it thereby, and set the Quotient of that division for the former Denominator, but reserve still the numerator, and so is the multiplication ended.

Scholar. Then fain this example $\frac{7}{20}$ to be multiplyed by 5, and because 5 will justly divide 20, therefore I take the Quotient of that division, which is 4, and set it in stead of 20, and so the Fraction will be $\frac{7}{4}$ that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$ that would have followed of the other sort of work.

Scholar. I perceive it very well.

Master. *Now then for the other sort, where the number is mixt, take this way: first to reduce the said whole number and fraction into one improper Fraction (as I shewed you in* Reduction*) and then multiply them together, as if they were proper Fractions.* How to multiply mixt numbers.

Scholar. $13\frac{3}{5}$ being set to be multiplyed by $\frac{5}{7}$ first I must reduce the mixt number, as in this Example appeareth, by multiplying 13 by 5, and that maketh 65, whereto I must adde the numerator 3,

$13\frac{3}{5}$	340
68 by	5
5 40	8

and

and ſo the Fraction will be $\frac{16}{5}$ which two Fractions now I ſhall multiply after the accuſtomed form, and it will be $\frac{192}{40}$, or $\frac{12}{4}$.

Maſter. You have done well: and ſo may you ſee, that although moſt part of the formes of Multiplication may be wrought without Reduction, yet ſome cannot, as namely, mixed numbers.

Duplation. *And yet one note more I will tell you of* Multiplication *before we leave it: That is, whenſoever you would multiply any* fraction *by* 2, *which commonly is called* Duplation, *you may doe it not only by doubling the* numerator, *but alſo by parting the* denominator *into half, if it be even.*

Scholar. When if I would double $\frac{5}{12}$ I may chuſe whether I will make it $\frac{10}{12}$ or elſe $\frac{5}{6}$. And indeed I ſee that is all one, but that the dividing of the Denominator ſeemeth the better way to make ſmaller termes of the Fraction, and ſo they ſhall need the leſſe Reduction.

Maſter. It is ſo: and now I ſhall not need to tell you that multiplication is proved by Diviſion, and Diviſion likewiſe by multiplication: but the like worke that I ſhewed you in multiplication, will I ſhew you in Diviſion.

Diviſion

Diviſion of Fractions.

Diviſion of Fractions.

Henſoever two fractions *be proponed, that one ſhould be divided by the other, I muſt ſet down firſt the* fraction *that ſhall be divided* (*which is called the* dividend) *and then after it the other which is the* diviſor; *Then ſhall I multiply the* numerator *of the* dividend *by the* denominator *of the* diviſor, *and that which amounteth I muſt put for a new* numerator. *Again, I ſhall multiply the* denominator *of the* dividend *by the* numerator *of the* diviſor, *and the* number *that amounteth thereof I muſt put for the new* denominator. *And this third* fraction *is the* Quotient *of the ſaid* diviſion.

Scholar. **This ſeemeth eaſie in form, as by example thus; If I would divide $\frac{2}{3}$ by $\frac{5}{8}$, firſt I multiply 5, (being the** Numerator **of the** Dividend) **by 8 which is the** Denominator **of the** Diviſor, **and thereof riſeth 30: then I multiply 8** (being **the** Denominator **of the** Dividend) **by 2, being the** Numerator **in the** Diviſor; **and ſo riſeth 16, the which I muſt make a third Fraction thus $\frac{30}{16}$.**

	30	
5		2
8		6
	16	

Maſter. **I ſee ſeemeth you are quicker in underſtanding now, then you were when I taught**

taught you the Art of whole numbers, but that is no marvell: for the more knowledge that a man getteth, the readier shall he finde his wit, and quicker in understanding: but yet of two things I will admonish you, which you might have observed here for the ease of work and lightnesse of understanding, the nature of the Quotient.

Whensoever you divide one Fraction by another, either they be both equall together, or else the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first Fractions be inequall, their Quotient shall declare the same by the inequality of the Numerator and Denominator, as in these Examples following shall appear.

First, if equall Fractions $\frac{2}{7}$ and $\frac{12}{27}$ bee equall together, and if the one be divided by the other, the Quotient will bee $\frac{189}{196}$, as you may perceive by that Rule aforesaid.

Now in the unequall Fractions, as $\frac{2}{9}$ and $\frac{1}{1}$, the Quotient will be $\frac{2}{7}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion between two numbers.

Master. The reason is this: when any Fraction is divided by another, the Quotient declareth what proportion the Dividend beareth to the Divisor. So $\frac{1}{2}$ divided by $\frac{1}{4}$, maketh 2, which must bee founded, not 2, but twice,

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the Quotient representeth the Dividend, and the Denominator representeth the Divisor. And this is alwayes true, whether the greater Fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be exactly known, till you have learned the Art of proportions: notwithstanding somewhat of it I have declared in the Rule of Reduction. But now for the easie remembrance of the Quotient in Division, as soone as you have set downe your two Fractions the one against the other, then make a straight line for the Quotient: and as soone as you have multiplyed the Numerator of the Dividend, by the Denominator of the Divisor, set the Number that amounteth over the said line, and then multiply the other two Numbers, and set their totall under the same line.

Scholar. I perceive you would not have me trust to memory till I were better expert, lest oftentimes I happen by misse-remembrance to bee abused. This Example I take for that declaration.

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$ I must set the numbers one against the other (as here doth appear) and then make another line for the Quotient in some good distance,

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

distance, where I may set the numbers of the Quotient, as soon as any of them is multiplied. So then as soon as I have multiplyed 2 by 4, which maketh 8, I shall set that 8 over that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set under the same line, and then will the whole Quotient appear thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I perceive that?

Master. Although you might better perceive it by the Rule of Reduction, yet this example may be declared in common coines, as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions do agree. And if you had taken this Example before when you took the Example of $\frac{1}{2}$ and $\frac{1}{3}$, your Quotient would appear (as this doth) more easie to understand; whereas that Quotient being $\frac{12}{16}$, is not an easie proportion for you to perceive, being yet little acquainted with proportions.

Scholar. If there be whole numbers to be divided by a Fraction, how shall I perform it?

To divide a whole number by a fraction.

Master. *When any whole number shall be divided by a fraction, you must multiply the said whole number with the Denominator of the Fraction, and set the totall thereof for the new Numerator, and for the Denominator set the*

the Numerator of the fraction.

Scholar. Then 20 divided by $\frac{1}{4}$ will make $\frac{20}{1}$, as here appeareth $\frac{80}{3}$.

20 / 1 by 1/4 = 80 / 3

Master. *Even so: but if you would divide the Fraction by the whole number, then multiply the Denominator by the same whole number, and set the total all for the Denominator, without changing the Numerator.* To divide the Fraction by the whole number.

Scholar. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this Example $\frac{20}{92}$.

20 / 23 by 4 = 20 / 92

Master. You say well. And by the same Example you give me cause to remember another brief way to doe the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not only as well done, but also in a Fraction of lesser terms. Another brief way.

Scholar. I guesse it to be even so, by a like work that you taught me in multiplication: And for proof thereof $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set for 20 over 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these termes by 4, and so reducing them to

to their least Denomination, I may easily prove: as appeareth by this example $\frac{12}{29} \frac{1}{13}$.

Master. You conceive it well. And if there be mixt numbers, (either one or both) you must first reduce that mixt number into an improper Fraction, and then work as you have learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: if the Denominators be like, then divide the Numerators, as it were in whole numbers, and the Quotient, whether it be Fraction, whole number or mixt, is a good Quotient for that Division. And generally, if one of the Numerators may justly divide the other by that Quotient, multiply the Denominator of the lesser Numerator, and set it that doth amount in the roome of the same Denominator, and then for a numerator to it, set the Denominator of the other Fraction.

Scholar. Then if I would divide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will divide 12, and the Quotient will be 4, by which I must multiply the other 4, that is the Denominator under 3, and then it is 16, which is set for the Denominator 4, and over it in stead of 3 I must set 17 the other Denominator, and so it is thus $\frac{17}{16}$.

Master. And so is $\frac{7}{16}$ in stead of $\frac{11}{17}$, which would have risen by the common work, as here appeareth.

	51	
3		12
4	by	17
	48	

And

And now for Mediation (which is to divide by 2) marke this, if the Numerator *be an even number, set the half of it in his place without the Divisor, and so have you done: and if the* numerator *be not even, then double the* Denominator.

Scholar. That is, if I would mediate $\frac{16}{11}$, I may make the Quotient $\frac{8}{11}$ and if I would mediate $\frac{7}{11}$, I must make it $\frac{7}{22}$.

Master. And thus will I make an end of the works of common fractions for this time, not doubting but you can apply them both to the Rule of Progression, and also to the Golden Rule, without any other teaching then you have learned before, which might seem tedious to repeat, in regard you have sufficient knowledge in Reduction, Addition, Subtraction, Multiplication, and Division: And therefore will I goe in hand with the Rule of Proportion, or Golden Rule, which now will appeare easie enough.

The Golden Rule direct in Fractions.

Master.

THerefore as touching the Golden Rule for the placing of the three numbers proponed in the question whereby to finde the fourth, and for the form of their worke, with other like notes, I referre you to that which you have already learned. The rule of proportion, in Fractions.

Note this for a generall Rule.

But this eaſie forme of working by Fractions ſhall you note, that if your three numbers be Fractions, for an apt work and certaine, multiply the numerator of the firſt number in the queſtion, by the Denominator of the ſecond: And all that againe multiply by the Denominator of the third number, and the totall thereof ſhall you keep for to bee the Diviſor. Then multiply the Denominator of the firſt number by the numerator of the ſecond, and the whole thereof by the numerator of the third, and the totall thereof ſhall be your Dividend.

Now divide this dividend by the Diviſor which you found out before, and that number ſhall be the fourth number of the queſtion which you ſeek for, as in this example.

A queſtion of velvet.

If $\frac{1}{4}$ of a yard of Velvet coſt $\frac{2}{3}$ of a ſoveraign, eſteemed at 20 ſhillings, what ſhall $\frac{1}{6}$ coſt?

Scholar. If it pleaſe you to let mee make the anſwere, I would firſt place theſe three numbers, as I learned in whole numbers thus: $\frac{1}{4}$ Z $\frac{2}{3}$ $\frac{1}{6}$

And then according to your new rule, I muſt multiply 3, being numerator in the firſt number, by 3, the denominator of the ſecond: and thereof cometh 9, which I multiply againe by 6, the denominator of the third number, and ſo have I 54, which I keep for the diviſor. Then multiply I 4 the denominator of the firſt, by 2 the numerator of the ſecond, and there ariſeth 8, which againe I multiply by

by 5 the numerator of the third, and it maketh 40. Then muſt I divide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in leſſer termes, and then the figure will ſtand thus.

But what that is in money I cannot tell, except I ſhall worke it by Reduction, as you taught me.

Maſter. It forceth not now, you may reduce it when you liſt, but it were diſorderly done here to mingle divers workes together, where we do not ſeek the value of the thing in common money, but in apt number, which yee have well done: and therefore will I yet ſhew you another like way of eaſineſſe in worke, how you may change your Three Fractions into three whole numbers, by which you ſhall worke, as if the queſtion were propoſed in whole numbers. The firſt number you ſhall finde as I taught you: now to finde the Diviſor of the ſecond number, take the numerator for the ſecond Fraction: and for the third number take that that ariſeth of the multiplication of the Denominator of the firſt, by the numerator of the third, and then worke your queſtion.

Scholar. *For example hereof: I put this queſtion. If* $\frac{11}{12}$ *of 1 pound weight of ſilver, be worth* $\frac{12}{4}$ *of a Soveraigne, what is,* $\frac{1}{2}$ *of 1 pound weight worth?* A queſtion of ſilver.

For the anſwer, firſt I place the Fractions in order thus:

Then to turne these Fractions into whole numbers, I multiply 11, which is the Numerator of the first by 4 (the denominator of the second) and there cometh 44, which I multiply by 2 the denominator of the third, and so amounteth 88, which I set for the Divisor in the first place. Then in the second place I set 12, which is the Numerator of the second Fraction, and in the third place, I set the sum that amounteth of 12 being the denominator of the first number multiplied by one, being Numerator in the third number, and so the figure will stand as here you see.

88 Z 12
12 Z

The proof of the golden rule.

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be $1\frac{16}{88}$, or in lesser termes, $1\frac{2}{11}$, and then the figures will stand thus:

11 Z 12
12 Z 4
1 Z $1\frac{2}{11}$
3

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shall you mark for the proof of this Rule, whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth; then multiply the second by the third, and mark what amounteth also. Now if those two numbers so amounting be equall, then is your worke well done, else you have erred. And this shall suffice for the former Rule.

The

The Backer Rule, or Reverſe Rule in Fractions.

BUt in the Backer Rule, this ſhall you note for eaſe of worke, that you multiply the numerator of the firſt by the numerator of the ſecond, and the whole thereof by the Denominator of the third, and that amounteth thereof, ſhall bee the dividend. Then multiply the denominator of the firſt, by the denominator of the ſecond, and that whole by the numerator of the third, and that that ariſeth thereof, ſhall be the diviſor. Example of this. The backer Rule in Fractions. Note this alſo for a generall Rule.

I did lend my friend $\frac{1}{4}$ *of a* Porteguiſe, *ſeven Moneths upon promiſe that he ſhould do as much for me again; and when I ſhould borrow of him, he could lend me but* $\frac{1}{12}$ *of a* Porteguiſe: *now I demand how long time I muſt keep of his money in juſt recompence of my loane, accounting* 13 *Moneths in the year.* A queſtion of Loan.

Scholar. **The firſt number muſt be the firſt money borrowed, that is** $\frac{1}{4}$ **of the** Porteguiſe: **the ſecond number the 7 moneths, that is** $\frac{7}{13}$ **of a** yeare: **and the** third nnmber **the** money **that was lent in recompence, that is** $\frac{1}{12}$ **of a** Porteguiſe: **then I ſet the numbers thus:**

$$\begin{matrix} \frac{1}{4} & \frac{7}{13} \\ \frac{1}{12} & \end{matrix} \quad Z$$

Then (as you taught me) I multiply 3 (being Numerator in the firſt number) by 7, the numerator of the ſecond number, and it maketh 21, which I multiply by 12 the Denominator of the third, and ſo have I 252 for the dividend: then I multiply 4, the Denominator of the firſt, by 13 the Denominator of the ſecond, and it yeeldeth 52, which I multiply again by 5, the Numerator of the third, and it will make 260, that is the Diviſor. Then muſt I divide 252 by 260, ſo it will be in the ſmall fraction $\frac{252}{260}$ of a year.

Maſter. And thus do you ſee ſome eaſe in working, better then to multiply and divide tediouſly ſo many Fractions.

Statute of Aſſiſe of bread and ale.

Another queſtion yet will I propoſe, to the intent you may ſee thereby the reaſon of the Statute of aſſiſe of Bread and Ale, which in all Statute Bookes, in Latine, French and Engliſh is much corrupted for want of knowledge in this Art; for the right underſtanding whereof, I propone this queſtion.

Queſtion of bread.

When the price of a quarter of Wheat is 2 ſhillings, the farthing white loaf ſhall weigh 68 ſhillings; then I demand what ſhall ſuch a loafe weigh, when a quarter of wheat is ſold for 3 ſhillings.

Scholar. This queſtion muſt be wrought as it is propoſed in whole numbers: and not in Fractions.

Maſter. You ſeem to ſay reaſonably, howbeit in the Statute of Aſſiſe, the rate is made by

by the proportion of parts in a pound weight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other moneyes doe: but this Statute being well understood, is a continuall Rule for ever, as I will anon declare by a new Table of Assise, converting the shillings into ounces, and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound weight, and so by a penny $\frac{1}{20}$ of an ounce: wherefore although you might worke this question proponed by whole numbers well enough, for that time when the Statute was made, yet to apply it to your time, and make it serve for all times generally, it is best to worke it by Fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings $\frac{68}{20}$, and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus.

$\frac{2}{20}$ Z $\frac{68}{20}$
$\frac{1}{20}$

In which question, because all the Denominators be like, you shall worke onely with the Numerators.

Scholar. Then shall I multiply 68 by 2, whereof cometh 136, which if I divide by 3, the quotient will be $45\frac{1}{3}$: but how shall I make a Fraction of that, to stand with the other?

Master. Have you so soon forgotten what was taught you so lately? this is his forme.

$45\frac{1}{3}$
20

Scholar. I remember it now, and then it signifieth 45 twenty parts, and the third deale of one twenty part.

Note what a shilling is.

Master. So is it that maketh in shillings 45 shillings 4 pence, whereby you may note one great errour in the Statute Books; which have constantly 48 shillings in that Assize. And by this Rule, if you examine the Statute, you shall finde many summes false. Wherefore for the true understanding of that Statute, and such like, as I have made mention of it, and somewhat recognized it, so do I wish that all Gentlemen and other Students of the Lawes would not neglect this Art of Arithmeticke, as unneedfull to their studies. Wherefore to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the Statutes in two Columnes, and in a third Columne, I will note the correction of those errours which have crept into it.

Here followeth the Table.

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Bookes.			The Correction by juft Aſſiſe.		
s.	d.	l.	s.	d.	l.	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	2	8	0	3	8	0
2	6	2	14	4 1/3	2	14	5 2/3
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2 1/2	1	7	2 1/11
5	6	1	4	8 1/4	1	4	8 1/22
6	0	1	2	8	1	2	8
6	6	0	16	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 2/3
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	0 3/4	0	14	3 11/19
10	0	0	13	7 1/2	0	13	7 1/7
10	6	0	12	11 1/2	0	12	11 1/2
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 22/22
12	0	0	11	4	0	11	4

In the common Books there is no further rate of Assise made, then into 12 s. the quarter of wheat, but in an ancient Copy of 200 years old (which I have) there is added the rate of Assise unto 20 s. the quarter, but yet was that Assise also either wrong cast at the first penning, or else corrupt since that time, for lack of just knowledge in the Rule of Proportion, *which I will adde here also to gratifie such as be desirous to understand truth exactly.*

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Bookes.			The Correction by just Assise.		
s.	d.	l.	s.	d.	l.	s.	d.
12	6	0	11	0	0	10	10 16/23
13	0	0	11	0 2/7	0	10	5 3/13
13	6	0	10	1 1/2	0	10	0 1/2
14	0	0	9	7	0	9	8 4/7
14	6	0	9	2 1/2	0	9	4 15/29
15	0	0	9	1 1/2	0	9	0 4/7
15	6	0	9	1 1/4	0	8	9 7/31
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	2 12/11
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	10 2/33
18	0	0	7	6	0	7	6 2/3
18	6	0	7	3	0	7	4 1/27
19	0	0	7	2	0	7	1 17/19
19	6	0	5	10	0	6	11 2/23
20	0	0	5	6	0	6	3 1

These

These two Tables I have set severall, because no man should think that I would either adde or take away from any Law those parts which might of right seem either superfluous, either diminute: but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but every good Subjects duty with sobriety to correct. And for avoyding of offence, I have rather done it in this private Booke, then in any Booke of the Statutes it self, trusting that all men will take it in good part.

Scholar. I would wish so, but I dare not so hope, sith never good man that would reforme error, could escape the venemous tongues of envious detractors, which because they either cannot or list not to do any good themselves, doe delight to bark at the doings of others, but I beseech you to stay nothing for their perverse behaviour.

Master. I consider many things that some may object, whereunto I am not unprovided of just answers, but I will not seem so hasty to make the answers before I hear their Objections, but I trust that men are of a better nature, and more gratefull now then some have been in times past. As I have done in the Statute of Assise for bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applyed to all times: But I mean not by this to alter one word of the Statute

Concerning the following Tables.

Statute (being ſo good an Ordinance, and of ſo great continuance) but onely to make it as a kinde of expoſition and declaration of the ſaid Statute, truſting that thereby the Statute may be better underſtood, and conſequently better put in execution. And here you ſhall note, that I have accounted the ſhillings after the rate of 60 ſhillings to the pound weight, becauſe I eſteem it the moſt apt for our time. Wherefore in the firſt Columne you finde the price of Wheat directly againſt it; in the ſecond Columne, you may finde the weight of a farthing white loafe, in this our time: and if you double the number (as I have done in the third Columne) then have you the weight of the halfe penny white loafe; and ſo in the fourth Columne) is ſet the weight of a penny white loafe; It needeth not to tell, that the light doth teſtifie how that every Columne is parted into three ſmaller pillars, whereof the firſt Columne hath theſe three titles; pounds, ounces and penny weights. And as in the firſt Columne 12 pence make a ſhilling, and 20 ſhillings make a pound, ſo in the other three Columnes 20 pence weight maketh an ounce, and 12 ounces doe make a pound.

A pound weight.

Gentle

GEntle Reader, touching the underſtanding of the *Table* following, wherein according to our time, *Maſter Record* alloweth 60 *pence* to the *ounce*, and 3 *pound* or 60 *ſhillings* to the *pound*, and thereupon after the rate of 60 *ſhillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *ſhillings the quarter*, till he come to 40 *ſhillings* 6 *pence the quarter*. And this his proportion (for that he hath not ſet down any one *Example* to continue the work) hath been hard for many to conceive or comprehend, and therefore the onely chief cauſe why I have written this *digreſſion*, for the better underſtanding of him therein.

The firſt thing therefore that is ſought for in this *Table*, as in the other aforeſaid, is a *Maxime* grounded upon the *Statute*, which is this. When the *quarter of Wheat* is ſold for *two ſhillings*, then the *farthing white loafe* ſhall weigh 68 *ſhillings*, where by a *ſhilling* is meant $\frac{1}{20}$ of a *pound*, and by a *penny*, $\frac{1}{20}$ of an *ounce*. Now therefore for a generall Rule, to finde what weight the *farthing white loafe* ſhall weigh at 3 *ſhillings the quarter*, till you come to 40 *ſhillings*, 6 *pence the quarter*, is thus to be wrought. Comming to the *firſt ground*, and working by the *Backer Rule*, ſay; If *two ſhillings the quarter* give, or allow the *farthing white loafe* to weigh 68 *ſhillings*, what weight ought the *farthing white loaf* to weigh at 3 *ſhillings the quarter*? Worke, and you ſhall find 45 *ſhillings* 4 *pence*, as before in the correction of the *firſt*

Table

Table is noted. Then for the *ſecond work*, ſay by the *Rule of Three Direct*, if 20 *pence* give one *ounce*, what giveth 45 *ſhillings* 4 *pence*? multiply and divide, and you ſhall finde 544 *ounces*, which 544 *ounces* being multiplyed by 3, for 3 *pounds*, or 60 *ſhillings*, yeeldeth 1632 *ounces*, which divided by 20, produceth 81 *ounces*, and $\frac{12}{20}$ or rather $\frac{1}{5}$ of an *ounce*, equall unto 12 *penny weight*, which is *halfe an ounce*, and 2 *penny weight*, and ſo maketh in all 6 *pounds*, 9 $\frac{1}{2}$ *ounces*, and 2 *penny weight*. Now the next way to continue this *Table* to know the weight of the halfe *penny white loafe*, is thus, multiply 1632 *ounces* by 2 and it bringeth forth 3264 *ounces*, and divided by 20, it yieldeth 163 *ounces*, and $\frac{4}{5}$, which is equall to 13 *pounds* 7 *ounces*, and 4 *penny weight*, as M. *Record his Table noteth*.

Thirdly, for the weight of the penny white loafe, multiply 1632 *ounces* by 4, and divide by 20, and after by 12 as before, and you ſhall finde 27 *pounds* 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or elſe by doubling the *farthing white loafe*, for the *weight of the halfe penny white loafe*, and ſo doubling the *halfe penny white loafe*, for the *weight* of the *penny white loafe*, is the order to continue the *Table* to the end thereof.

The

The price of a Quarter of Wheat.

li.	s.	d.
0	3	0
0	4	0
0	6	0
0	7	0
0	9	0
0	10	0
0	12	0
0	13	0
0	15	0
0	16	0
0	18	0
0	19	0
1	1	0
1	2	0
1	4	0
1	5	0
1	7	0
1	8	0
1	10	0
1	11	0
1	13	0
1	14	0
1	16	0
1	17	0
1	19	0
2	0	0

The weight of a farthing white loaf.

Po.	ounces.	Penny weight.
6	9	12
4	6	8
3	4	16
2	8	12 4/7
2	3	4
1	11	6 4/7
1	8	8
1	6	2 1/7
1	4	6 2/7
1	2	16 8/11
1	1	12
1	0	11 8/13
1	0	4 1/7
	10	17 1/2
	10	4
	9	12
	9	1 1/2
	8	11 11/19
	8	3 2/7
	7	15 6/7
	7	8 8/11
	7	1 12/21
	6	16
	6	10 1/35
	6	5 7/15
	6	0 3/4

The price of a Quarter of Wheat.

li.	s.	d.
0	3	0
0	4	6
0	6	0
0	7	6
0	9	0
0	10	6
0	12	0
0	13	6
0	15	0
0	16	6
0	18	0
0	19	6
1	1	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

The weight of a half penny white loaf.

Po.	ounces.	Penny weight.
13	7	4
9	6	16
6	9	12
5	5	$5\frac{3}{5}$
4	6	8
3	10	$12\frac{4}{7}$
3	4	16
3	0	$13\frac{1}{3}$
2	8	$12\frac{4}{9}$
2	5	$13\frac{3}{11}$
2	3	4
2	1	$2\frac{3}{13}$
1	9	$9\frac{1}{5}$
1	8	$15\frac{1}{5}$
1	7	8
1	6	$2\frac{2}{3}$
1	5	$1\frac{11}{19}$
1	4	$6\frac{2}{5}$
1	3	$10\frac{6}{3}$
1	2	$16\frac{5}{11}$
1	2	$3\frac{12}{21}$
1	1	12
1	1	$1\frac{1}{25}$
1	0	$11\frac{1}{13}$
1	0	$0\frac{6}{42}$

The weight of a half penny white loaf.

Po.	ounces.	Penny Weight.
27	2	8
18	1	12
13	7	4
10	10	$11\frac{1}{3}$
9	0	16
7	9	$5\frac{1}{7}$
6	9	12
6	0	$10\frac{1}{2}$
5	5	$5\frac{3}{7}$
4	11	$6\frac{12}{11}$
4	6	8
4	2	$4\frac{4}{13}$
3	0	$19\frac{1}{5}$
3	7	$10\frac{1}{9}$
3	4	16
3	2	8
2	0	$5\frac{5}{8}$
2	10	$7\frac{1}{30}$
2	8	$12\frac{2}{7}$
2	7	$1\frac{1}{7}$
2	5	$13\frac{1}{11}$
2	4	$7\frac{12}{33}$
2	3	4
2	2	$2\frac{6}{15}$
2	1	$2\frac{1}{13}$
2	0	$0\frac{1}{45}$

HAving ſpoken before for the underſtanding of the *Table* placed by M. *Record*, a man indued with rare knowledge in *Arithmeticall* and *Geometricall proportions*, touching the *Statute* of *Coynage*, *and the ſtandard thereof*, as appeareth in his Epiſtle of this *Book dedicated* to *King Edward the ſixth*, inſinuating unto his Highneſſe that the *Standard of Coyne* is much altered from the 14. year of *King Edward the third* (when this *Satute and Aſſiſe* was confirmed) to the *Standard of this our time*. For it appeareth that in *King Edward the thirds time*, when the *Aſſiſe of Bread and Drink* was eſtabliſhed, that a *Sterling penney*, *round without clipping*, did then weigh 32 *cornes of wheat drie*, and taken out of the middle of the *eare*, and 20 of theſe *pence* made an *ounce*, and 12 *ounces* made a *pound Troy*. And ſo from the *weight of a penny* to 20 *ſhillings ſterling*, which then weighed 12 *ounces*, tooke *Bread his weight* and proportion: And now finding 60 *pence* is an *ounce*. That only cauſe (I perceive, for the zeal of a Commonwealth) moved him to ſet down the ſame *Table in this private Book*; meaning not thereby to alter any word of the *Statute* being ſo good an ordinance and of ſo long continuance, but as a kinde of expoſition by the way, that thereby the *Statute* may be better underſtood, and ſo conſequently better put in execution. Which *Aſſiſe of his*, is three times greater then the *Statute* now alloweth. Therefore alſo, to gratifie ſuch as are deſirous of knowledge, according to theſe *prices of a quarter of wheat*) I have added to this *Author theſe three other new Tables following*, and reduced their *prices* into their juſt proportions of *Sterling money*, and alſo reduced the *Money* into known *weight Troy*, according to the Statute. And thereafter according to proportion in my othe three *Tables*, have I noted the *juſt weight* that a *Farthing*, *Half peny*, *and Peny white-Loafe*, ought to weigh by the *Statute*.

The price of a Quarter of Wheat.

l.	s.	
0	3	
0	4	
0	6	
0	7	
0	8	
0	9	
0	10	
0	12	
0	13	
0	15	
0	16	
0	18	
0	19	
1	1	
1	2	
1	4	
1	5	
1	7	
1	8	
1	10	
1	11	
1	13	
1	14	
1	16	
1	17	
1	19	
2	0	

The weight of a Farthing white loaf in Sterling money by Assise.

l.	s.	d.	
2	5	4	
1	10	2	$\frac{2}{3}$
1	2	8	
0	18	1	$\frac{1}{5}$
0	15	1	$\frac{1}{3}$
0	12	1	$\frac{5}{7}$
0	11	4	
0	10	0	$\frac{8}{9}$
0	9	0	$\frac{4}{5}$
0	8	0	$\frac{2}{11}$
0	7	6	$\frac{2}{3}$
0	6	6	$\frac{1}{13}$
0	6	0	$\frac{9}{13}$
0	5	8	
0	5	4	
0	5	0	$\frac{4}{9}$
0	4	9	$\frac{1}{19}$
0	4	6	$\frac{2}{5}$
0	4	3	$\frac{11}{21}$
0	4	2	$\frac{1}{11}$
0	3	11	$\frac{2}{23}$
0	3	9	$\frac{2}{3}$
0	3	7	$\frac{11}{25}$
0	3	5	$\frac{11}{13}$
0	3	4	$\frac{8}{27}$

The weight of a farthing white loaf in Troy weight by Assise.

Ounces.	Pounds.	Penny weight.	
2	3	4	
1	6	2	$\frac{2}{3}$
1	1	12	
0	10	17	$\frac{1}{5}$
0	9	1	$\frac{1}{3}$
0	7	15	$\frac{1}{7}$
0	6	16	
0	6	0	$\frac{8}{9}$
0	5	8	$\frac{4}{5}$
0	4	18	$\frac{2}{11}$
0	4	10	$\frac{2}{3}$
0	4	3	$\frac{1}{13}$
0	4	1	$\frac{1}{3}$
0	3	12	$\frac{1}{15}$
0	3	8	
0	3	4	
0	3	0	$\frac{4}{9}$
0	2	17	$\frac{1}{19}$
0	2	14	$\frac{2}{5}$
0	2	11	$\frac{11}{21}$
0	2	9	$\frac{1}{3}$
0	2	7	$\frac{1}{23}$
0	2	5	$\frac{2}{3}$
0	2	3	$\frac{11}{25}$
0	2	1	$\frac{11}{13}$
0	2	0	$\frac{1}{27}$

The Price of a Quarter of Wheat.

li.	s.	d.
1	0	0
1	1	0
1	2	0
1	3	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	12	0
1	13	0
1	14	0
1	15	0
1	16	0
1	17	0
1	18	0
1	19	0
2	0	0

The Weight of a half penny white loaf in Troy weight by Assise.

Po.	ounces.	Penny weight.
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0
0	4	0

The weight of a half penny white loaf in Troy weight by Assise.

Po.	ounces.	Penny Weight.
0	9	16
0	6	10
0	4	6
0	3	7
0	3	0
0	2	7
0	2	3
0	2	0
0	1	9
0	1	7
0	1	6
0	1	4
0	1	4
0	1	2
0	1	1
0	1	0
0	1	0
0	0	11
0	0	10
0	0	10
0	0	9
0	0	9
0	0	9
0	0	8
0	0	8
0	0	8

Scholar. Sir, I doe thanke you moſt heartily for this, not onely in mine owne name, and in the name of all ſtudents, but alſo in the name of the whole Commons, to whom the reſtitution of this Aſſiſe (I truſt) ſhall bring reſtitution of the weight in Bread, which long time hath been abuſed. And if you know any like thing more, wherein you would vouchſafe to declare the errours, and ſet forth the truth, you cannot but obtain great thankes of all good hearted men that love the Commonwealth.

Maſter. I have ſundry things to declare, but I have reſerved them for a private Booke by it ſelf, yet notwithſtanding becauſe the ſtatute of the rate of meaſuring of grounds is ſo common, that it toucheth all men, and yet no more common then needfull, but ſo much corrupt, that is, too farre out of all good rate; not onely in the Engliſh Bookes of ſtatutes, commonly Printed, but alſo in the Latine Bookes, and in the French alſo, (for I have read of each ſort, and conferred them diligently) I will give you a Table for the reſtitution of thoſe errours, as may ſuffice for this preſent time. And firſt I will propoſe one queſtion to you touching the uſe of that ſtatute, whereby you may perceive the order how to examine the whole ſtatute, and every parcell thereof, and the queſtion is this.

A queſtion of meaſuring of ground.

When the Acre of ground doth contain ſixe Perches in breadth, then muſt it contain 40 Perches

ches in length. Then do I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declar unto you another Statute which is the ground of the former Statute. And that Statute is this.

Statute measure.

It is ordained, that three Barly-cornes dry and round, shall make up the measure of an inch: 12 inches shall make a foot, and 3 foote, shall make a yard, (the common English books have an Elne) five yards and a halfe shall make a Perch, and 40 Perches in length, and 4 in breadth, shall make an Acre. This is that Statute, whereby you may perceive, that the intent of the statute is, that one Acre should contain 160 square Perches. Now let me hear you answer to the question.

An Acre.

Scholar. As I perceive by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, so could I make those numbers all in fractions, and so work the question: but seeing I may doe it also in whole numbers, I take that forme for the most ease; therefore thus I set the question in forme.

Then doe I multiply 40 by 4, and it maketh 160, which I divide by 13, and the Quotient is $12\frac{4}{13}$.

$$\begin{array}{r|l} 4 & 40 \\ 13 & 12 \end{array}$$

Master. Now turn that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former statute: Howbeit it shall be best to take one of the least parts in Denomination

for avoyding of much labour, as Feet, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to return $\frac{4}{13}$ into Feet, I multiply $16\frac{1}{2}$ by 4, and it maketh 66. which I must divide by 13, and the Quotient is $5\frac{1}{13}$.

Master. So I find, that if the Acre hold in breadth 13 Perches, it shall contain in length 12 Perches, 5 Foot, and $\frac{1}{13}$ of a foote, which is not fully an Inch, for the Inch is $\frac{1}{12}$ of a foote, But here all the Statute Books in Latine and English (that I have seen) doe note it to be 13 Perches, 5 foote, and one Inch, which maketh above 13 Perches, too many in the Acre: so that I would have thought the errour to have crept into the Printed Books, by the great negligence that Printers in our time do use, save that in written copies of great antiquity, I doe finde the same; yet have I one French copy which hath 12 Perches, and one foote, and that misseth very little of the truth.

Note this errour.

Scholar. Then I see it is true that I have often heard say, that the truest Copies of the Statutes, be the French copies.

Master. That is often true, but not generally, as I have by conference tried diverslie: but in this Statute the French Book is most corrupt in all other places lightly.

But now to performe my promise, I will set forth the Table for measuring of an Acre of ground, only by such parts as the Statute doth mention, because at this time I doe of purpose

purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table *following, I have not done as in the other* Statute *before compared by restitution with the faults crept into the* Statute, *but onely have written that true measure, which the equity of the* Statute *doth pretend. For it were vile to judge of so noble Princes and worthy Counsellors, as have authorised and set forth this* Statute, *that they would make one* Acre *in any forme greater then another, but every one to be just and equall with each other, which is the ground also of my work: and hereby may all men perceive how needfull* Arithmetick *is to the* Students of Law. *But now I think best to make an end of these matters for this present time, sith the* Table *hath in it none obscurity that I should need to declare.*

The bredth of the Acre.	The length of the Acre.			
Perches	Perches	Feete.	Inches.	Parts of an Inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{2}{17}$
18	8	14	8	0
19	8	6	11	$\frac{2}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{2}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{7}$
26	6	2	7	$\frac{6}{13}$
27	5	51	3	$\frac{1}{3}$

The breadth of the Acre.	The length of the Acre.			
Perches	Perches	Feet.	Inches.	Parts of an Inch.
28	5	11	9	$0\frac{1}{7}$
29	5	8	6	$0\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$0\frac{22}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$0\frac{11}{17}$
35	4	9	5	$0\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$0\frac{8}{37}$
38	4	3	5	$0\frac{12}{19}$
39	4	1	8	$0\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$0\frac{22}{41}$
42	3	13	4	$0\frac{2}{7}$
43	3	11	10	$0\frac{12}{43}$
44	3	10	6	0
45	3	9	2	0

Scho-

Scholar. Indeed Sir, I understand the Table (as I think) by those other which you set forth before. For in the first columne is set the Perches of the breadth of an Acre, and then in the two Columnes following appeareth how many Perches, and how many foote that same Acre must have for his length.

Master. You take it well: howbeit to speak exactly of breadth and length, and the first Columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the Title, to let the name remain, although the proportions of the numbers do change: for still by the first Columne is expressed the measure of the one side, and by the two other Pillars in one Columne, is set forth the measure of the other side, And this shall be sufficient now for the use of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certain other Rules, which for their severall names may seem divers Rules, and distinct from this, but indeed they are but branches of it: yet because they have severall workings in appearance, but also

also pleasant in use, I will give you a taste of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I have already said in teaching the works of whole numbers: yet an example or two will we have to refresh the remembrance of the same, and to declare certain proper uses and applications of it, as this for one.

The Rule of Fellowship without time.

Four men got a booty or prize in time of war, the prize is in value of money 8190 *pound, and because the men be not of like degree, therefore their shares may not be equall; but the chiefest person will have of the booty the third part, and the tenth part over: the second will have a quarter, and the tenth part over; the third will have the sixth part: and so there is left for the fourth man a very small portion, but such is his lot (whether he be pleased or wroth) he must be content with one* 20 *part of the prey: Now I demand of you what shall every man have to his share?*

Question of inequall society.

Scholar. You must be fain to answer to your own question, else it is not like to be answered at this time.

Master. The forme to understand the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of some other of them, for all such parts you may overpasse and take for them all those numbers, whose parts they be: as in this example the shares be these $\frac{1}{3}\ \frac{1}{12}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{6}\ \frac{1}{20}$, if

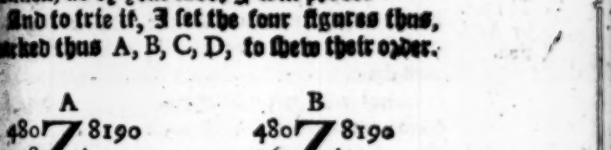
I multiply all the Denominators together, beginning with 3, and so go on unto 20, it will make 144000: but considering that 3 is a part of 6, I will omit that 3, and likewise 10, which is a part of 20, I may overpasse all, and then is there but 3 Denominators to multiply, that is 4, 6, & 20 which make 480, which summe I take for my work, because all the Denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, & it maketh 208. Then for the second mans share, which is $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must have $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had been but 480 pound, then were the question answered: but because the summe was of greater value, by this means now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty, And look what proportion is between the first number and the second, the same proportion shall be between the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third

The reason of this rule.

third place, one of the parts or ſhares, and then worke by the former Rule of Proportion or Golden Rule. And becauſe I have 4 ſeverall parts of the firſt number, by which I would finde out foure like parts of the ſecond number, therefore muſt I make foure ſeverall figures.

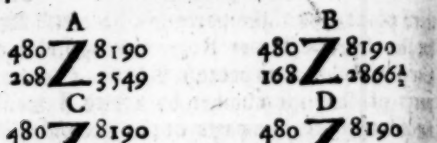
Scholar. Now I truſt I can anſwer to your queſtion, as by your favor I will prove.

And to trie it, I ſet the four figures thus, marked thus A, B, C, D, to ſhew their order.



And then in each of them I multiply the ſecond number by the third, and divide their totall by the firſt, and ſo amounteth the fourth ſumme which I ſeek for: For if I doe multiply 8190 by 208, it maketh 1703520, which being divided by 480, maketh in the Quotient 3542 for the firſt mans portion.

And ſo working with the other three figures, I finde for the ſecond man 2866 $\frac{1}{2}$, and for the third man 1365, and then for the fourth man 409 $\frac{1}{2}$, and ſo every mans ſhare is ſet forth in the figure here annexed.



And thus I think I have done well.

The proof by Addition.

Master. If you misdoubt your working, and list to prove it, adde all the shares together and if they make the totall, then seemeth it well done.

Scholar. **I may set them thus: and then by** Addition **the just summe doth amount, that is,** 8190, **and therefore (as you say) it seemeth to be well wrought.**

	3549
	2866½
	1365
	409½
	8190

But I beseech you, is there any doubt in this triall, that you use that word seemeth?

Master. **You may easily conjecture, that if you did assigne the** first mans share to the last, **and so change all the rest, and one had another share, yet would the** Addition **appear all one, and therefore is not the proof exact.**

But if you will make a just proof for the first mans part. take $\frac{1}{3}$ and $\frac{1}{10}$ of the whole summe, **and if it agree with the** number in the figure, **then it is well done. And so do for the** second, third, **and** fourth summes, **and this proof faileth not. Now will I propound certaine other** questions, **which have been set forth**

The just proof.

lyth by certain learned men, albeit notwithout some oversight, which questions I protest heartily, I do not repeat to deprave those good men, whose labours and studies I much praise and greatly delight in. But onely according to my profession, to seek out truth in all things, and to remove all occasions of errour as much as in me lieth: and for that cause I will onely name the Questions without hurting the Authors name.

The first question is this.

A question of building

Four men did build an house, which cost them 3000 *crownes, their shares were such that one man should pay* $\frac{1}{2}$ *of the summe, and six crownes over: the second should pay* $\frac{1}{3}$ *and* 12 *crownes over: the third man must lay out* $\frac{2}{5}$ *abating* 8 *crownes: and the fourth man should pay* $\frac{1}{4}$ *and* 20 *crownes more. Can you answer to this question?*

Scholar. No, I cannot sir, and that you know best of any man, for I know no more then you have taught me.

Master. Then I dare say you cannot do it, neither yet the best learned man that ever did propose it: for the question is impossible. For declaration whereof, I will be bold to use first the representation of the Numbers in their aptest forme, (although I have not yet taught that manner of work) because it may appear plainly that the question is not possible. For here I have set the parts, and added them, and they make the whole summe, and $\frac{1}{2}$ and 30 more. Now, how is it possible to

An impossible question.

to divide truely either gaines, either charges, so that the particulars $1\frac{1}{4}$ shall be more then the totall?

$\frac{1}{2} + 6$
$\frac{1}{3} + 12$
$\frac{1}{4} + 20$
$1\frac{1}{4} + 30$

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceive it the better) I will trie it after the vulgar forme, as in this figure you see where the $\frac{1}{2}$ with 6 over, is 1506, for the totall, as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{1}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, doe make 5280, where the totall summe should be but 3000, which summe of 3000 if you divide by $\frac{1}{4}$, you shall have $\frac{1}{4}$ of it, that is 1250, and thereto adde 30 more, then will those three sums make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more then the whole summe by $\frac{1}{4}$ and 30 more, and therefore can that question not be accepted as a possible thing, but yet doe certaine learned men propound such questions, and answere to them:

	1506
	1012
	1992
	770
	5280

	£
	2000
	2250
	5280

There-

Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anone declare what may be said for their defence: but in the mean season I will propound the Question as it may be wrought by good possibility.

As if four men build a house together, and it cost them 3000 *Crowns, and then for the partition they agree thus: that as often as the first man doth pay* 6 *Crowns, so often the second man shall pay* 4, *the third man* 8, *and the fourth man* 3. *Or else thus, that the first man shall pay double so much as the fourth, and the second man shall pay* ⅓ *of the first mans charge: the third man shall pay double so much as the second: (and these two waies are to one end) but further for their agreement it is appointed also, that the first shall give* 6 *Crowns overplus, and the second* 12, *and the fourth shall give* 20, *but the third man shall give no overplus, but shall have* 8 *Crowns abated of his charge.*

The former Question of Buildings now possible.

Now is the Question possible to be assoiled, and this is the way to doe it. Mark the proportion of the severall charges, and set out small numbers in that Rate, by which you may reduce the work to the Golden Rule, as here in the first form the numbers are already named, 6, 4, 8, 3: and in the second form, although they bee but plainly named, yet they may bee the same numbers: for 6 is double to 3, and 4 is ⅓ of 6: and again 8 is double to 4. Now adde these together, and they make 21, which 21 must be set for the first number in the Gol-

den Rule: for if it with the overplus of each mans charge would make the totall summe of the charges, then were those severall summes the charges of each man, besides his overplus: but now it is not so.

But yet this is true: (so excellent are conclusions Arithmeticall) that look what proportion each of their severall summes doth bear to 21, the same proportion doth the just charges of every man (besides his overplus) bear to the totall of the charges, the overplus being deducted: wherefore this may you note, that before you do apply the totall of his charges to the Golden Rule, you must deduct the overplus, which is 6, 12, and 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule, for the second summe: and for the third summe, I must put each of the small numbers before mentioned, which although they be not several charges, yet they represent them in proportion. And so making for every mans charge a several Question, the Figures will be 4, which I mark with four letters, A, B, C, D, thus.

A

21 Z 2970
6 Z 848 4/7

B

21 Z 2970
4 Z 565 1/7

C

21 Z 2970
8 Z 1131 3/7

D

21 Z 2970
3 Z 424 3/7

Where I have ſet for briefneſſe the ſumme of every mans charge in the fourth place, preſuppoſing that you can tell how to trie out that fourth ſumme by ſo many Examples as yet have bin.

Scholar. As I truſt that I underſtand this ſorme, ſo I deſire much to know what may be ſaid for them that miſtook this Queſtion.

Maſter. You ſeem ſo deſirous to know this errour, that you have forgotten to examine, whether this work be without fault.

Scholar. He ſeemeth this work to be well done, becauſe the Addition of the four ſeverall numbers doth make the totall ſumme of 1970, which was to be divided into ſuch four parts.

Maſter. But then have you forgotten that the firſt man muſt pay ſix Crowns more beſides his ſhare, and the ſecond man 12 Crowns more, the third man 8 Crowns leſſe, and the fourth man 20 Crowns more; for without theſe, your firſt totall of 3000 Crowns will not be made.

Scholar. Then muſt I adde to the firſt mans ſumme 6 more, and it will be $854\frac{4}{7}$, and to the ſeconds ſumme, I muſt adde 12, and it will be $577\frac{1}{7}$: from the thirds ſumme I muſt abate 8, and then will the ſumme be $1123\frac{6}{7}$: then adding unto the fourths ſumme

20, it will be $444\frac{3}{7}$, and these four ſummes will make 3000, which is the whole charge, as in this Example it may appear, where firſt I gather that $\frac{12}{7}$, that maketh 2, and ſo proceed I in the Addition to the end.

	$854\frac{6}{7}$
	$577\frac{1}{7}$
	$1123\frac{1}{7}$
	$444\frac{3}{7}$
	3000

Maſter. Now have you well done, and this work in the ſame ſummes is brought of other Learned for the true ſolution of the Queſtion, as it was proponed, which (as I ſaid) was impoſſible: and now examine by theſe ſeverall ſummes, and ſee whether it doeth agree with the ſummes in the Queſtion proponed.

The firſt man muſt pay $\frac{1}{2}$ and 6 over of the totall ſumme: how think you, is $854\frac{6}{7}$ the half and 6 more of 3000?

Scholar. No that it is not, for it ſhould be 1506: and for the ſecond man 1012, and for the third man 1992, and for the fourth man 770: whereof not one ſumme agreeth to this work. But I marvell, that ſo wiſe men could be ſo much over-ſeen.

Maſter. It is commonly ſeen, that when men will receive things from elder Writers, and will not examine the thing, they ſeem rather willing to erre with their Ancients for company, then to be bold to examine their Workes or Writtings. Which ſcrupulosity hath ingendred infinite errors in all kindes of know-

knowledge, and in all civill adminiſtration, and ſo in every kinde of Art. But theſe Learned men did not mean any other thing by this Queſtion, then to finde ſuch numbers as ſhould bear the ſame proportion together, as thoſe numbers in the Queſtion propoſed did bear one to another: which thing you ſhall perceive more plainly by another Queſtion of theirs, that is this:

A man lying upon his Death-bed, bequeathed his goods (which were worth 3600 Crowns) in this ſort: becauſe his Wife was great with childe, and he yet uncertain whether the Childe were Male or Female, he made his bequeſt conditionally, that if the Wife bare a Daughter, then ſhould the Wife have half his goods, and the Daughter $\frac{1}{3}$, *but if ſhe were delivered of a Son, then that Son ſhould have* $\frac{1}{2}$ *of the goods, and his Wife but* $\frac{1}{3}$. *Now it chanced her to bring forth both a Son and a Daughter; The Queſtion is, How ſhall they part the goods agreeable to the Teſtatour his Will?* A Queſtion of a Teſtament

Scholar. If ſome cunning Lawyers had this matter in ſcanning, they would determine this Teſtament to be quite void, and ſo the man to die intestate, becauſe the Teſtament was made inſufficient, ſith this condition was not expreſſed in it, and alſo it might have chanced that ſhe ſhould have brought forth neither Son nor Daughter, as often hath been ſeen: ſo is the Will inſufficient in that point alſo.

Maſter. Such Scanners ſhould ſeem to cunning, and yet not ſo cunning as cruell: for the minde of the Teſtator is to bee taken favorably for the aid of the Legatories, when there ariſeth ſuch doubt. But let us trie this work, not by force of Law, but by proportion Geometricall, ſeeing the Teſtator did minde to provide for each ſort of them.

Scholar. If the Son ſhall have $\frac{1}{2}$ by force of the Teſtament, ſo muſt the Mother have $\frac{1}{3}$. Again, becauſe ſhe hath a Daughter alſo, therefore ought ſhe to have $\frac{1}{3}$, and the Daughter $\frac{1}{3}$, that is both wayes $\frac{11}{27}$, and $\frac{11}{33}$, which cometh to the whole goods, and $\frac{2}{3}$ more.

Wherefore it ſeemeth alſo impoſſible.

Maſter. In this matter the minde of the Teſtator is to be underſtood that ſuch proportion ſhould be betweene the portion of the Wife, and the Son, as is betweene $\frac{1}{3}$, and $\frac{1}{2}$, that is, the Son muſt have $\frac{1}{2}$, for $\frac{1}{3}$ to his Mother, ſo ſhall he have 3 to 2, that is as much as his Mother, and halfe as much more; and the Mother muſt have the like rate in compariſon to her Daughter. Then muſt I finde out 3 numbers in ſuch proportion, that the firſt may have as much as the ſecond, and halfe as much more (that is) in proportion ſeſquialtera, and the ſecond to the third, in that ſame proportion: ſuch numbers be 9, 6, 4.

Scholar. I pray you Sir, how ſhall I finde out theſe numbers?

Maſter. That will I gladly tell you.

What

Whatsoever the proportion be of any three numbers, multiply the Terms of that proportion together, and the number that amounteth, shall be the middle number of the three: then multiply that middle number by the lesser term, and divide that totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreme, and divide the totall by the lesser extreme, then will the greatest number of that Progression amount. To finde 3 numbers in any proportion.

Scholar. Then in this Example to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must divide (as you taught me in Division) $\frac{1}{2}$ by $\frac{1}{3}$, and the Quotient will be $\frac{3}{2}$, that is, $1\frac{1}{2}$, whereby I perceive that the proportion in this Question, is as 3 to 2. Therefore as you taught me even now, I multiply 3 by 2, and the summe is 6, which must bee the middle number: then I multiply the middle number 6 by 2, which is the least term, and the summe is 12, that I doe divide by 3, being the greater term, and the Quotient is 4: so is 4 the least number, of the three. Then I multiply 6 by 3, whereof cometh 18, and that I divide by 2, and so have I 9 which is the greatest number of the three. To finde the proportion between 2 numbers.

Master. Another way yet may you finde the third number in any Progression, if you have two of them: for if the middle number bee one of them which you have, then multi-

ply it by it ſelf (as in this Example, 6 by 6, maketh 36) and that totall divide by the other number which you have, and the third number will be the Quotient.

Scholar. Then I divide 36 (which cometh of 6 multiplied by it ſelf) by 4 the Quotient will be 9: and if I divide 36 by 9, the Quotient will be 4. But what if I know the firſt number, and the third, and would have the middle number?

Maſter. Multiply the two numbers together, and in their totall you muſt ſeek the root of that number, and it ſhall be the middle number: but becauſe as yet you have not learned to extract Roots, therefore uſe the firſt form which I have taught you, till I teach you to extract Roots. And now go forwards with the Anſwer to the ſame Queſtion.

Note.

Scholar. I perceive then, that the Sonne muſt not have ½ of the goods, neither the Mother ⅓, nor yet the Daughter ⅓, but yet muſt the goods be divided into ſuch proportion, that the Sonne ſhall have 9 Crowns for 6 to his Mother, and the Mother ſhall have 6 Crowns for every 4 to her Daughter. Then I apply it to the Golden Rule in three Examples, as followeth.

Where the firſt number is the Addition of thoſe three numbers 9, 6, 4: and the third is one of them ſeverally: the ſecond is the totall of the goods in that Teſtament: and then by the work of the Golden Rule, I finde out the

fourth

fourth number in every worke: that is for the Son 1705 $\frac{1}{19}$, for the Mother 1136 $\frac{16}{19}$, and for the Daughter 757 $\frac{12}{19}$; the which ſummes added together, doe make the ſumme of the whole goods as may be ſeen by this Example:

19 Z 3600
9 Z 3600
19 Z 3600
4

1705 $\frac{1}{19}$
1136 $\frac{16}{19}$
757 $\frac{12}{19}$
3600

And this (me thinketh) I doe perceive, becauſe in this Caſe there is a neceſſary remedy deviſed againſt an urgent incontenience: therefore thoſe learned men thought they might uſe the like liberty in that other Queſtion.

Maſter. Your gueſſe is good, but they had ſo good reaſon for them in the one, as they have in the other: As in another example of theirs, it may better appear, as in this.

A man left unto his three Sons 7851 *Crowns to be parted in ſuch ſort, that the firſt Son ſhould have* $\frac{1}{2}$, *the ſecond Son* $\frac{1}{3}$, *and the third Son* $\frac{1}{4}$, *which is not poſſible: for* $\frac{1}{2}$ *and* $\frac{1}{3}$ *and* $\frac{1}{4}$ *doe make* $\frac{13}{12}$, *or* $\frac{12}{12}$, *that is,* $1\frac{1}{12}$, *ſo it is more then the whole, but reduce theſe* Fractions *into one denomination, the leaſt that they will come to, and they will* Another Queſtion of a Teſtament.

be

be $\frac{6}{11}$, $\frac{4}{13}$, $\frac{3}{13}$, *and so may you part the goods into such proportion as these three Numerators bear together, that is, the first to have 6 for every 4 to the second, and the second to have 4 as often as the third hath 3: and so their portions will be for the first,* 3623 $\frac{7}{13}$, *for the second* 2415 $\frac{2}{13}$, *and for the third* 1811 $\frac{12}{13}$: *and these three shares added together, will make the totall summe of the whole goods, as you may easily see in this Example.*

	3623 $\frac{7}{13}$
	2415 $\frac{2}{13}$
	1811 $\frac{12}{13}$
	7851

Another like Question.

Another Question is there proponed thus: *There are* 450 *Crowns to be divided between three men, so that the first man must have* $\frac{1}{2}$ *and* $\frac{1}{4}$, *the second man* $\frac{1}{3}$ *and* $\frac{1}{4}$, *the third man shall have* $\frac{1}{4}$ *and* $\frac{1}{5}$.

Scholar. I marvell that any man should be so overseen, to propone that Question as a thing possible, seth $\frac{11}{29}$, $\frac{11}{34}$, $\frac{11}{45}$, doe make 1 $\frac{13}{19}$, that is almost double the whole summe.

But I perceive it might be thus proponed: that as often as the first man did receive 50 Crowns, so often the second man should receive 35, and the third man 27: for $\frac{1}{2}$ and $\frac{1}{3}$ is equall to $\frac{10}{60}$: and so is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{15}{60}$, and $\frac{1}{4}$ and $\frac{1}{5}$ to $\frac{27}{60}$: and so working the Question the three Figures will appear in this form: whereby the first mans portion is found to be 200 $\frac{12}{16}$: the second mans part is

112	Z	450
50	Z	200 $\frac{12}{16}$
112	Z	450
35	Z	140 $\frac{11}{16}$
112	Z	450
27	Z	108 $\frac{27}{16}$

140 $\frac{11}{16}$

140 11/12 the third mans ſhare 108 1/12 : which in the whole doth make 450 Crowns to be divided betweén them.

Maſter. And thus you are (I think) ſufficiently inſtructed in the Rule of Fellowſhip.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation; *which hath his name, for that by it there are divers parcels of ſundry prices, and ſundry quantities alligate, bound or mixed together: whereby alſo it might be well called the* Rule of Mixture; *and it hath great uſe in compoſition of Medicines, and alſo in mixtures of Metalls, and ſome uſe is hath in mixtures of Wines: but I wiſh it were leſſe uſed therein then it is now a daies. The order of this Rule is this:* The Rule of Mixture.

When any ſummes are propoſed to bée mixed, ſet them in order one over another, and the common number (whereunto you will reduce them) ſet on the left hand; then mark what ſummes bée leſſer then that common number, and which bée greater, and with a draught of your penne evermore link two numbers together, ſo that one bée leſſer then the The reaſon of this Rule.

the common number, and the other greater then he: (for two greater or two ſmaller cannot well be linked together) and the reaſon is this, that one greater and one ſmaller, may be ſo mixed, that they will make the mean or common number very well: but two leſſe can never make ſo many as the common number, being taken orderly: no more can two ſummes greater then the mean, ever make the mean in due order, as it ſhall appear better to you hereafter. And as it is of neceſſity to link every ſmaller (once at the leaſt) with one greater, and every greater with one ſmaller, ſo it is at liberty to link them oftner then once, and ſo many there be to one Queſtion, many Solutions. When you have ſo linked them, then mark how much each of the leſſer numbers is ſmaller then the mean or common number, and that difference ſet againſt the greater numbers, which be linked with thoſe ſmaller, each with his match ſtill on the right hand, and likewiſe the exceſſe of the greater numbers above the mean, you ſhall ſet before the leſſer numbers, which be combined with them.

Then ſhall you (by Addition) bring all theſe differences into one ſumme, which ſhall be the firſt number in the Golden Rule, and the ſecond number ſhall be the whole maſſe that you will have of all thoſe particulars: the third ſumme ſhall be each difference by it ſelf, and then by them ſhall be found the fourth number, declaring the juſt portion of every particular in

that

that mixture: As now by these Examples I will make it plain.

There are four sorts of Wine, of severall prices, one of 6 pence *a Gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence *the Gallon. Of all these Wines would I have a mixture made to the summe of* 50 *Gallons, and so the price of each Gallon may be* 9 pence. *Now demand I, how much must be taken of every sort of Wine?*

A question of Mixing of Wines.

Scholar. If it shall please you to work the first Example, that I may mark the applying of it to the Rule: then I trust I shall be able not only to doe the like, but also to see the reason in the order of the work.

Master. Mark then this form, and the placing of every kind of number in it.

Here (you see) I have set down the severall

rall prices, which be 6, 8, 11, 5, and have linked together 6 with 15, and 8 with 11. The common price 9, I have set on the left side, and the difference between it, and every particular price, I have set on the right hand, not against the summe (whose difference it is) but against the summe that it is linked withall, to the difference of 15 above 9, is 6, which I have set, not against 15 but 6, that is linked with 15, and the difference between 6 and 9 (that is 3) I have set against 15. So likewise the difference between 8 and 9, is but 1, that I have set against 11; and the difference of 11 above 9 (which is 2) I have set against 8. Then adde I all those four differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I should have, and the third summe is every particular difference. Now if you work by the Golden Rule, you shall finde the number of Gallons that shall bee taken of each sort of Wine: For the better distinction whereof, I have set these Letters, A, B, C, D, both against the Numbers for which the workes doe serve, and over the worke also, which severally serve for each of them. And now (if you list to examine the truth of these workes) adde these four summes together, and they will make 50, that is the totall which I would

The proof of this Rule.

would have, as by this Example you may easily perceive. And (for to prove how the prices doe agree) doe this: Multiply the totall summe 50, by the common price 9, and it will make 450: then keep that sum by it self, & afterward multiply every severall summe of Gallons, by the price belonging to the same Gallons, and if that summe doe agree with this, which you have kept first, then is your worke well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set down, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{2}{3}$: so again $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And last of all, $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{1}{2}$: and these added together do make 450, as in the Example annexed you may see, wherefore seeing it doth agree with the former summe of 50, multiplied by 9, I may justly affirm this worke to be good, and well done.

25	
$8\frac{1}{3}$	
$4\frac{1}{6}$	
$12\frac{1}{2}$	
50	
150	
$66\frac{2}{3}$	
$45\frac{5}{6}$	
$187\frac{1}{2}$	
$187\frac{1}{2}$	

And now to prove how you can doe the like, I propound the same Question, only willing you to use some other form of combinnig or linking the sums. The variation of this Question.

Scholar. That shall I prove with your favour, and therefore I combine 8 with 15, & 6 with 11, & then the form will be as followeth.

9

Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I understand the reason by your former work. And therefore here appeareth no strange thing, but that now I have $8\frac{1}{2}$ gallons of 6 pence, and 25 gallons of 8 pence, and 12 gallons and $\frac{1}{2}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{2}$ of 15 pence: so that multiplying $8\frac{1}{2}$ by 6, it maketh 50, and then 25 multiplied by 8, maketh 200: likewise $12\frac{1}{2}$ multiplied by 11 yeld $137\frac{1}{2}$, and $4\frac{1}{2}$ multiplied by 15, maketh $62\frac{1}{2}$, which four summes added into one, will yeld in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of gallons) be 9 the common or mean price.

50	
200	
$137\frac{1}{2}$	
$22\frac{1}{2}$	
450	

Master. Seeing you conceive this work so well, I will propound another Example unto you of more variety in the Alligations or combinings, as thus:

A Merchant being minded to make a bargain for ſpices, in a mixt maſſe (that is to ſay) of Cloves, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloves being at 6 ſhillings, Saffron at 10 ſhillings, Pepper at 3 ſhillings, Ginger at 2 ſhillings, and Almonds at 1 ſhilling. A queſtion of Spices.

Now would hee have of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another; to bear in price 5 ſhillings: How much ſhall hee have of each ſort?

Scholar. That will I trie thus.

Firſt I ſet down thoſe 6 ſeverall prices, and at the left hand I ſet the common price 5 ſhillings. Then I link them thus, 1 with 10, 2 with 6, and 3 with 8: As in the Example following:

Maſter. I had minded to have combined them in more variety: but I am content to ſee your owne worke firſt, and then more varieties in combination may follow anon.

Scholar. Then to continue as I began I seek the difference betweene 1 and 5, (which is 4) and that I set against 10, then against 1 I set 5, which is the excesse of 10 above 5, so I gather the difference betweene 2 and 5, which is 3, and that I set against 6, because it is combined with 2: and likewise the difference of 6 above 5, (which is 1) I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 above 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden Rule, and so appeareth by those workes, that of Almonds I must take 83 $\frac{1}{3}$ pound, of Ginger 16 $\frac{2}{3}$ pound, Pepper 50 pounds, of Cloves 50 pounds, of Nutmegs 33 $\frac{1}{3}$ pounds, and of Saffron 66 $\frac{2}{3}$ pounds.

	83 $\frac{1}{3}$
	33 $\frac{1}{3}$
	150
Then for triall hereof, I multiply every parcell by his severall price, as 83 $\frac{1}{3}$ which is the summe of Almonds, I multiply by one which is their price.	300
	266 $\frac{2}{3}$
	666 $\frac{2}{3}$
	1500

Also 19 $\frac{2}{3}$ the summe of Ginger, I multiply by 2, which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them all together I finde the totall to bee 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, where-

for

fore it appeareth well wrought.

Master. Now I will make the Alligation to prove your cunning somewhat better: but because you shall not think your self pressed so much, I will also note the differences, as by this Example you may see, where I have al-

ligated 1 with 6 and 8, and therefore have I set against 1 both their differences, that is 1 and 3: Likewise, because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I have set only 5, which is the difference of 10, with whom 3 is combined onely. Likewise 6 is onely alligate to 1, and therefore is the difference of 1 from 5, which is 4 onely set against it: 8 is linked with 1 and 2, and therefore hath set 4 against him, both their differences, 4 & 3: and 10 is joyned with 2 & 3, therefore hath he their differences, 3 and 2. And because of ease for you, in another column I have set the differences reduced into one number, for every severall sort,

and have also added them together, whereby appeareth that they make 33, and so consequently you see the works of the Golden Rule set forth. For the six Drugs I have added the letters A, B, C, &c. as before.

Note. But I would not wish you to cleave still to these Elementary aids, but accustome Memory to trust her self: so shall occasion of negligence best be avoided. And as for the proof trie it at more leisure, because the time now is short, and you sufficiently instructed in that point. And there resteth divers things behinde yet, of which I would gladly give you some taste, before your departure.

Scholar. But if it may please you to let me see all the variations of this Question, before you go from it, for me thinketh I could vary it two or three waies more yet.

Master. I am content to see you make two or three variations: but I would be loth to stay to see all the variations: for it may be varied above 300 waies although many of them would not well serve to this purpose.

Scholar. I thought it impossible to make so many variations.

Note. Master. Marvell not thereat, for some Questions of this Rule, may bee varied above 1000 waies; but I would have you forget such fantasies till a time of more leisure. And now goe forward with some variation of this Question.

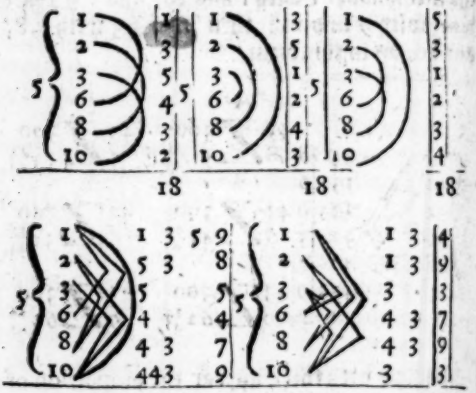
Scholar. For the first variation, I link the

the first number 1 with 1 and 10, and 2 & combine with 9 and 10: then joyn & 3 with 6, 8, and 10, as in this form.

And so doth there appear the proportion of weight for every kinde of Drug in this mixture. Now for the triall.

Master. Nay stay there: you shall not need to make triall in one Example so often, or if you list to doe it by your self, I am content. But now set forth (for declaration that you conceive the Rule) two or three Examples of severall Combinations, and then will we passe to some other Example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these bee the varieties: in which, as the



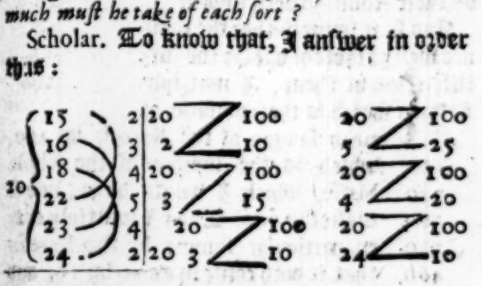
combinations are ſeverall, ſo doth it plainly appear, that the differences by which the proportion of each ſeverall kinde is taken are alſo ſeverall. And yet I ſée in the three firſt of theſe five varieties, and in the one other before, the totall ſumme of the differences to be one, that is to ſay, 18, whereby I perceive that the variety of their mixture doth depend on the variety of their differences ſeverall and not of the variety of their totall ſumme.

Maſter. So is it: And ſéeing you conceive it ſo well, I will make an end of this Rule, onely exhibiting unto you one Queſtion or two of the mixture of Metals, that by it you may deviſe others like, and exerciſe your ſelf therein alſo, becauſe the uſe of it ſerveth often

in

in buſineſſe of charge, not ſo much for Goldſmiths, as of coynage in Mints. *Firſt, I demand of you this Queſtion: If a Mint-maſter have Gold of 22 Karects, and ſome of 23 Karects, ſome of 24: Again, ſome 15, ſome 16, and ſome of 18 Karects, and would mix them, ſo that he might have 100 Ounces of 20 Karects: How much muſt he take of each ſort?*

Scholar. To know that, I anſwer in order this:



Maſter. You have wrought the Queſtion well: But how chanced you make no doubt of that new name Karect?

Scholar. Becauſe I thought it out of time to demand ſuch Queſtions now, ſeeing you make ſo much haſte to end: and again in this caſe the proportion of the number is ſufficient for my purpoſe in this work, truſting that another time you will inſtruct me as well of this, as of ſundry other things, which as I have heard you talk of, ſo I have a great deſire to them.

Maſter. Your anſwer is reaſonable, and your requeſt and truſt (with Gods help) I in-

tend to ſatisfie. And now to goe forward with this matter, let mée ſée your examination of this laſt work.

Scholar. Firſt for the one part I adde together all the particular ſummes, as they appear in the work, and they make 100, as here by their Addition doth appear.

And ſo it ſéemeth that the ſums are well gathered: but for the further triall of them, I multiply firſt 20 which is the common or mean ſumme of the Karects by 100, which is the ſumme of the whole Maſſe, which I would have, and it maketh 2000. Then I multiply every particular ſumme by the Karects that it doth contain as 10 by 15, and that maketh 150.

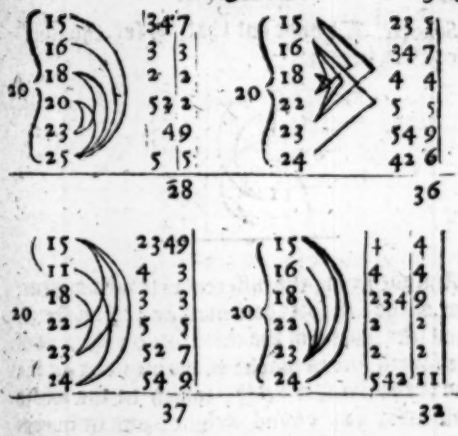
Likewiſe I multiply 15 by 16, and it yéeldeth 240: ſo 20 by 18, maketh 360. And 25 by 22, yéeldeth 550: likewiſe 20 by 23, bringeth forth 460: and laſt of all, 10 multiplied by 24, yéeldeth 240: which ſummes all ioyned together make 2000, that doth agrée with the like ſumme before, wherefore I may well ſay, that the work is good. And now (if it pleaſe you) I would ſet forth ſome varieties of this Queſtion, to prove my wit.

10	
15	
20	
25	
20	
10	
100	
120	
240	
360	
550	
460	
240	
2000	

Maſter. Goe to, let mée ſée.

Scholar. Here be four varieties.

And



And more yet could I make, but not like to the number that you ſpeak of in the variation of the other Queſtion.

Maſter. That will I teach you at more leiſure, ſeeing it is a thing rather of pleaſure then of any neceſſity.

But now for your exerciſe in this Rule, one other Queſtion I will propoſe. A Mint-maſter hath ſix Ingots of Silver, of ſundry fineneſſe, ſome of four Ounces fine, and ſome of five Ounces, ſome of ſix, and other of eight, ſome of eleven, and other of twelve, and his deſire is to mix 500 pounds weight, ſo that in the whole maſſe every pound weight ſhould bear nine Ounces of fine Silver: How much ſhall he take (ſay you) of every ſort of ſilver?

Queſtion of mixing of Silver.

Scholar.

Scholar. To finde out that, I ſet the numbers thus in order.

And gathering the differences it will appear, that, of the firſt ſort there muſt be $43\frac{11}{23}$: of the ſecond like much: of the third ſort $65\frac{1}{23}$: and of the fourth ſort as much: of the fift ſort $195\frac{11}{23}$, and of the ſixt ſort $86\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of the firſt ſort, $173\frac{21}{23}$, and of the ſecond ſort $217\frac{2}{23}$, of the third ſort $391\frac{7}{23}$, of the fourth ſort $521\frac{17}{23}$, of the fift ſort $2152\frac{14}{23}$, & of the ſixt ſort $1045\frac{11}{23}$, which all together doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Maſter. This is well done of you, therefore now make three or four varieties, and ſo an end of this Rule.

Schol. Theſe 4 varieties I ſet for Examples.

Master. And by these it appeareth, that you can finde out more, with which I will not now meddle, save onely (for to shew you an easie help drawing the lines of Combination) I will set forth two varieties here.

And this shall suffice now for the Rule of Alligation or mixture: for by these Examples may you easily conjecture such other as doe appertain to it, as well for the due working, as for variety of drawing the lines of Combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my musing at the many varieties that may fall in these Combinations, and

& termed them phanſies, yet my phanſie giveth mé, that the conſideration of this ſhould in many other Examples and Caſes of importance be very néedfull, and the knowledge of it moſt profitable: Therefore yée may well think, that at another time convenient I will requeſt you to aid mée herein.

Maſter. Truth it is, that this conſideration may fall in practiſe as well Politick as Philoſophicall, and ſundry wates in them bée applied: Therefore when time ſhall fall fit, for the diſcuſſing of this conſideration, you ſhall not want my helping hand.

The Rule of Falſhood.

The occaſion of the name.

NOw will I briefly alſo teach you ſomewhat of the Rule of Falſhood, *which beareth his name, not for that it teacheth any fraud or falſhood, but for that by falſe numbers taken at all adventures, it teacheth how to finde thoſe* true numbers *you ſeek for.*

Scholar. So might any other Rule bée called the Rule of Falſhood, for they worke by wrong numbers, and by them finde out the right numbers: ſo doth the Rule of Alligation, the Rule of Fellowſhip, and the Golden Rule partly.

Maſter.

Maſter. In the Golden Rule, the Rule of Fellowſhip, and the Rule of Alligation, although the numbers that you work by, be not the true numbers that you ſeek for, yet are they numbers in juſt proportion, and are found by orderly work, whereas in this Rule the numbers are not taken in any proportion, nor found by orderly work, but taken at all adventures.

And therefore I ſometimes being merry with my friends, and talking of ſuch queſtions, doe call unto them ſuch Children or Idiots, as hapned to be in the place, and ſo take their anſwer, declaring that I would make them ſolve thoſe queſtions, that ſeemed ſo doubtfull.

And indeed I did anſwer to the Queſtion, and work the Triall thereof alſo by thoſe anſwers which they hapned at all adventures to make: which numbers ſeeing they be taken as manifeſt falſe, therefore is this Rule called the Rule of falſe Poſitions, and for briefneſſe, The Rule of Falſhood: which Rule for readineſſe of remembrance, I have compriſed in the few verſes following, in form of an obſcure Riddle.

Gheſſe at this work as hap doth lead,
By chance to truth you may proceed,
And firſt work by the queſtion,
Although no truth therein be done.
Such falſhood is ſo good a ground,
That truth by it will ſoon be found.

From

From many bate too many moe,
From too few take too few alſo:
With too much joyn too few again:
To too few adde too many plain;
In croſſe-wiſe multiply contrary kinde,
And all truth by falſhood for to finde.

The ſenſe of theſe Verſes, and the ſumme of this Rule is this:

The expoſition of the Rule.

When any Queſtion is proponed appertaining to this Rule, firſt imagine any number that you liſt, which you ſhall name the firſt Poſition, and put it in ſtead of the true number, and then work with it as the Queſtion importeth: and if you have miſſed, then is the laſt number of that work either too great or too little: that ſhall you note as hereafter ſhall be taught you, and you ſhall call it the firſt Errour.

Then begin again, and take another number, which ſhall be called the ſecond Poſition, and work by the Queſtion: if you have miſſed again, note the exceſſe or default as it is, and call that the ſecond Error. Then multiply croſswiſe the firſt Poſition by the ſecond Errour, and again, the ſecond Poſition by the firſt Errour. and note their totalls ſeverally by the names of totalls: Then mark whether the two Errours were both alike, that is to ſay, both too much, or both too little: or whether they be unlike, that is, the one too much, and the other too little:

little: for if they be like, then ſhall you ſubtract the one totall from the other (I mean the leſſer from the greater) and the remainer ſhall be your Dividend: ſo muſt you abate the leſſer errour out of the greater, and the reſidue ſhall be the Diviſor. Now divide the Dividend by that Diviſor, and the Quotient will ſhew you the true number that you ſeek for. But, and if the errours be unlike, then muſt you adde both thoſe totalls (which you noted) together, and take that whole number for the Dividend, ſo ſhall you adde both errours together, and that whole number ſhall be the Diviſor, and the Quotient of that Diviſion ſhall give you the true number that the Queſtion ſeeketh for, and this is the whole Rule.

Scholar. This Rule ſeemeth ſo unlike any other, that without ſome Example I ſhall not eaſily underſtand it.

Maſter. With a good will: propoſe half a ſcore ſundry Queſtions & Examples of variety, for the better underſtanding of the work hereof: and for the firſt, take this Example:

A Maſon was bound to build a wall in 40 daies, and it was covenanted ſo with him, that every day that he wrought, he ſhould have for his wages 2 ſhillings 1 peny, *and every day that he wrought not, he ſhould be amerced* 2 ſhillings 6 pence, *ſo that when the wall was made, and the reckoning taken of the daies that he wrought, and of the other that he wrought not, the Maſon had clearly but* 5 ſhillings, 5 pence, for the work. *Now doe*

Aqueſtion of Maſonry, the firſt Example.

doe I demand how many daies did he work of thoſe 40, and how many did he not work?

Scholar. I pray you expreſſe the order of the work, that I may partly by imitation, and partly by comparing it with the Rule, bee able again to doe the like.

Maſter. This order ſhall you keep in the work of this Rule: Firſt take ſome number (as you liſt) at adventure; As for example, I ſay he played 12 daies, and wrought 28 daies. Now caſt you the wages of every day, and ſee whether it will agree with the ſumme of 5 ſhillings 5 pence.

Scholar. The 28 daies that he wrought after 25 pence the day, yeeldeth 700 pence: Then 12 daies that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there reſteth 340: but you ſay he had not ſo much.

Maſter. He had but 65 pence, and by this ſuppoſition he ſhould have had 340: therefore is this ſumme too much by 275, which ſumme I muſt ſet down after this ſort as you ſee here, where firſt I have made a croſſe, (commonly called S. Andrews croſſe) and at the over corner on the left hand I have ſet the firſt poſition 12: at the other corner under it I have ſet 275, which is the firſt error, with this figure †, which betokeneth too much, as this line —— plain without a croſſe line betokeneth too little.

12

X

975 †

The

On the right hand of the croſſe I have left two like rooms for the ſecond poſition and his error. Therefore to proſecute the worke, I ſuppoſe he played 16 daies, and wrought 24.

Scholar. I was a while in doubt why you named the daies of his working, ſeeing they be not ſet in the figure: and I doubted how you knew them, or elſe whether that you did ſuppoſe them at all adventures, as you did the daies that he played: but now I gather that ſeeing 40 daies is the whole time limited, then the daies that he played being ſuppoſed, the reſt of 40 muſt needs be the daies that he wrought, and therefore 28 followed 12 of neceſſity, and 24 followeth 16 alſo of neceſſity, but yet I ſcarce perceive why you ſet not in the figures as well 28 as 12.

Maſter. It forceth not which of them I take, ſo that in the ſecond poſition I take the numbers of the ſame nature that is here both of working daies, or both of idle, but now examine you this ſecond poſition.

Scholar. If he played 16 daies, then abating 16 times 30 pence, the ſumme will be 480 pence, and for 24 daies that he wrought, every day yelding 25 pence, the totall is 600 pence: ſo that abating 480 out of 600, there reſteth 120, and as you ſay, it ſhould be but 65: therefore it is too much by 55: that muſt be ſet on the right hand of the figure, at the neather part, and over it on the ſame ſide 16, which is the ſecond poſition, thus:

12 16 And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

27† 55† Master. So were it. But this shall you mark, that, of what nature soever the two positions bee, of the same nature is the quotient. Therefore when the positions in this question are 12 and 16, which both being numbers of the playing daies, the quotient shall declare the true number of playing daies: whereas if the positions had been 28 and 24, which are supposed to bee the working daies, then would the quotient declare the true number of the working daies, and not of playing daies, as it will doe now. And therefore to continue the work of this Question, and to finde the true number of playing daies, I must multiply crosse-wise the first position by 55, that is the second errour, and the totall will bee 660. Then I multiply 275 and 16, and it yeelds 4400. Now because the errors are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the dividend. Again, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remain 220, which will bee the Divisor: then dividing 3740 by 220 the quotient will bee 17. Wherefore I say now constantly, that 17 is the true number of daies that the Mason played: and then it followeth that he wrought 23 daies: and so is the Question answered.

Now

Now for the order of triall of this work, there needeth none other triall but only this, to work with this number according to the Queſtion, and if it agree, then appeareth the number to be it that you would have.

The proof of this Rule:

And here now ſeeing he wrought 23 daies, and muſt have for every day 25 pence, the whole ſumme cometh to 757. Then again, ſeeing he played 17 daies, and muſt abate 30 pence for every day, the whole ſumme of the abatement will be 510. Therefore I ſubtract 510 out of 575, and there will remain 65, which maketh 5 ſhillings 5 pence, the cleer wages of the Maſon, for his work, according to the Queſtion.

Scholar. Now I truſt I underſtand the work and the Rule ſo well (and the better by this proof) that I can be able to do the like: And for a proof, I take the ſame Queſtion, all ſave the laſt number, where I will ſuppoſe that he had 10 ſhillings for his wages cleer. And now to ghelle at the number of the daies he wrought, I ſuppoſe, firſt, that he wrought 20 daies: then ſay I, if he wrought 20 daies, his wages muſt be 500 d. then for he play other 20 daies, for which muſt be abated 600 d. and then he loſeth 100 d. And ſo am I at a ſtay, for it is not like to your former work.

Maſter. You ſhould have required of me ſome Queſtion, and not have taken a Queſtion of your own fantying, until you were more expert in this Art, for ſo might you as well

happen to an impossible Question, as on a possible: but now to go forward, consider that this number is too little by 220, seeing he should gain by your supposition 120 pence, and in this position he loseth 100 those both make 220, which you shall set down for the first errour, with this signe —, betokening too little, as here in this form following doth appear.

	20
	X
	200

And now for the rest goe forward your selfe once again.

Scholar. As my errour hath uttered my folly, so it hath procured mee better understanding.

Now therefore considering this position not to solve the Question, I take another, supposing that he wrought 30 daies. Then for his wages he must be allowed 750 pence, and for the 10 daies which he wrought not, he must abate 300 pence, and so remaineth cleer 450 pence, but it should be only 120 pence, therefore it is too much by 330, which I set down in the figure with the former position and his errour, and the figure appeareth thus:

Now first, I multiply in crosse-waies 20 by 330, and it will be 6600.

Then again I multiply 30 by 220, and it will be also 6600. Wherefore if I shall subtract the one out of the other, there will

	20	30
	X	
	220-	3304

will remain nothing to be the Dividend.

Maſter. In this you forget your ſelf again; for in as much as the ſignes in the errours bee unlike, therefore muſt you work by Addition, adding together thoſe two totalls to make the Dividend, and alſo adding the two errours to make the Diviſor. And becauſe you ſhall no more forget this part of that Rule, take this briefe remembrance.

Unlike require Addition.
And like deſire Subtraction.

Scholar. You mean, that if the errours have like ſignes, then muſt the Dividend, and the Diviſor bee made by Subtraction, as is taught before: And if thoſe ſignes be unlike (as in this laſt Example they be) then muſt I by Addition gather the Dividend and the Diviſor. Therefore muſt I adde 6600 to 6600, and it will be 13200, which will be the Dividend. Then again I adde 220 to 330, and it will be 550, which muſt be the Diviſor: wherefore dividing 13200 by 550, the Quotient will bee 24, whereby I know that the Maſon wrought 24 daies, and then it followed, that he played 16 daies.

Maſter. Examine your work, whether it be agreeable to the Queſtion or no.

Scholar. For 24 daies work, the wages muſt bee 600 pence, and for 16 daies which the Maſon wrought not, there muſt bee abated 480 pence, and then remaineth clear to the

Maſon 120, as the Queſtion importeth: wherefore it is evident that 24 is the true number of daies that he wrought.

Maſter. Although you ſeem now to underſtand this work, yet to acquaint your minde the better with the new Trade of this Rule, I think it good to propone to you 5 or 6 Examples more before I make an end of it.

Scholar. Sir, I thank you that you doe ſo conſider my commodity and profit in knowledge, for undoubtedly it is practiſe and exerciſe that maketh men prompt and expert in every kinde of knowledge.

Maſter. You ſay well, ſo that they follow ſome certain precepts to govern and rule their practiſe by, elſe may practiſe procure cuſtome of errour, and a repugnance to exactneſſe of knowledge: namely, as long as the errour is not plainly known to the vulgar ſort. But to return to your work.

A queſtion of Wares, the ſecond Example.

There is a Servant that hath bought of Velvet and Damask for his Maſter 40 yards, the Velvet at 20 ſhillings a yard, and the Damask at 12 ſhillings, and when he cometh home, his Maſter demandeth of him, how much he hath bought of each ſort: I cannot tell (ſaith he) exactly: but this I know, that I paid for Damask 48 ſhillings more then I paid for Velvet: now muſt you ſhew how many yards there is of each ſort.

Scholar. Although the queſtion ſeemeth difficult, yet I will prove what I can doe: for I remember your ſaying, that it foreth not how

how fond or falſe the gheſſe be, ſo it be ſomewhat to the Queſtion, and not an anſwer of a contrary matter.

Therefore firſt I imagine that he bought 20 yards of Damask, for which he ſhould pay after the former price 240 ſhillings: then muſt he needs have of Velvet other 20 yards, (to make up the 40 yards) and that would coſt 400 ſhillings. So that the totall of the price of the Damask is leſſe then the ſumme paid for Velvet 160 ſhillings, and ſhould bee more by 48. Therefore the firſt errour is 208 too little. Then begin I again, and ſuppoſe he bought of Damask 30 yards, that coſt 360 ſhillings, then had hee but 10 yards of Velvet, which coſt 200 ſhillings: and now the price of the Damask is greater then the price of the Velvet by 160 ſhillings, and ſhould be but 48, therefore is the ſecond errour 112 too much, which I ſet in form of figure, as here doth appear. Then doe I multiply in croſſe-wales 208 by 30, and the ſumme will bee 6240. Alſo I multiply 112 by 20, and there will amount 2240. And in as much as the ſignes of the errours be unlike, I know I muſt work by Addition, therefore adde I theſe two totalls together, and they make 8480, which is the Dividend: then adde I alſo the two errours together, 208, & 112, and they make 320, which is the Diviſor: wherefore

20	30
X	
208.	112 †

dividing 8480 by 320, the Quotient will be 26 ½, which is the true ſumme of yards of Damask that he bought, and in Velvet 13 yards ½, and that appeareth by examination, thus: 26 ½ yards of Damask at 12 ſhillings the yard maketh 318 ſhillings: then in Velvet he had but 13 yards and ½, and coſt 270 ſhillings, at 20 ſhillings the yard. Now ſubtract 270 out of 318, and there will remain 48, which is the number of ſhillings that the Damask did coſt more then the Velvet.

Maſter. Now ſhall you have a Queſtion of another kinde.

Aqueſtion of Debt, the third Example.

There are three men that doe owe money to me, and I have forgotten what the totall ſumme is, and what the particulars be.

Scholar. Why, then it is impoſſible to know the debt.

Maſter. Peace, you are too haſty, there is more help in it then yet you ſee, I have three ſeverall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the firſt and the ſecond to amount to 47 pound, the debt of the firſt man and the third man did make 71 pound, and the ſecond man his debt with the third, did riſe to 88 pound. Now can you tell what every man did owe, and what was the whole ſumme?

Scholar. Nay, in good faith, but as I perceive that it muſt bee found by conjecture, ſo will I ghelle at it, ſuppoſing that the firſt man did owe 20 pound, and the ſecond man 30, and the third.

Maſter.

Maſter. Nay, ſtay there, you are too farre gone already: you may not ſuppoſe a ſeverall ſumme for every man, for it is enough to ſuppoſe one ſumme for the firſt man, and let the other riſe as the Queſtion importeth. Therefore ſeeing you ſet the firſt man his debt to bee 30 pound, the ſecond man cannot owe 30 pound, for the declaration is, that their debts added together did make 47 pound, ſo muſt the ſecond man his debt bee but 27 pound. Now the ſecond debt with the third, muſt make 88: therefore ſubtract 27 out of 88, and there will remain 61, as the third man his debt. Then ſaith the declaration, that the firſt and third mans debts doe make 71: but by this ſuppoſition they make 81, that is 10 too much, which I muſt ſet for the firſt error. Now worke you the ſecond poſition.

Scholar. I ſuppoſe the firſt mans debts to be 24 pound: then muſt the ſecond mands debt (by your declaration) be but 23 pound, ſeeing both they make but 47 pound. And the ſecond man his debt with the third, doe make 88 pound, and the ſecond man oweth but 23: therefore the third man muſt owe 65 pound. Now the third mans debt with the firſt, ſhould make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound too much, and that is the ſecond errour, which I ſet down with the firſt, and their poſitions in this form, and then I doe multiply in croſſe-waies 20 by 18, and it is 360.

And

And 10 by 24 maketh 240. Also because the signes of the errours be like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Dividend: then doe I subtract 10 out of 18 by the same reason, and so is the Divisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15 li. and then the second man must owe 32 li. for those two do make 47 li. and the third mans debt is 56: for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88.

10 24
X
10† 18†

The 4th Example.

Master. *For the fourth Example, take this easie question for the variety in work: Two men having severall summes, which I know not, doe thus talk together: the first saith to the second, if you give me* 2 shillings *of your money, then shall I have three times so much money as you. The second man answereth: It were more reason that our summes were made equall, and so will it be if you give me* 3 shillings *of your money. Now ghesse what each of them had.*

Scholar. I imagine that the first had 9 s.

Note.

Master. Consider evermore in your imagination that you take a likely summe, as in this Question, take such a summe, that having 3 added unto it, may be divided into three parts even.

Scholar. Why? I remember you said before, it forceth not how fondly soever I ghessed.

Master.

Maſter. As for the poſſibility of the ſolution, it is truth: but for eaſineſſe in work, the apteſt numbers are moſt convenient.

Scholar. I thought no leſſe, and therefore I tok 9 as an apt number to bee parted into three: but I perceive I ſhould have conſidered the aptneſſe of that partition after the addition of two unto it, and then 7 had been more meet.

Maſter. That is truth, and then ſhould the ſecond mans ſumme be 5: for although he have now but the third part of 9, that is 3, yet you muſt remember that he lent the firſt man 2, and ſo had he 5.

Scholar. Then to go forward: if the ſecond man had three of the firſt man, then ſhould he have 8 and the firſt man but 4; ſo hath hee double to the firſt man, yet he ſaid in the Queſtion they ſhould have equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that errour with his ſuppoſition, and ghelle again that he hath 10 s. whereunto I adde 2 s. borrowed of the ſecond man, and then he hath 12 s. ſo the ſecond man hath remaining but 4, whereunto if I adde the 2 that he lent to the firſt man, ſo had he but 6 s. at the beginning.

Then take 3 s. from the firſt man, and gite to the ſecond, then hath the firſt man but 7, and the ſecond hath 9, which are not equall, but there are 2 too

7	10
X	
4 †	2 †

many

many, wherefore I set down both the positions with their errours, as before you see, and multiply a crosse, so cometh there 40 and 14: and because the signes be like, I take 14 out of 40, and so resteth 26 to be divided: then likewise I take 2 out of 4, and there resteth 2, by which I divide 26, and the Quotient will be 13, which is the summe that the first man had. And so appeareth that 2 being added thereto, the summe will be 15, so hath the second man but 5, and before he had 7: then take 3 from the first, and put to his 7, and so have each of them 10, and that is equall as the Question would.

The fift Example: a question of Lambs.

Master. *For the fift Example, take this Question: One man said to another, I think you had this year two thousand Lambs: so had I said the other; but what with paying the tythe of them, and then the severall losses, they are much abated: for at one time I lost half as many as I have now left, and at another time the third part of so many, and the third time $\frac{1}{4}$ so many. Now shew you how many are left.*

Scholar. Because here is mention made of certain parts, I must take a number that may have all these parts, that is to say, $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$ which will bee 24, howbeit 12 hath the same parts: Therefore I take first 12 to bee the number that doth remain, so hath he lost 6, 4, and 3, that is 13, and the whole 23, but it should be 2000.

Master. We are deceived yet still, you have for-

forgotten the 10 part, which muſt be defalked, that is 200, ſo there remaineth but 1800, and now go on again.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 to few, which I ſet for the errour. Then for the ſecond poſition I take 24, whoſe half is 12, the third part 8, and the quarter 6, whereby riſeth 50, which is too little by 1750, therefore I ſet down both the poſitions, with their errours, thus:

And multiply in croſſe-waies 1775 by 24, whereof cometh 42600. Alſo I multiply 1750 by 12, and there ariſeth 21000. And becauſe the ſignes are like, I doe ſubtract the one from the other, and ſo remaineth the Dividend 616c00. Then doe I ſubtract 1750 out of 1775, and there reſteth 25, by which I divide 21600, and the Quotient is 864, whereof the half is 432, and the third part is 288 the quarter is 216; which all being added together, will make 1800: And if you adde thereto the tenth which was abated before, then will the whole ſumme bee 2000.

12	24
X	
1775	1750

864
432
288
216
1800

And now doth there come a Queſtion to my memory which was demanded of me, but I was not able to anſwer to it: and now me thinketh I could ſolve it.

Maſter. Propone your Queſtion.

Scholar.

Aqueſtion of Sheep & Tillage, the ſixt example.

Scholar. *There is ſuppoſed a Law made, that (for furthering of Tillage) every man that doth keep Sheep, ſhall for every ten Sheep ear and ſow one Acre of ground: and for his allowance in Sheep-paſture, there is appointed for every four Sheep one Acre of paſture. Now is there a rich Sheep-maſter which hath 7000 Acres of ground, and would gladly keep as many as he might: by that Statute I demand how many Sheep ſhall he keep?*

Maſter. Anſwer to the Queſtion your ſelf.

Scholar. Firſt, I ſuppoſe he may keep five hundred ſheep, and for them he ſhall have in paſture after the rate of four ſheep to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I gheſſe again that he may keep 1000 ſheep, that is in paſture 250 Acres, and in tillage 100 Acres, which make 350, that is too little by 6650. Both theſe errours with their poſitions, I ſet down as you ſee; and multiply them croſs 6825 by 1000, and it maketh 6825000, alſo I multiply 6650 by 500, and there cometh 3325000, which ſumme I ſubtract out of the former, and there remaineth 3500000 for the Dividend: likewiſe I ſubtract the leſſer errour out of the greater, and there reſteth 175, by which I divide 3500000 (the Dividend aforeſaid) and the

500 1000
X
6825 6650

the quotient will be 20000, ſo that by this rate he that hath 700 Acres of ground, may keep 20000 ſheep.

Maſter. You have done well, notwithſtanding both this laſt queſtion and the next before might be wrought without the ſecond Poſition by the Rule of Proportion, as this: When in this queſtion you found in the firſt errour that for 500 ſheep there muſt be 175 Acres, then might you reduce it to the Golden Rule, thus: Another way of working.

175 Z 500
7000 Z 20000

If 170 Acres will admit in allowance 500 ſheep, then 7000 will have 2000. And ſo by one poſition, with the help of the Golden Rule may you anſwer that Queſtion.

Likewiſe for the Queſtion of Lambs, when you had found that 12 came of 25, you might have ſet the Figure as followeth, and have ſaid: If 25 doe leave but 12, what ſhall 1800 leave? and it would appear to be 864.

25 Z 12
1800 Z 864

Scholar. Sir, I thank you for this aid, for it both much ſhorten the work of this Rule. Another way yet.

Maſter. Yet again, I will ſhew you another way to anſwer this laſt queſtion without the Rule of falſe poſition, and that by the Rule of Fellowſhip, for it appeareth in the proponing of the Queſtion, that 10 ſheep

must

must have in pasture 2 Acres and $\frac{1}{2}$, and for them must there be eared but 1 Acre; so it followeth, that for 2 Acres eared, there must be 5 set to pasture: and if you put them both into one summe, they will make 7. Therefore look what proportion 7 being this totall, doth bear to 5 and to 2, such proportion shall any totall in this Question bear to the pasture ground, and the eared ground.

Scholar. This serveth wondrous aptly. Therefore to prove it, I demand this by the former supposition: If a man have 300 Acres, how much shall he leave in pasture, and how much shall he turn to tillage? You say, that as 7 is to 5, so shall 300 be to the Acres of pasture: and as 7 is to 2, so is 300 to the Acres of tillage, whereof for both I have set Examples here following, whereby appeareth that of Pasture, there shall be 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{1}{7}$, which both summes added together, doe make 300.

7 Z 5 :: 300 Z 214 $\frac{2}{7}$

7 Z 2 :: 300 Z 85 $\frac{1}{7}$

Another Question, the 7th Example.

Master. *Now take another Example: A man hath three silver Cups with one Cover, the Cover weigheth 18 ounces, the second Cup weigheth one half the weight of the first and the third. Now if the Cover be put to the first Cup, they weigh just as much as all the three Cups doe weigh: and if the Cover be joyned with the second Cup, they weigh as much as the second twice, and the third: and if the*

Cover

Cover be put to the Cup they will make twice as much as the firſt and ſecond Cup. Now trie you what was the juſt weight of every Cup.

Scholar. I doe ſet the weight of the firſt Cup to bee nine Ounces, then in as much as theſe two (that is to ſay, the Cover and the firſt Cup) doe weigh the weight of the three Cups, I ſee that the three Cups muſt weigh 27 Ounces, for ſo much is 18 and 9. Alſo becauſe the firſt and the third doe weigh double ſo much as the ſecond, therefore it is the third part of that weight, that is 9, and then would it follow, that the third Cup alſo ſhould weigh nine Ounces; but then the Queſtion ſaith that the Cover being joyned to the ſecond Cup, they weigh as much as the ſecond twice, and the third once, that ſhould bee 27, and ſo it doth; that being joyned with the third Cup, they ſhould weigh twice as much as the firſt and the ſecond, that ſhould bee 36, and they weigh but 27, ſo is that errour nine to little. Then begin I again, and ſay, that the firſt Cup doth weigh twelve Ounces which I joyn with the Cover, and they make thirty Ounces: then ſeeing the ſecond is of that weight, it muſt needs weigh ten Ounces, and the third muſt weigh 8 Ounces, ſeeing the firſt & the third muſt weigh 20 Ounces. Now put I the Cover to the ſecond Cup, and they weigh 28 Ounces, which ſhould bee even ſo: then joyn I the Cover with the third Cup, and ſo ſhould it weigh twice the firſt & the ſecond,

that is 44 ounces, and they weigh but 26, that is 18 too little: those Errours with their Positions I set down, & multiply in crosse-wayes 9 by 12, whereof cometh 108: Also 9 by 18, and that yeeldeth 162: and in as much as the signes bee like, I abate the lesser out of the greater, and there doth remain 54. Then doe I also abate the lesser errour from the greater, and so remaineth 9, by which I divide 54, and the quotient is 6, which I take for the true weight of the first Cup, which being joyned with the Cover, must weigh as much as the three Cups, so doe they weigh but 24 ounces. Then seeing the second Cup is the third part of that weight, for the other two Cups (you say) must weigh double his weight, the weight of the second Cup is 8 ounces, and so the weight of the third Cup must be 10 ounces. Now put the Cover to the second Cup, and it will make 26 ounces: that must bee the weight of the second twice, and the third once, that is, twice 8, and once 10, and so is it. Again put the Cover to the third Cup of 10 ounces, and they must weigh twice as much as the first and the second, that is, 28, and so is all agreeable.

9	12
X	
9-	18-

Master. Then answer to this Question:

Aquestion of Water: the eighth Example.

There is a Cistern with four Cocks, containing 72 Barrels of Water: and if the great Cock be opened, the water will avoid clean in six hours; at the second Cock it will ask eight hours:

hours: at the third Cock it will avoid in no leſſe then 9 hours: and at the ſmalleſt it will require 12 hours. Now I demand, in what ſpace will it avoid all the Cocks being ſet open?

Scholar. Firſt, I imagine it will avoid in two hours.

Maſter. Then muſt there avoid by the firſt Cock $\frac{1}{3}$ of the water, that is 24 Barrels, and by the ſecond Cock $\frac{1}{4}$, that is 18, and by the third Cock $\frac{2}{9}$, that is 16 Barrels, and by the ſmalleſt Cock $\frac{1}{8}$, that is 12 Barrels, all which ſummes put together doe make 70, as by their Addition it doth appear, but it ſhould be $72\frac{1}{8}$ therefore the errour is too few.

Scholar. Then will I begin again by your favour, becauſe I think I underſtand the work, and put 3 hours for the due time: ſo ſhall there run out at the greateſt Cock $\frac{1}{3}$, that is, 36 Barrels, and at the ſecond hole $\frac{1}{8}$, that is 27, and at the third Cock $\frac{1}{3}$, that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 Barrels, which all together doe make 105, and ſhould bee but 72, ſo is it too much by 33: therefore doe I ſet the errours in order of the figure with their poſitions, and work by Multiplication, in croſſe, ſaying, two times 3 is 6, and two times 33 maketh 66, and becauſe the ſignes are unlike,

24	
18	
16	
12	
70	

2 3

2 33

I muſt adde theſe two totalls together, which make 72: alſo I adde the two errors, and they make 35, by which I divide 72, and the Quotient riſeth $\frac{2}{35}$, whereby I ſee that all the Cocks being ſet open, the Water will avoid in two hours, and $\frac{2}{35}$ of an hour.

Maſter. This exerciſe maketh you to grow expert in the Rule. Therefore I will inure you ſomewhat more with a Queſtion or two.

A Queſtion of Partners. The ninth Example.

There were two men that had been Partners, and had in account between them 300 *Duckets; whereof the one ſhould have for his part* 180, *and the other* 120: *but in the parting of them, they fell at variance; ſo that each of them catched as many as he could: yet afterward being reconciled they agreed that he which had gotten moſt part of them, ſhould lay down* $\frac{1}{4}$ *of them again, and he that had gotten leaſt, ſhould lay down* $\frac{1}{3}$ *of thoſe which he had taken, and then parting them into two equall parts, each man to have half thereof, and ſo had they their juſt portions, as they ought: now I demand of you what each of them had gotten by the ſcambling?*

Scholar. I ſuppoſe he that had leaſt, got 108 Duckets, then the other had 192: wherefore in laying down again of the 192, there was put down $\frac{1}{4}$, that is, 144, and ſo had he left but 48. Alſo of the 108, there was laid down 36, that is $\frac{1}{3}$, and ſo he had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts even, and ſo cometh 90 to be given to each of them:

them: which summe put to 72 maketh 12 and joyned to 148, it maketh 238: and now I doubt how I shall go forward.

Master. You need not to take but one of them, which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other summes, remembring that you make comparison to the same in the second work: As for Example of the first part: If you compare 138 with the lesser summe due, that is, 120, so is it 18 too much, and if you compare it with the greater summe, then is it 42 too little. Again, if you compare 162 to the greater summe, the errour will be 18, as it was in the other: but it will have a contrary signe: and if you compare it with the lesser summe, it will be 42 too much: so that the errour both wales is either 18 or 42: and as for the signes it little forceth, for in them is nothing considered here, but likenesse & unlikenesse, which in this case doth neither further or hinder: But now go on with the work. Note.

Scholar. If it be so, then am I out of my greatest doubt. Then I joyn that 90 (which I found as the half of the latter partition) unto 48, which is left with the one man, and so hath he, 138, which (I may say) is 18 too many, for the least should be but 120, that errour do I note, and then make a new position, supposing the one man to have 204, and the other to have 96, wherefore of the 204, there

must bee laid down 153, and so remaineth with him 51. Also of the 96, there must bee laid down $\frac{1}{3}$, that is 32, and so resteth with that man 64: Now of the 153 and 32, I make one summe, as 185, which I must divide into two equall parts, and so each man shall have $92\frac{1}{2}$, whereunto if I adde their former portions reserved, then the one shall have $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore take the lesser summe now again, as I did before, that is, $143\frac{1}{2}$ and finde that he hath too many by $23\frac{1}{2}$, for he should have but 120, and so have I for my two positions two errors, which I set down as here may bee seen, each error under his position, and then by the Rule I doe multiply in crosse-wates 108 by $23\frac{1}{2}$,

and there riseth 2538,	108	96
which I note, then again		X
I multiply 96 by 18,		
and thereof amounteth	18 †	$23\frac{1}{2}$ †
1728.		

Now because the signes are both like, that is, both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the Dividend 810: then for the Divisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I divide 810, and the quotient will be $147\frac{1}{11}$, which is the just portion of him that had the least summe. And if I doe subtract it out of 300, being the totall summe, then will there remain $152\frac{1}{11}$, as the portion that the other did get.

Master

Maſter. For the proof of this work, you may chuſe whether you will examine thoſe numbers according to the form of the Queſtions, or elſe work by other two poſitions for to finde the ſecond number: and if thoſe poſitions bring the ſame numbers that did amount by the firſt two poſitions, then doth each work confirm other.

Scholar. By your patience, I will prove both waies, not only to ſeek their agreement but alſo to accuſtome my minde to thoſe works, for I perceive it is exerciſe that muſt be the chief engraver of theſe Rules in my memory.

Maſter. You conſider it well: then go to.

Scholar. Firſt, I will by two other poſitions trie to finde the portion of him which had moſt.

Maſter. Although you may doe it with any poſitions, yet to ſee the agreement of your work the better, take the ſame poſitions that you did before, comparing them now to the greater, as you did before unto the leſſer.

Scholar. Then I ſuppoſe that he that had moſt, had 192, ſo had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144 and there will reſt to that man but 48. And from the ſecond which had 108, if I take $\frac{1}{3}$, that is 36, there will remain to him 72: then joyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to each of thoſe

two mens portions remaining with them, the one shall have 138, and the other 162, of which two I take the greater (that is 162) and see it to be 18 too few; for it should be 180, that errour I note under this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{1}{4}$ of 204, and it will be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$, that is, 32, and there remaineth to him 64; now put I that 32 to 153, and it yeldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that doe I note also, and set both these errours under their positions, as in this Example following doth appear.

And then multiplying 102 by $23\frac{1}{2}$, there doth arise, 4512.

Again, I multiply 204 by 18, and it maketh 3672, which I doe subtract out of 4512, because the signes be like, and there resteth 840 for the dividend, then subtracting 18 out of $28\frac{1}{2}$ there will remain $5\frac{1}{2}$, which I must take for the Divisor. And so dividing 840 by $3\frac{1}{2}$ the quotient will be $152\frac{8}{11}$, whereby I have found an agreeable summe to that which I found by the former positions, for him that had

102	204
X	
18	$23\frac{1}{2}$

most

moſt which I doe ſubtract out of 300, that is the totall, there will reſt 147 $\frac{1}{11}$, which was the portion of him that had the leaſt part.

Maſter. So by divers poſitions, you ſée, that one doth confirm the work of the other. Now examine thoſe two numbers by the forme of the Queſtion, and ſo ſhall you prove your work good alſo.

Scholar. If that hée which gat moſt, had 152 $\frac{4}{11}$, then muſt he lay down $\frac{1}{4}$ of this ſumme. That is 114 $\frac{6}{12}$, and ſo ſhall remain with him but only 38 $\frac{5}{11}$. The other which had leaſt, that is 147 $\frac{1}{11}$, muſt put down of his ſumme $\frac{1}{3}$, that is 49 $\frac{1}{11}$, and ſo doth there remain with him yet 98 $\frac{10}{11}$. Then doe I adde together 114 $\frac{6}{11}$, & 49 $\frac{1}{11}$, and it will make 163 $\frac{7}{11}$ which I muſt part into equall parts, and that will bée 81 $\frac{7}{11}$, to bée given to each of them, putting 81 $\frac{7}{11}$ unto 38 $\frac{5}{11}$, there doth amount 120 juſt, which is the true portion of him that ſhould have the leſſer ſum: & adding 81 $\frac{4}{11}$ 98 $\frac{10}{11}$ the totall will be 180, the true portion of the other. And ſo is the work by this proof alſo tried to bée good. And this I mark by the way, that in their ſcambling, he got moſt (as it chanceth often) that ought to have had leaſt by juſt partition.

Maſter. Let your ſtudy be to learn truth and juſt Art of proportion, and to diſtribute and part according thereunto, as often as occaſion ſhall be miniſtred. And here would I make an end of this Rule, ſave that I remember one pleaſant queſtion which I cannot overpaſſe,

paſſe which I will declare ſomewhat largely, becauſe you ſhall as well underſtand ſome reaſon in the pleaſant invention, as apt proceeding in the witty working thereof.

The tenth Example of Gold and Silver.

Hiero *King of the Syracuſans in Sicilia had cauſed to be made a Crown of Gold of a wonderfull weight, to be offered for his good ſucceſſe in warres: in making whereof the Gold-ſmith fraudulently took out a certain portion of Gold, and put in Silver for it, ſo that there was nothing abated of the full weight, although there was much of the value diminiſhed.*

Which thing at length being uttered (as no evil can alwaies lie hid) the King was ſore moved: and being deſirous to know the truth without breaking of the Crown, proponed the doubt to Archimedes, unto whoſe wit nothing ſeemed unpoſſible, which although preſently he could not anſwer unto, yet he had good hope to deviſe ſome policy for that invention, and ſo muſing thereon, as he chanced to enter into a Bain full of water to waſh him, he obſerved, that as his body entred into the Bain, the water did run over the Tub, whereby his ready wit of ſuch ſmall effects conjecturing greater works, conceived by and by a reaſon of ſolution to the Kings queſtion, and therefore rejoicing exceedingly, more then if he had gotten the Crown it ſelf, forgat that he was naked, and ſo ran home, crying, as he ran, Εὕρηκα, Εὕρηκα, I have found, I have found. And thereupon cauſed two maſſie pieces,

pieces, one of Gold, and another of Silver, to be prepared, of the ſame weight that the ſaid Crown was of: and conſidering that Gold is heavier of nature then Silver, and therefore Gold of like weight with Silver, muſt needs occupie leſſe rom by reaſon it is more compact and ſound in ſubſtance, he was aſſured that putting the maſſe of Gold into a veſſell brim full of water, there would not ſo much water run out, as when he ſhould put in the ſilver maſſe of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but alſo the difference or exceſſe of the one above the other, whereby he learned what proportion in quantity is betwéen Gold and Silver of equall weight. And then putting the Crown it ſelf into the veſſell of water brim full (as before) marked how much water did run out then, and comparing it with the water that ran out when the Gold was put in, noted how much it did excéd that: and likewiſe comparing it to the water that ran out of the Silver, marking how much it was leſſe then that, and by theſe proportions found out the juſt quantity of Gold that was taken out of the Crown, and how much Silver was put in ſtead of it: but ſéeing Vitruvius which writeth this Hiſtory, hath not declared the particular work of this triall, it ſhall be no inconvenience to ſuppoſe an Example for declaration ſake, wherein although the true and juſt pro-

proportion be not expressed, yet the form of triall shall be truly set forth. And for an Example, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ranne out two pound of water: and when the Masse of Silver was put in, I suppose there ranne out three pound $\frac{1}{2}$. Again, when the Crown was put in, there ranne out two pound $\frac{1}{4}$: Now to know what quantity of Silver was in the Crown, work by the Rule of false position, and imagine that there was two pound of Silver, then must there bee six pound of Gold, then say thus by the Rule of Proportion: If eight pound of Gold doe expell two pound of Water, what shall six pound expell, and it will be 1 pound $\frac{1}{2}$. Again, for the Silver: If eight pound of Silver expell three pounds $\frac{1}{2}$ of water, what shall two pound of Silver put out, it will be $\frac{7}{8}$, now adde those two weights of water together, and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{3}{4}$, is it too much by $\frac{1}{8}$.

Scholar. Now doe I understand the work as I think, therefore I pray you let me work the rest of the Question. And because this first supposition did erre, I note that position and his errour, and take a new position, esteeming the Silver to be but one pound, so must there bee in Gold seven pound. Then say I: If eight pound of Gold doe yeeld two

pound

pound of water, what ſhall 7 pound yeld? and it will bee 1 pound $\frac{1}{4}$. Again, if 8 pound of ſilver expell 3 pound $\frac{1}{2}$ of water, what ſhall 1 pound expell? and it will be $\frac{1}{28}$. Now muſt I adde thoſe two ſummes together, and they make two pound $\frac{1}{16}$, and they ſhould make 2 pound $\frac{1}{4}$, ſo is it too little by $\frac{1}{16}$. Therefore I ſet the poſitions, with their errors in order as here followeth: And then I multiply in croſſe-waies 2 by $\frac{1}{16}$ and it maketh $\frac{1}{8}$: Likewiſe 1 multiply by $\frac{1}{8}$ maketh $\frac{1}{8}$. And becauſe the ſignes be unlike, I muſt adde theſe two ſums which make $\frac{1}{4}$: and that is the dividend.

2 1
X
$\frac{1}{8}$ † $\frac{1}{16}$

Again, I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the Diviſor. Now I ſhall divide $\frac{1}{4}$, by $\frac{3}{16}$ and the quotient will be $\frac{16}{12}$, that is 1 $\frac{4}{3}$: whereby I know that there was put 1 pound and $\frac{1}{3}$ of ſilver into the Crown, and ſo much gold taken out for it.

Maſter. Prove it now by examination, according to the Queſtion.

Scholar. If there were 1 pound $\frac{1}{3}$ of ſilver, then was there of gold 6 pound $\frac{2}{3}$. Now ſay I by the Rule of Proportion:

If 8 pound of gold expell two pound of water, what ſhall 6 $\frac{2}{3}$ pound $\frac{1}{3}$ expell 7.

8 2
Z
6 $\frac{2}{3}$ 1 $\frac{1}{3}$

It will bee 1 pound $\frac{2}{3}$. Again, if 8 pound of Silver expell three pound $\frac{1}{2}$ of Water, what shall $1\frac{1}{3}$ expell? It will be $\frac{1}{12}$. Now must I adde together 1 pound $\frac{2}{3}$ and $\frac{1}{12}$, and they will make 2 pound $\frac{3}{36}$, that is, 2 pound $\frac{1}{4}$, according to the supposition of the Question: whereby I perceive the work to be well done. And I cannot but much rejoice of this excellent invention, so my desire is kindled vehemently to be perfectly instructed in every part thereof, and namely in this point, whether the proportion between water and Gold be such that for 8 pound of Gold put into a vessell full of water, there shall run out 2 pound of Water, and for as much Silver, whether 3 pound $\frac{1}{2}$ of Water would avoid.

Master. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the exact proportion between Gold and Silver, and Water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I have reserved for another work that intreateth of such matters especially. And at this time you must consider that you learn Arithmetick, which intreateth of the manner to solve doubtfull Questions touching number, without regard what matter is signified by that number: else were it necessary in Arithmetick to teach all Arts, seeing in it may bee moved Questions of all Arts.

In

But ſeeing you are ſo deſirous to know theſe things, I will tell you in ſuch a ſort, that you ſhall practiſe your Art in finding it, and propound it in form of a Queſtion. Gold beareth a greater proportion to water, then Silver doth, and their two proportions be in proportion together, as 48 to 25. But to help you ſomewhat in this Riddle, you ſhall note that the proportion of Quick-ſilver unto water, is the juſt middle number proportionall in progreſſion Geometricall between the proportion of Gold and Silver unto Water.

A queſtion of the proportion of Gold, Silver, and Quick-ſilver, unto Water.

And this proportion is $\frac{220}{21}$. Now if you will know the juſt numbers of theſe three proportions, then muſt you finde out three numbers in Progreſſion Geometricall, whereof the middlemoſt muſt be $\frac{220}{21}$, and the firſt muſt be unto the laſt, as 25 to 48. And thus I will leave you to finde thoſe numbers when you bee at leiſure.

Scholar. Yet Sir, I thank you heartily for thus much, for now I ſee the poſſibility to finde them out. Howbeit, becauſe this Queſtion ſeemeth ſtrange, if it might pleaſe you to inſtruct me ſomewhat in the order of working for it, I ſhould the more eaſily finde the true working.

Maſter. You deſire too much if you will ſtudy for nothing: Therefore to occaſion you to ſtudy the better, I will leave this doubt wholly to your owne ſearch: But as touching the generality of the Rule, Archimedes needed not to take two Maſſes of Gold

and

and silver equall in weight with the Crown, for the proportion might as well be found in any other weight, yea, although the Masse of Gold were of one weight, and the Masse of Silver of another. As for Example: if the Crown were of 18 pound weight as I did suppose, and I have not so much other fine Gold but onely 1 pound, and trying that by water, and finding that it doth expell but $\frac{1}{18}$ of an ounce of water, yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewise of silver, whereof if I had but 2 pound, and finde that it doth expell 3 ounces of water, then might I affirm that 8 pound would expell 12 ounces, that is, 1 pound weight: and so is it good as if the 3 Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmetick.

Scholar. Although I cannot sufficiently thank you for this, yet your promise made me to look for the Art of Extraction of Roots, whereof hitherto I have learned nothing.

Master. I will not break my promise, but intend (God willing) to perform it within this three or four moneths, if I perceive this my pains to be well taken in the mean season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the mean time you shall take this Addition, not for the second Part of Arithmetick which I promised, but for an any-

augmentation of the firſt Part, unto which I would have annexed the extraction of Roots ſquare and cubike, namely for examples of the Statute of Aſſiſe of wood, but that in the ſecond Part I muſt write of divers other Roots, and thought it beſt to reſerve thoſe Rules alſo with their Examples unto the ſame ſecond Part.

Scholar. Sir, although I cannot recompenſe your gwdneſſe, yet I ſhall alwaies doe mine endeavour to occaſion you not to repent your benefit on me thus imployed.

Maſter. That recompenſe is ſuffitient for your part.

FINIS.

The third Part,

OR,

Addition to this Book:

Entreateth of *brief Rules*, called *Rules of Practiſe*, of rare, pleaſant, and commodious effects abridged into a briefer method then hitherto hath been publiſhed.

With divers other neceſſary *Rules*, *Tables*, and *Queſtions*, not only profitable for Merchants, but alſo for Gentlemen, and all other Occupiers whatſoever, as by the Contents of this Book, may appear.

Set forth by JOHN MELLIS, *School-maſter.*

The first Chapter of this Addition, entreateth of brief Rules, called *Rules of Practise*, with divers necessary Questions, profitable not only for Merchants, but also for all other Occupiers whatsoever.

THe working of Multiplication in Practise, is no other thing then a certain manner of Multiplying of one kinde by another: whereupon is brought forth, the Product of the proponed number, which is accomplished by the means of Division in taking the half, the third, the fourth, the fift, or such other parts of the summe, which is to be multiplied. Rule 1.

And for the better understanding of such conversions, you shall understand that in the manner and use of these *Rules of Practise*, you ought first to know the even or aliquot parts of a shilling, which in this Table following doth appear.

Item { 6, 4, 3, 2, 1 } pence is the { 1/2, 1/3, 1/4, 1/6, 1/12 } of a shilling.

Wherein as you see according to the order

of theſe Rules of Practiſe: at 6 d. the yard of any thing, you muſt take ½ of your number which is to be multiplied, and the product that cometh thereof ſhall be ſhillings, if any Unite doe remain it is 6 d.

For 4 d. take the ⅓ of the number that is to be multiplied, and the product alſo produceth ſhillings, if any unites doth remain, each one ſhall be worth in value 4 d. The like is to be underſtood of the other 3, &c.

I *Example.*

At 6 d. the yard, What 379 yards?
189 s.——6 d.

II.

At 4 d. the yard, What 104 yards?
34 s.——8 d.

III.

At 3 d. the yard, What 5014 yards?
1253 s.——6 d.

IV.

At 2 d. the yard, What 532 yards?
88 s.——8 d.

V.

At 1 d. the yard, What 409?
341 s.——1 d.

Here you may ſee in the firſt Example, that 379 yards at 6 d. the yard, are worth 189 s. 6 d. in taking the ½ of 379. And in the ſecond Example the 104 yards at 4 d. the yard, are worth 34 s.— 8 d. in taking the ⅓ of 104. Likewiſe in the third Example, 5014 yards at

3 d. the yard, bringeth forth 1253 s. 6 d. in taking the $\frac{1}{4}$ of 5014. Also in the fourth Example at 2 d. the yard, maketh 88 s. 8 d.

And lastly, in the fift Example: 409 yards at 1d. the yard amounteth to 34s. 1d. in taking the $\frac{1}{12}$ of 409, and so is to be done also of all other Questions the like, when the number of the pence is any of the even or aliquot parts of 12 d.

Item to bring the Products of these shillings, and all other the like into pounds is very easie in dividing of it in your minde by 20, for it is to be understood, that as often as 20 is found in that Product, so many pounds doth it contain: which with facility to perform, alwaies strike off the figure towards your right hand, with a right down dash of your pen, for the o that appertaineth to the 20. And then begin at the left hand, in taking the half of the rest. And if that at the last any Unite doe remain, the same shall be joyned with the figure that is cut off, which shall represent the odde shillings contained in that work.

As for example, in your third Question at 3d. the yard, which amounteth to 1253 s. 6 d. the Product whereof maketh, 62 li. 13 s. 6 d. as here you may see is easily performed by this Example.

	1	
	125	3
	02	13—0

Also for the working of 1 peny the yard, it is something harsh and hard to take the $\frac{1}{12}$ of some Products: therefore to ease that hard worke, you shall first bring your delivered

ſumme into groats by taking $\frac{1}{4}$ part of the product, and if any unites remain of that $\frac{1}{4}$ part, as ſometimes there may, they are pence: and muſt be ſignified with a line from the groats with their title of pence; and becauſe that 60 groats make a pound or 20 s. ſtrike off the firſt figure toward your right hand, for the 0 that appertaineth to 60 (as you did even now for the 0 that belongeth to 20:) Then in taking the $\frac{1}{3}$ of that product, if there doe remain any unites, the ſame ſhall you joyn with the figure that you cut off, eſteeming them as groats, which keep in your minde, and by taking the $\frac{1}{3}$ part of them, you ſhall turn them into ſhillings, and ſo have you done: As for Example, by a Queſtion or two hereafter proponed, ſhall more plainly by the work appear.

At 1 d. the yard, What	54368 yards?
	13592 groats.
	$\frac{1}{3}$ li.——226-10 s.-8 d.

Here in taking the $\frac{1}{4}$ part of 13 59, in coming to the laſt work, the $\frac{1}{4}$ part of 39 being taken, the remainder is 3, which joyned with the 2 that was cut off, maketh 32 groats, which converted into ſhillings, by taking the $\frac{1}{3}$ part, maketh as appeareth 10 s. 8 d. Many other waies there are, but none more apt for a young learner to underſtand then this: wherefore, this one way well impreſſed in memory, is better then twenty waies doubtfully underſtood.

At

At 2 d. the yard, What 4533 yards?

1133 groats-1 d.

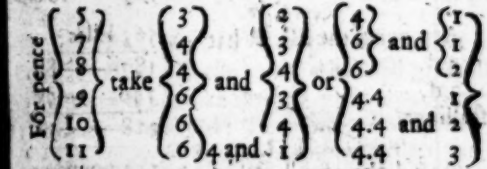
½ li. 18——17——9 d.

At 1 d. the yard, What 64768 yards?

1619 groats.

li. 269——17——4 d.

NOw followeth also to be understood that if the number of pence be not an aliquot part of 12, you must reduce them into some aliquot part of 12; and after the aforesaid manner, you shall make of them two or three Products, as need shall require, and adde them together into one summe. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.



Here in the first note of this Table at 5 d. you shall first take for 3 d. the ¼ of the number that is to be multiplied: and likewise for 2 d. the ⅙ of the same number, adding together both the Products: But if you will work by 4 and 1, you must for 4 d. first take the ⅓ of the number that is to be multiplied: and for 1 d. take the $\frac{1}{12}$ of the whole summe, or rather, which is more better, for 1 peny you may take the ¼ of the product which did come of the 4 pence:

pence: because that 1 d. is the ¼ of 4 d. The totall summes of these two numbers shall be the solution to the Question. And in like manner it is to be done of all others, as by these Examples following shall appear.

I.

At 5 d the yard, What	758 yards?
3 d.	189——6 d.
2 d.	126——4 d.
shillings,	325——10 d.

Otherwise,

At 5 d. the yard, What	758 yards?
4 d.	252——8 d.
1 d.	63——2 d.
shillings,	315——10 d.

II.

At 7 pence the Ell, What	563 Elles?
4 d.	187——8 d.
3 d.	140——9 d.
shillings,	328——5 d.

III.

At 8 d. the pound, What	112 pound?
4 d.	37——4 d.
4 d.	37——4 d.
shillings,	74——8 d.

Otherwise,

At 8 pence the pound, What	112 pounds?
6 d.	56——0
2 d.	18——8
shillings,	74——8 d.

IV.

IV.

At 9 pence the Elle, What	356 Elles?
6 d.	178——0
3 d.	89——0
shillings.	267——0 d.

V.

At 10 d. the piece, What	795 pieces?
6 d.	397——6
4 d.	265——0
shillings.	662——6

VI.

At 11 d. the pound, What	7576 pounds?
6 d.	2788——0
4 d.	2525——4
1 d.	631——4
	6944——8 d.

Pounds. 347—4 s.—8 d.

1 Here in this first Example, where it is demanded (at 5 d. the yard) what will 758 cost? First, for 3 d. I take the $\frac{1}{4}$ of 758: and thereof cometh 189 s. 6 d: Then for 2 d. I take the $\frac{1}{6}$ of the same 758, which amounteth to 126 s. 4 d. these two summes added together, doe make 315 s. 10 d: and so much are the 758 yards worth at 5 d. the yard.

Item also for the same again: First for 4 d. I take the $\frac{1}{3}$ of 758; and thereof cometh 252 s. 8 d: then for 1 d. I take the $\frac{1}{4}$ of the same 758, that is to say, of 252 s. 8 d. and it yeildeth me 63 s. 2 d. which both added together maketh 315 s. 10 d. as before.

2 *Item*, for 7 d. there is taken the $\frac{1}{3}$ and the $\frac{1}{4}$ of the whole summe which is to be multiplied,

and adde them together, that is to say, first, for 4 d. there is taken $\frac{1}{3}$ of 563 : which comes to 187 s.——8 d. as appeareth by the work, and for 3 d. there is taken the $\frac{1}{4}$ of the whole summe, which amounteth to 140 s.——9 d. Both which Products added together, doe make 328 s.—5 d. and so much comes 563 Ells to at 7 d. the Ell.

3 *Item*, for the first 8 d. there is taken for 4 d. the $\frac{1}{3}$ of the whole summe, and another $\frac{1}{4}$ for the other 4 d. which added together, as in the Example doth evidently appear, amounteth to 74 s.—8 d.

Again, for the second work of 112 li. there is taken first the $\frac{1}{2}$ of the whole summe for 6 d. which comes to 56 s. then for that 2 d. you have to take $\frac{1}{3}$ of the whole summe, or if you will, the $\frac{1}{3}$ of the product that came of 6 d. either of which maketh 18 s.—8 d. These two summes being added together, doe make 74 s. 8 d. as in the third Example appeareth.

4 *Item*, for 9 d. there is taken for 6 d. the $\frac{1}{2}$ of the whole summe : and the $\frac{1}{4}$ of the whole summe for 3 d. or otherwise for the 3 d. you may take the $\frac{1}{2}$ of the product that came of 6 d. because 3 d. is the $\frac{1}{2}$ of 6 d. which added together, as plainly appeareth in the fourth Example, amounteth to 267 s. 0 d.

Item, for 10 d. first there is taken for 6 d. the $\frac{1}{2}$ of the whole summe, which amounteth to 397 s.—6 d. Then for 4 d. there is found 265 s. both which added together, make 662 shillings,

shillings, 6 d. as appeareth in the fift Example: It may also be wrought, as appeareth by the second note in the Table, by 4 d. twice taken, and the $\frac{1}{2}$ of the product of 4 d. or else by the $\frac{1}{3}$ of the whole summe, &c.

Item, for 11 d. there is first taken the $\frac{1}{2}$ for 6 d. then the $\frac{1}{3}$ of the whole summe for 4 d. Lastly, the $\frac{1}{4}$ of the last product for 1 d. All which three summes added together, maketh in shillings 6944 s.-8 d. and in pounds 347 l.-4 s.-8 d.

Item, likewise by the same reason, when you will multiply (by shillings) any number that is under 20, you shall have in the Product pounds, if you know the even or aliquot parts of 20, which are here in this little Table set down to sight.

Item, s. { 10, 5, 4, 2, 1 } is the { $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$ } of one pound.

So that for 10 s. which is the $\frac{1}{2}$ of a pound, you may take the $\frac{1}{2}$ of the number which is to be multiplied, and you shall have in your Product pounds, if an Unite doe remain, it shall be worth 10 s.

Likewise for 5 s. you must take the $\frac{1}{4}$ of the number which is to be multiplied, and if there doe remain any Unites, they shall be fourth parts of a pound, every Unite being in value 5 s.

For 4 s. take the $\frac{1}{5}$ of the number which

is

is to be multiplied: and if there doe remain any Unites, they shall be fift parts of a Pound, each Unite being in value 4 s.

For 2 shillings you must take the $\frac{1}{10}$ of the number to be multiplied, wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is neareſt your right hand) from all the other figures with a small right down line or dash with a Pen, and so have you done: for all the other figures which doe remain toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a Pound: and that figure so separated towards your right hand, shall be so many pieces of 2 s. the which figure you must double to make thereof the true number of shillings, as by the Example shall appear.

Finally, for 1 shilling needeth small work, for it is so many shillings as be proponed in the summe, which to bring into pounds, hath been alread taught in the first Rule.

Example.

At 10 s. the piece, What	6543 pieces?	
$\frac{1}{2}$ li.	3271 10 s.	
At 5 s. the Elle, What	4373 Elles?	
$\frac{1}{4}$ li.	1093 5 s.	
At 4 s. the yard, What	7839 yards?	
$\frac{1}{5}$ li.	1567 16 s.	
At 2 s. the pound weight, What	7521 pound?	
$\frac{1}{10}$ li.	752 14 s.	

At

At 1 s. the piece, What	757	3 pieces?	
1/20 li.	387	13 s.	

NExt, followeth in order to be understood, that if the number of shillings be not some even or aliquot part of 20, you must then convert the same number of shillings into the aliquot parts of twenty, and thereof make two or three products as need shall require: which done, adde them together, and bring them into Pounds. And here for thy furtherance, I have set down a note of the order of their parts, as they are to be taken.

s.					s.	
3	2 ℔ 1			13		10.2 ℔ 1
6	4 ℔ 2		5 ℔ 1	14		10. ℔ 4
7	5 ℔ 2			15		10. ℔ 5
8 of	4 ℔ 4	of	5.2.1	16	of	10. 5. 1
9	5 ℔ 4		4.4.1	17		10. 5. 2
11	10 ℔ 1			18		10. 4. 4
12	10 ℔ 2			19		10. 5. 4

For 3 s. according to the tenure that you see is expressed in the Table, you must first take for 2 shillings the 1/10 of the number that is to be multiplied. Then for 1 shilling you must take the 1/2 of the Product which did come of the same 1/10 part: which two summes added together produceth the effect desired.

Item, for 6 shillings according to the note set forth in the Table first for 4 s. I take the 1/5 of the number

number that is to be multiplied: Then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s. and adde them together.

Or else as appeareth also in the Table, for 5 s. you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 s. and adde them together.

Item, for 7 s. first for 5 s. take $\frac{1}{4}$ of the product that is to be multiplied, then for 2 s. take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, &c.

Item, for 8 s. according to reason, and the intent of the Table, for the first 4 s. take the $\frac{1}{2}$ of the product, and the same number again for the other 4 s. and adde them together.

Item, for 9 s. first for 5 s. take the $\frac{1}{4}$ then for 4 s. take the $\frac{1}{5}$: and adde them together.

Otherwise, as you see by the intent of the Table, work twice for 4 s. as was taught even now for 8: and then take the $\frac{1}{4}$ of the last product for the 1 s. but 5 and 4 is the shorter.

Item, for 11 s. first dispatch 10 s. for which you must take the $\frac{1}{2}$ of the product: then lastly, for 1 s. take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 s. where I will end with the first part of my Table. For take the $\frac{1}{2}$ for 10 shillings. And then for 2 shillings, take the $\frac{1}{10}$ of the summe that came of 10 shillings, take and

adde

adde them together, or else if you please for 2 s. you may take the $\frac{1}{10}$ of the whole given number.

To write more of the manner of taking the true parts, I omit. The desirous practitioners will (no doubt) conceive it. Also the Table is some aid to help the unperfect, whereupon by and by I will set down three or four of these notes in Examples, and the rest I will leave to thine own industry and practise, to labour upon.

This is the order most commonly used in practise, when the number of shillings is not an aliquot part of a pound. But (loving Reader) after I have touched the even or aliquot parts of a pound that falleth out in pence and shillings, I will deliver two new Rules that shall drown this common order quite and clean: wherein shall be comprehended in one line, or working both of even and odde parts of shillings under 20 without regard whether it be an aliquot, or not an aliquot part; which two Rules (when they come in place) I commit to thy friendly judgment in working.

Now follow the Examples upon the Notes aforesaid.

At 6 s. the yard, What	3215 yards?
4 shillings	643
2 shillings	321——10
pounds	964——10 s.

Otherwise by Multiplication of 6.

3215

6 shillings 1929 | 0

li. 964 10 shillings.

At 7 shillings the Ell, What 4563 Ells?

5 shillings 1140 15

2 shillings 456 6

li. 1597 1 shilling.

Otherwise by Multiplication of 7.

4563

7 shillings 3194 | 1

1597 1

At 8 shillings the piece, What 7563 pieces?

4 shillings 1512 12

4 shillings 1512 12

pounds 3025 4

Otherwise by Multiplication.

7563

8 shillings 6050 | 4

pounds 3025 4 shillings.

At 13 shillings the piece, What 401 pieces?

10 shillings 200 10

2 shillings 40 2

1 shilling 20 1

pounds 260 13

Other-

Otherwise by Multiplication.

	401
13 shillings	1203
	401
	5213

pounds 260 — 13 s.

These and such like Questions of compound numbers, which I have here in this fourth Rule for orders sake set down, for that it hath been heretofore a common course of work, I account but superfluous. For in the eight and ninth Rules of this my simple Addition shall appear, that the given price of any even or odde number of shillings, either under or above 20 shall be wrought at one or two workings at the most, how difficult soever the Question be.

3 Rule. To reduce pence into pounds at one operation.

IT rem, there resteth yet a kinde of practise, how to bring pence into pounds at the first working, whereupon you must understand that 240 pence maketh one pound, or 20 s. In consideration whereof I cut off the last figure or 0, and there remaineth but 24 (of which 24) 8 d. is the $\frac{1}{3}$ part thereof, 6 d. is the $\frac{1}{4}$ part, 4 d. is the $\frac{1}{6}$ part, and 2 d. is the $\frac{1}{12}$ part thereof.

Whereupon if it were demanded what 1486 yards or pounds of any thing cometh to, at 8 pence the yard, in pricking or cutting off the first figure towards your right hand, for

the o that appertaineth to 240. There is remaining of the said summe 148, whereout I taking the $\frac{1}{3}$ part, and it cometh to 49 li. and there resteth 1, which I put to the 6, that I prick or cut off, and it maketh 16 pieces of 8 pence, which I double to make into groats, and they make 32, whereof the $\frac{1}{3}$ part maketh 10 s. and there remaineth $\frac{2}{3}$ s. which is 8 d. whereby it followeth, that the 1489 yards at 8 d. the yard maketh 49 li. 10 s. 8 d. as by the Example shall appear.

Item, for 6 d. take $\frac{1}{4}$ part of the number from the prickt figure, and if any Unites remain, they are so many six pences, whereof taking the $\frac{1}{2}$ they are shillings, if there doe remain yet one, it is in value 6 d.

Item, for 4 d. take the $\frac{1}{3}$ part of the number from the prickt figure, if any unites doe remain, they are so many groats, which to convert into shillings, take the $\frac{1}{3}$ part. And if any yet remain, they are thirds of shillings, each one in value being worth 4 d.

Item, for 3 d. take the $\frac{1}{4}$ part from the prickt figure, if any Unites remain, they are so many pieces of 3 d. whereof in taking the $\frac{1}{4}$ part maketh shillings: if any thing yet remain, they are the fourth parts of shillings, each one being in value 3 d.

Item, for 2 d. as appeareth also by the Table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remain, they are so many pieces of 2 d. which by taking the $\frac{1}{6}$ part, you shall

ſhall turn into ſhillings, and if any Unites remain, they are ſo many ſix parts of ſhillings, or pieces of 2 d. whether you will.

If one coſt 8 d. What maketh pounds	1486? 49—10—8 d.
If one coſt 6 d. What maketh pounds	7865? 196—12—6 d.
At 4 d. the yard, What maketh pounds	8736 yards? 145—12—0
If one coſt 3 d. What maketh pounds	9874 worth? 123—8—6 d.
At 2 d. the Ell, What maketh pounds	7894 Ells to? 65—15—8 d.

BUt if your number of pence be not an aliquot or even part of 24, then muſt you bring them into the aliquot parts of 24, and make thereof divers products which muſt be added together, as by the Queſtion hereafter following ſhall appear. 6 Rule.

Item, for 5 d. firſt take for 3 d. then for 2 d. and adde them together, according to the inſtruction of the ſecond Rule: or elſe firſt take for 4 d. then 1 d.

Item, for 7 d. firſt take for 4 d. then for 3 d. and adde them together.

Item, for 9 d. first take for 6 d. then for 3 d. and adde them together.

Item, for 10 d. first take for 6 d. then for 4 d. and adde them together.

Item, for 11 d. first take for 8 d. then for 3 d. and adde them together: As by these Examples.

Examples.

1 If one yard cost 5 d. What 749 | 6?

4 pence	126——12
1 peny	31——13
maketh pounds	158——5

Otherwise.

1——5——759 | 6

3 pence	94——19
2 pence	62——6
maketh pounds	158——5s.

2 If one cost 7 d. What 98 | 7 worth?

4 pence	16—9
3 pence	12—6—0
maketh pounds	28—15—9d.

Otherwise.

1——7——98 | 7

6 pence	24—13—6
1 peny	4—2—3
maketh pounds	28—15—9d.

3 If one cost 9 d. What 98 | 7 worth?

6 pence	24—13—6
3 pence	12—6—9
maketh pounds	37—0—3d.

Other-

Otherwise.

	1—9—98 | 7.	
6 pence	24—13—6	
3 pence	12—6—9	
maketh pounds	37—00—3	

4 If one cost 10 d. What 98 | 7?

6 pence	24—13—6	
4 pence	16—9—0	
maketh pounds	41—2—6	

5 If one cost 11 d. What 98 | 7?

8 pence	32—18—0	
3 pence	12—6—9	
maketh pounds	45—6—9	

But if you have any shillings and pence to be multiplied together, then are you to take for the shillings according to the instruction of the third Rule: and for the pence according to the first Rule before mentioned: unless you can spie the advantage thereof, and thereby help your self; as appeareth in this second Example, where first I work for 6 d. which is to be rebated out of the given number, and I have 719 li. 11 s. my desire.

At 19 s. 9 d. the yard, What 738 yards?

	748	*Otherwise by Rebating:* 738
10 shillings	369——0	
5 shillings	184——10	
4 shillings	147——12	6 d. 18——9 s.
6 pence	18——9	li.--7—19—11 s.
pounds	719——11 s.	

The like again is done by rebating, as by these two Examples appeareth:

At 18 s. the Ell, What	418 Ells?
2 shillings	41——16
pounds	376——4 s.
At 16 s. the Ell, What	517 Ells?
4 shillings	10——8
pounds	413——12 s.

7 Rule. AND now I will touch a little the even part of a pound, that falleth out in pence and shillings, whereof for those parts you shall take such like part out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound which also for their better aid is here set down.

1 s. 8 d.	is the	$\frac{1}{12}$
2		$\frac{1}{8}$
3		$\frac{1}{6}$
6		$\frac{1}{3}$

part of a pound.

Item,

Item, first for 1 s. 8 d. take the $\frac{1}{12}$ part of the given number, and if any thing doe remain they are twelve parts of a pound, each one being in value 1 s. 8 d.

Item, for 2 s. 6 d. take the $\frac{1}{8}$ part of the number that is to be multiplied; and if any thing doe remain, they are eight parts of 1 li. each one being in value 2 s. 6 d.

Item, for 3 s. 4 d. as appeareth by the Table, you must take the $\frac{1}{6}$ part of the given number, and if any thing doe remain, they are 6 parts of a pound, each one being in value 3 s. 4 d.

Item, for 6 s. 8 d. take the $\frac{1}{3}$ part of the number that is to be multiplied: And if any Unites doe remain, they are thirds of a pound, every one being worth 6 s. 8 d.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a pound, as 16 s. 8 d. maketh $\frac{5}{6}$: which 16 s. 8 d. is easily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then set 60 the groats of a pound under 50: cutting off the two Cyphers as is here performed.

And then have you brought 16 s. 8 d. into the known parts of a pound, which maketh

16—8
3
5|0
6|0

But

But yet (gentle Reader) for thy further instruction, I have hereunto annexed in a Table, how pence and shillings bear proportion to a pound, which I commit to thy friendly benevolence; it will be some aid unto the ungrounded Practitioner: but I count him the best workman that can presently reduce his given price into the known and proportionate parts of a pound.

A Table of the Aliquot parts of a pound, or 20 shillings.

s.	d.	li.
0	2	1/20
0	3	1/30
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	1/20
3	4	1/6
3	9	1/16
4	0	1/3
5	0	1/4
6	0	1/10
6	3	1/16
6	8	1/3
7	0	1/10
7	6	1/8
8	0	2/3

s.	d.	li.
8	4	1/15
8	9	1/10
9	0	3/20
10	0	1/2
11	0	11/20
11	3	3/10
12	8	2/3
12	0	1/10
13	0	13/20
13	4	2/3
13	9	11/10
14	0	7/10
15	0	1/4
16	0	4/5
16	8	5/6
17	0	11/20
17	6	7/8
18	0	9/10
18	4	11/15
18	9	11/10
19	0	19/20

Here follow four Examples upon the four Notes delivered.

At 1 s. 8 d. the yard, What maketh pounds — 3884 yards? — 323—13—4 d.

At 2 s. 6 d. the yard, What maketh pounds — 4563 yards? — 570—7—6 d.

At 6 s. 8 d. the Ell, What maketh pounds — 7562 Ells? — 2520—13—4 d.

Now by custome you are able to work by all sorts of summes being delivered in shillings and pence, as 1 s.—1 d. 2 s.—2 d. 3 s.—3 d. and so of all other: wishing you to have some consideration of your Questions, when they are set down, for there are many subtile abbreviations, and great advantages to be gotten, and easily to be perceived.

As of 3 s. ——— 8 d. of 2 s. and 1 s. 8 d.

Of 4 s. ——— 2 d. of 3 s. ——— 4 d. & 10 d. which 10 d. is $\frac{1}{4}$ of 3 s. ——— 4 d.

Of 5 s. ——— 8 d. of 4 s. 1 s. ——— 8 d.

Of 5 s. 10 d. of 5 s. and 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this mean when you have taken one product, you may oftentimes upon the same take

take another more briefly then upon the summe which is to be multiplied, &c.

NOw (gentle Reader) that you have seen the vertue of the even or aliquot parts of a pound in shillings alone, and also in the aliquot part of shillings and pence: according to my promise hereafter followeth a brief and easier method for any even number of shillings either under or above 20, then ever yet hath been published; Notwithstanding Mr. *Humphry Baker*, whose travell is worthy commendation, and whom for knowledge sake I reverence, hath in some part touched this first Part, though not in this method. The work of the Rule is both pleasant, ready, and brief, as by the variety of the Examples delivered thereupon shall appear. And first I will set forth a Question, thereby the better to express or teach you the order thereof; which is this: 8 Rule.

If one cost 6 s.	What	8574?
1	6	8574
maketh pounds		2572.——4 s.

To the understanding of this Example, after you have set down your given number in form of the Rule of Three, with a line drawn under it, you shall presently set a prick under your first figure 4. toward your right hand drawing from the prick, as heretofore hath been practised, a little short line, thereto set down Mr *John Mellis* his first Rule.

the

the ſhillings anon, which done, multiply the firſt figure 4 by 6, the value of your price; (which here you ſee ſtandeth in ſight above the line) it maketh 24, which is one pound four ſhillings. The one pound keep to carry to the next place, and the four ſhillings ſet down at the end of the preſcribed line towards your right hand. Thus have you done now with 6 above the line, and alſo with 4 in the firſt place (for the prick under 4 doth ſignifie that 4 hath done his office.) Then ſecondarily for a generall Rule take but the ½ of the given price, which here is 3, which 3 is the number that ſhall now continue the reſt of the multiplication, and end the work, whereupon I multiply 3 into 7, ſtanding in the ſecond place it maketh 21, and with the 1 pound I kept in 22, ſet down 2, and keep 2 in minde, working according to the Rule of Multiplication, delivering the tenths in mind in their due place, which done, the product from the prick to your left hand repreſenteth the pounds, and the other at the end of the ſhillings, as appeareth by the Examples:

Note a generall Rule.

If one yard coſt 2 s. What	7536?	
1 2	7536	
maketh pounds	753	12 s.
If one yard coſt 4 s. What	8792?	
1 4	8792	
maketh pounds	1758	8 s.

If

If one piece coſt 6 s. What 9537 ?

1 6 9537

maketh pounds 2861 —— 2 s.

If one piece coſt 8 s. What 7509 ?

1 8 7509

maketh pounds 300 —— 12 s.

If one coſt 12 s. What 5794 ?

1 12 5794

maketh pounds 3476 —— 8 s.

If one coſt 14 s. What 3705 ?

1 14 3705

maketh pounds 2593 —— 10 s.

If one coſt 18 s. What 5703 ?

1 18 5703

maketh pounds 5132 —— 14 s.

If one coſt 22 s. What 953 ?

1 22 953

maketh pounds 1048 —— 6 s.

Let theſe ſuffice (gentle Reader) for an entrance into even numbers. And now I will ſhew the like Rule for any odde or uneven part of a pound.

TO help you to the underſtanding of theſe other Queſtions that hereafter follow: where in my firſt Example the given number is

6487

6487 at 3 s. the yard: I multiply 3 above the line into 7, it maketh 21. The 1 s. is set down, and the 1 li. I keep. Now am I to take the ½ of 3, which because it is an odde number I cannot.

Mr John Kellis his second Rule.

Therefore I shall keep and continue my multiplication by three still, and work by the ½ of the rest of the given figures or number, to wit, 648. And first the ½ of 8 which is 4 multiplied into 3, maketh 12, there to joyn the 1 li. in minde, it maketh 13, set down 3, keep 1. Then again multiply by 2 the ½ of 4 it maketh 6, and with 1 in minde it maketh 7. Then lastly, take the ½ of 6, which is 3, saying, 3 times 3 is 9, which 9 set down, and so is the Question answered, as appeareth by the practise, and Examples following:

At 3 s. the yard, What 6487?

1 3 6487

maketh pounds 973 — 1 s.

If one yard cost 5 s. What 4269?

1 5 4269

maketh pounds 167 — 5 s.

At 7 s. the Ell, What 6489?

1 7 6489

maketh pounds 2271 — 3 s.

If one Ell cost 9 s. What 2807?

1 9 2807

maketh pounds 1263 — 3 s.

At

At 11 s. the Pistolet, What		8263?
1	11	8263
maketh pounds		4544 —— 13 s.

If 1 piece cost 13 s. What		4629?
1	13	4629
maketh pounds		3008 —— 17 s.

But now note (gentle Reader) when the given price falleth upon an odde number, as 3, 5, 7, 11, 13, &c. then it is to be presupposed that the given summe to be multiplied, must be a summe made of even numbers, 2, 4, 6, 8, 10, &c. else cannot that Question be wrought at one line or working.

Providing alwaies that it may bear an odde figure in the first place towards your right hand, as appeareth in these six Examples, which last were wrought, and such like, &c. which may bear an odde number for the price, and be done at one line, or working very well.

But if the given price be an odde number, and the summe to be multiplied, odde numbers also: then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall Rule. First, work for the even number, contained in that Question, or given price, according as you have learned, and then afterwards for the one odde shilling, take the ½ of A generall Rule.

the ſumme given to be multiplied, omitting the firſt priekt place, as was taught for the working of 1 s. in my firſt Rule of Practiſe, and adde thoſe two together, and you ſhall have your deſire.

Examples.

At 3 s. the yard, What	7539 yards?
2 ſhillings	753——18
1 ſhilling	376——19
maketh pounds	1130——17 s.
At 7 s. the Ell, What	7539?
5 ſhillings	7539
2 ſhillings	2261——14
1 ſhilling	376——19
maketh pounds	6238——13
At 13 s. the yard, What	7534?
10 ſhillings	3767——0
2 ſhillings	751——8
1 ſhilling	376——2
maketh pounds	4897——2

Note this well. And thus have I abridged into theſe two Rules how to bring any number of ſhillings, whatſoever they be, into pounds, with a briefer method, then ever yet hath been publiſhed, which I commend unto thy friendly cenſure and judgment in the uſe and practiſe thereof.

If

If 1 yard coſt 6 s. 5 d. What	1231 ?
6 ſhillings	369——6
4 pence	20—10—4
1 peny	5—2—7
maketh pounds	394—18—11
At 14 s. 2 d. What	2825 ?
14 ſhillings	1977——10
2 pence	23—10—10
maketh pounds	2001——0—10
At 16 s. 4 d. What	2531 ?
16 ſhillings	2024——16
4 pence	24——3—8d.
maketh pounds	2066—19——8
At 3 s. the Piſtolet, What	8325 ?
maketh pounds	1248——15 s.
At 7 s. the Crown, What	6529 ?
	2285——35 s.
At 9 s. the piece, What	6567 ?
maketh pounds	2955——35 s.

Theſe three laſt Queſtions may ſeem ſomething harder, yet they are eaſie enough, if you mark them well; If I ſhould explain them, then are they too eaſie. Therefore I leave them to whet the mindes of the deſirous.

10 Rule. ITem when any one of the ſummes, which is to be multiplied, is compoſed of many Denominations, and the given number but of one figure alone; then ſhall you multiply all the Denominations of the other ſumme by the ſame one figure, beginning firſt with that ſumme which is leaſt in value toward your right hand, and bring the Product of thoſe pence into ſhillings, and the Product of the ſhillings into pounds, As by this Example appeareth.

At 3 li. 7 s. 4 d. a yard, What are 9 worth?
maketh pounds 30——6 s.——0 d.

11 Rule. BUt if any of the ſummes that are to be multiplied, there be a broken number; Firſt work for the whole according to the inſtructions that you have learned, and then take ſuch part of the given price, as that broken number beareth in proportion to the price, as in the Examples following. After you have wrought for 3 s. and for 6 d. then are you to take the $\frac{1}{2}$ of 3 s. 6 d. for the $\frac{1}{2}$ yard, and adde that to the ſumme: So adding all the 3 Products together, which make 43 li. 2 s. 9 d. the juſt price of 245$\frac{1}{2}$ Ells, and thus muſt you doe of all other.

At 3 s. 6 d. the Ell, What	245 $\frac{1}{2}$?
3 ſhillings	36——18
6 pence	6——3
$\frac{1}{2}$	1——9
maketh	43——2——9

At

At 16 s. 4 d. the piece, What 14 ¼?

16 shillings	11——4
4 pence	0——4——3
¼	12——3
maketh pounds	12——0——11

If one piece cost 4 li. 3 s. 6 ½ d. What 12 pieces?

4 pounds	48——
3 shillings	1——16
6 pence	6——6
½	6
maketh pounds	50——2——7

The Proof.

If 12 pieces cost 50 li. 2s. 6 d. What 1 piece?

maketh pounds	4——3——6 ½

Item, touching the manner how to understand the order of this Question, and others the like, first seek how many times 12 is contained in 50, which is 4 times, and so resteth 2 li. which 2 li. converted into shillings, and joyned with the other 2 s. maketh 42 shillings: wherein is found 12, three times resteth 6 s. which turned into pence, putting thereto the 6 d. in the first place, it maketh 78, wherein 12 is found six times, resteth 6 d. which containeth 12, but ½ a time, put that ½ to the 6 d. and then the solution is 4 li. 3 s. 6 ½ d. as appeareth by the Practise thereof. 14 Rule.

13 Rule. *Item*, The like is to be done of any thing that is bought or sold after five score to the hundred, or the Quintall. As for Example.

If 100 pound cost 27 li. 13 s. 4 d. What one pound?

27 li.—13 s.—4 d.

2

s. 5 | 53
12
1 | 10
53

d. 6 4 0
10 0. or $\frac{2}{5}$

Maketh 5 s. $6\frac{2}{5}$ d.

But to work it more neatly, it is by a little understanding, ended thus.

27 li.—13 s.—4 d.

20

s. 5 | 53
12

d. 6 | 40
100

Maketh 5 s. $6\frac{4}{5}$ d.

I have wrought this at length for the aid of the young learner, because he should understand how all the Multiplication is set down.

Item, To the understanding of this and such like Questions, the right down line is all the guide, which is pulled down close by 20. as you see in the Example, where 27 li. 13 s. is reduced all into shillings, and maketh 553 s.

The 5 towards the left hand being separated with the hanging or right down line, is the just number of shillings, that answereth the Question. Nextly, 53 s. is multiplied by 12, to reduce them to pence, putting to the 4 d. it yeeldeth for

for the multiplication of the first figure two 110. the one beyond the line towards the left hand, is 1 d. towards the rest of the price: then 53 also multiplied by 1 yeeldeth 53: but the 5 behind the line toward the left hand, is also 5 d. more, towards the price, which 1 and 5 I adde together under the line, it maketh 6 d. So is there found now as appeareth by the Titles of shillings and pence, 5 s. 6 d.

Finally, I come now on this side the line towards the right hand, and under 12 I finde first 10, and then 3, which added together, maketh 40. under which 40, you must put the 100, and it maketh $\frac{40}{100}$ which abbreviated, cometh to $\frac{2}{5}$. So the just price of 1 pound after 5 score to the hundred, maketh 5 s. 6 $\frac{2}{5}$ d.

One Example more, and so I will leave this Rule.

If 100 cost 10 $\frac{3}{4}$ d. What 9874?

6 d.	246	17	
4 d.	164	11	4
$\frac{2}{4}$ d.	20	11	5
$\frac{2}{4}$ d.	10	5	8 $\frac{1}{2}$

	li.	4 | 42	5	parts of a peny, 5 $\frac{1}{2}$
		| 20		
Maketh	s.	8 | 45		
		| 12		
		| 45 $\frac{1}{2}$ 91		
	d.	5 | 100 | 100		

Alſo the like may be done of the uſuall weights here in *England*, (which is 112, for every hundred weight) in caſe you know the aliquot parts of a hundred weight, which are theſe, 56 li. 28 li. 14 li. and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li. 28 li. is the $\frac{1}{4}$ of 112 li. 14 li. is the $\frac{1}{8}$ and 7 li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the ſumme of the money that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the ſumme of money that 112 li. weight is worth.

For 14 li. take the $\frac{1}{8}$ of the ſumme that 112 li. is worth.

And for 7 li. the $\frac{1}{16}$ of the ſumme of money that 112 li. is worth.

As for Example; at 17 li. 19 s. the hundreth pounds weight, that is to ſay, the 112 li. what ſhall 3 quarters and 7 pound coſt?

1 C——17 li.——19 s.	3 q;——7 li.
2 quarters 8——19——6	
1 quartern 4——9——9	
7 pounds 1——2——5 $\frac{1}{4}$	
Maketh pounds 14——11——8 $\frac{1}{4}$	

The

The ſecond Chapter intreateth of the Reduction of divers meaſures to others value by *Rules of Practiſe.*

NOw will I ſhew a few Examples of Practiſe in reducing of Meaſures, as Ells, Yards, Braces, Pawns of Genee, &c. Much more would I have touched, but that I fear the Book will riſe to too great a Volume. 18 Rule.

In 864 Ells of *Antwerp*, how many yards of *London?*

864	864
432	216
216	648

maketh 648 yards of *London.*

ITem, in theſe and ſuch like Queſtions of Flemmiſh meaſure, to be brought into yards Engliſh, firſt take the ½ of the given number, as appeareth in the firſt Example towards your left hand, Then take half of that Product, or the ¼ of the given number, and adde thoſe two Products together, as they ſhall be yards Engliſh; as by the Example you may perceive.

The

The second Example toward your right hand is yet briefer then the first, whose work is this; Take the $\frac{1}{4}$ of the delivered number, and that Product subtract out of the given number, and the rest sheweth your desire. Of these two waies use which you think best.

The Proof.

In 648 yards *London*,
How many Ells of *Antwerp*?

648
216

maketh 864 Ells of *Antwerp*.

15 Rule. ITem, for the understanding of this work, first take the $\frac{1}{3}$ part of the yards of *London*, which found, adde that $\frac{1}{3}$ part and the yards together, as appeareth by the Practise, and the Product sheweth the Ells of *Antwerp*.

Item, in 20 yards of *London*,
How many Ells of *Antwerp*?
maketh 426 $\frac{2}{3}$ Ells.

320 yards
106 $\frac{2}{3}$
426 $\frac{2}{3}$ Ells $\frac{2}{3}$

Proof.
426 $\frac{2}{3}$ Ells.
106 $\frac{2}{3}$
320 yards.

Other Reductions.

16 Rule. ITem, you shall understand, that forasmuch as 6 Braces of Millain, make 5 Ells of *Antwerp*, whereupon according to the *Rules of Practise*, you may reduce the one into the other, by the like reasons aforesaid, in taking the $\frac{1}{6}$ part and

and then subtract the same, to make Ells of *Antwerpe*. And again, by the contrary, taking the $\frac{1}{3}$ part with adding the given number, to turn the Ells to Braces. As for Example.

In 876 Braces, how many Ells of *Antwerpe*?

876

146

Ells 730 *Antwerpe*

The contrary.

730 Ells Flemmish.
146 Braces.
816 Braces.

Ells 739 *Antwerpe*.

182 $\frac{1}{2}$.

Yards 547 $\frac{1}{2}$ English

Thus appeareth, that 876 Braces by Practise, make 730 Ells Flemmish, which Ells Flemmish reduce into English yards.

So again upon the same first Question of Braces, I would know how many yards English they make.

After the rate that 100 Braces are are worth 62 $\frac{1}{2}$ yards.

876 Braces.

438

109 $\frac{1}{2}$

I answer, 547 $\frac{1}{2}$ yards.

Item, To the understanding of this work, and such like, first take the $\frac{1}{2}$ of the given Braces, and after take the $\frac{1}{4}$ of that half, or the $\frac{1}{2}$ of the given number, and adde them together, and the Products are also yards English.

Item,

ITem, three Ells of *Rochell* make 5 Ells at *Lisbone*. So likewise three Ells at *Lions* make 5 Ells at *Antwerp*.

To work these and such like, double the Ells of *Lions*, and the Ells of *Rochell*, and from their Products subtract the $\frac{11}{21}$ and the rest shall be the Ells of *Antwerp*, or the Ells of *Lisbone*.

I. *Example.*

In 63 Ells of *Lions*, how many Ells of *Antwerp*?

63
63
126
$\frac{1}{3}$ 21
Answ. 105 Ells *Ant.*

In 100 Ells of *Rochel*, how many Ells of *Lisbone*?

110
200
$\frac{1}{2}$ $32\frac{2}{3}$.
Answ. $166\frac{2}{3}$ Ells of *Lisb.*

Touching the proof or return of these and such like Questions for a generall Rule, you shall first take the $\frac{1}{3}$ of the given number: and adde that $\frac{1}{2}$ and the given number together, and the $\frac{1}{2}$ of that Product shall be your desire.

Example.

In 105 Ells of *Antwerp*, how many Ells of *Lions*?

105
$\frac{1}{3}$) 21
$\frac{1}{2}$) 126
Answ. 63 Ells of *Lions.*

In $166\frac{2}{3}$ Ells of *Lisbone*, how many Ells of *Rochell*?

$166\frac{2}{3}$
$\frac{1}{3}$) $33\frac{1}{3}$
$\frac{1}{2}$) 200
Answ. 100 of *Roch.*

Quest.

Queſtions of Factoridge & Intereſt, brief and truly reſolved by the *Rule of Practiſe*, without Time.

1 *Queſtion.*

AT 5 ſhillings *per centum*, What comes 8860 li. 15 s. 4 d. unto?

Anſwer. Note that 5 s. is the fourth of 20 s. I take the ¼ part of 8860 li. 15 s. 4 d. which makes 2215 li. 3 s. 10 d. Now the Root is 100, which you ſhould divide by, ſo cutting the two laſt figures away of the pounds, you have 22 li. then multiply 15 li. by 23 s. ſo adde the 3 unto, you ſhall have 303 s. cut away the two laſt figures, there reſteth 36 s. Laſtly, there remains 3 s. which I multiply by 12 to bring into pence, and ſo I finde 0 d. and $\frac{44}{100}$ remaining, which being abbreviated makes $\frac{11}{50}$ parts of a peny, ſo I finde that there is gained 22 li. 3 s. 0 d $\frac{11}{50}$ parts of a peny.

li. 22. 15 s. 03. 10

20
s. 3 | 03
12
06
0 d. | 4
4 | 23
100 | 50

Queſt.

2. *Queſt.* At 10 s. *per centum*, what comes unto? 1448 li. 16 s. 8 d.

Anſw. Note that 10 s. is the $\frac{1}{2}$ of 20 s I take the $\frac{1}{2}$ of 1448 li. 16 s. 8 d. which makes 724 li. 8 s. 4 d. cut off the two laſt figures, and there reſteth 7 li. then multiply the 24 li. by 20 s. and adde the 8 s. and it maketh 488 s. cut the two laſt figures off, and there reſteth 4 s. then multiply 88 s. by 12 d. and take in 4 d. and there reſteth 1060 d. cut off the two laſt figures, and there reſteth 10 d. and $\frac{60}{100}$ which is $\frac{3}{5}$ of a peny: ſo the whole ſumme is 7 li. 4 s. 10 $\frac{3}{5}$ which is the Anſwer to the Queſtion.

li. 7. 24. 8. 4.
20.
s. 4 | 88
12
180
88
d. 10 | 6 | 0 | 3
10 | 0 | 5

3. *Queſt.* At 15 s. *per centum*, what comes unto? 1008 l. 12 s. 0 d.

Anſw. Note 15. that is $\frac{1}{2}$ and $\frac{1}{4}$ of 20. take the $\frac{1}{2}$ of 1008 lib. 12 s. there reſteth 504 li. 6 s. then take the $\frac{1}{4}$ adde them together, the totall will be 756 li. 9 s. cut off the two laſt figures, reſteth 7 li. then multiply by 20 s. and take in your 9 s. it maketh 1129 s. cut off the two laſt figures, there reſteth 11 s. then multiply by 12 d.

504. 6 ——— 0
252. 3 ——— 0
756. 9 ——— 0
20
11 | 29
12
58
29
3 | 48 | 24 | 12
100 | 50 | 25

there

there cometh 348 d. cut off the laſt two figures, there reſteth 3 d. and $\frac{48}{100}$ which being abbreviated maketh $\frac{12}{25}$ parts of a 1 d. ſo ſhall you finde 7 li. 11 s. 3 d. $\frac{12}{25}$, which is the Anſwer to the Queſtion.

4. *Queſt.* At 1 li. *per centum*, what comes unto?

Anſwer. Cut away the two laſt figures, & multiply by 20, & 12, and take in your ſhillings and pence. And you ſhall finde 8 li. 13 s. 8 d. $\frac{4}{5}$ as doth appear by this work.

li. 868 li. 13 s. 4 d.

20

s. 13 | 73

12

1 50

7 3

d. 8 8 | 0 4 |

10 0 | 5 |

5. *Queſt.* At 2 li. *per centum*, what comes unto?

Anſw. Multiply the whole ſumme by 2 lib. thus, then cut off the two laſt figures of your pounds, as you did before, and you ſhall find 112 li. then multiply by 20 and by 12, taking in your ſhillings & pence, and you ſhall finde 112 li. 3 s. 4 d. which is either for Factor or Broker, &c.

5608 li. 6 s. 8 d.

2

112 16. | 13. 4

20

3 | 33

12

7 0

3 3

4 00

6. *Queſt.*

6. *Queſtion.* At 3 li. *per centum*, what comes

Anſwer. Multiply the ſumme by 3 l. thus; then cut off the two laſt figures, and you ſhall finde 24 li. then multiply by 20, and by 12, taking your ſhillings & pence, and you ſhall finde 0 s. 6 d $\frac{27}{50}$ parts of a peny, which is ſomething above a half-peny.

800 li. 18s. 2 d. unto?

3

24 02. 14. 6.

20

0 54

12

114

54

6|54|27

100 50

7. *Queſtion.* At four *per centum*, what comes

Anſwer. Multiply by 4 li. thus, cut off the two laſt figures, Multiply by 20, and by 12, taking in your ſhillings and pence, and you ſhall finde 11 li. 19 s. 0 d. $\frac{03}{25}$ parts of a peny, which is ſomething above a farthing.

298 li. 15 s. 9 d. unto?

4

11 95. 3. 0.

20

19 03

12

06

03

036|18|9

100|50|25

8. *Queſtion.*

8 *Quest.* At 5 li. $\frac{1}{4}$ *per centum*, What comes *Answer.* Multiply by 5 li. thus, then take the $\frac{1}{2}$ of the whole summe, and place the figures even, then take the $\frac{1}{2}$ of that $\frac{1}{2}$, and adde all three summes together, cut off the two last figures, then multiply by 20, and by 12, taking in your shillings, and pence, and you shall finde 210 li. 7 s. 7 d. $\frac{2}{10}$ parts of a peny, which is the Answer to the Question.

3658 li. 16 s. 8 d.
unto ? 5

18294	03	4	
1829	08	4	
914	14	2	
210	38	50	10
	20		
	765		
	12		
	130		
	66		
	7190		
	100		

9 *Quest.* At 6 li. $\frac{1}{2}$ *per centum*, What comes *Answer.* Multiply by 6 li. and then take $\frac{1}{2}$ of the whole summe, adde them both together, then multiply by 20, and by 12, taking in your odde shillings & pence, and you shall find 369 l. 10 s. 10 d. $\frac{1}{20}$ parts of a peny, which is the Answer to your Question.

5684 li. 12 s. 6 d.
unto ? 6

34107	15	0	
28	2	06	2
369	50	01	3
	20		
	05		
	01		
	15	3	
	100	20	

10 *Queſt.* At 7 li. $\frac{1}{2}$ *per Centum*, What comes 3868 li. 13 s. 4 d. unto?

Anſwer. Multiply by 7 li. then take the $\frac{1}{2}$ adde them together, cut off the two laſt figures, then multiply by 20, you ſhall finde 290 li. 3 s. The Anſwer to the Queſtion.

7

37080. 13. 4
1934. 06. 8

290 1500. 0

20

3 00

184

11 *Queſt.* At 8 li. *per Centum*, What comes 2560 li. 17 s. 6 d. unto?

Anſwer. Multiply 8 li. cut off the two laſt figures, multiply by 20, and by 12, and you ſhall finde 204 li. 17 s. 5 d. $\frac{1}{25}$ parts of a peny.

8

204 87. 02. 0

20

17 42

12

84

42

5 04 | 2 | 2

100, 50, 25

Queſtions of Intereſt with Time, wrought by Practiſe.

1 *Queſtion.*

AT 6 *per Centum*, What comes unto for 1 moneth

368 li. 16 s. 8 d.

2813. 00 0

Anſwer. Multiply by 6 li. there cometh 2813 li. 00 s. 0 d. then take for 1 month the $\frac{1}{12}$ of the Totall, and you ſhall find 234 l. 8 ? 4d. of the two laſt figures of the pounds, Multiply by 20 and by 12, taking in your odde money, and you ſhall finde 2 li. 6 s. 10 d. $\frac{1}{3}$ parts of a peny, which is the Anſwer to the Queſtion.

li. 234. 08. 4

20

s. 6 | 88

12

180

88

d. 10 | 6 | 0 3

10 | 0 5

2 *Queſtion.* At 7 li. $\frac{1}{2}$ *per cent.* What comes unto for 2 months.

Anſwer. Multiply by 7 li. then take $\frac{1}{2}$, adde them two together, then for your 2 months take the $\frac{1}{2}$ of the totall, multiply by 20 and 12, taking in your odde ſhillings & pence, and you ſhall finde 47 li. 10 s. 1 d. $\frac{2}{10}$ parts of a peny, which is the Anſwer to the Queſtion.

3800 li. 12 s. 8 d.
7
26604. 08 8
1900. 06 4
28504. 15 0
47 50. 15 10
20
10 15
12
30
1 6
1 | 9 0
10 | 0

3 *Queſt.* At 8 li. *per centum*, What comes unto for 3 months.

Anſwer. Multiply by 8 li. then take for your 3 months $\frac{1}{4}$ of the totall, multiply by 20, and by 12, adding in your odde ſhillings & pence, and you ſhall finde 197 li. 5 s. 11 d. $\frac{1}{25}$ parts of a peny, your demand.

9864 li. 16 s. 4 d.
8
78918. 10. 8
197 29. 12. 8
20
5 92
12
192
93
11 | 12 | 6 | 3
100 | 50 | 25

4 *Queſt.*

4. *Queſt.* At 6 li. $\frac{1}{2}$ *per centum*, what comes unto for 4 months.

Anſwer. Multiply by 6 li. then take $\frac{1}{3}$ adde both together, then for your 4 months take $\frac{1}{3}$ part of the whole, cut away your two laſt figures, multiply by 20, and by 12: adde in your odde ſhillings & pence, and you ſhall finde 131 li. 14 s. 10 d. $\frac{42}{53}$ parts of a peny, your demand.

6080 li. 13 s. 0 d.
8

36483 18 0
3040 06 6

39524 04 6

131 | 74 11 2
| 20

14 | 91
| 12

184
91

10 | 94 | 47

| 100 | 50

5. *Queſt.* At 8 li. *per Centum*, What comes unto for 5 months.

Anſwer. Multiply by 8 li. then for 5 months take $\frac{2}{4}$ of the totall, cut off the two laſt figures of your pounds, Multiply by 20 and by 12, adde in your odde ſhillings and pence, and you ſhall finde 100 li. 13 s. 4 d. your demand.

3020 li. 00 s. 00 d.
8

24160 00 00

6040 00 00
4026 13 04

100 | 66 13 04
| 20

13 | 33
| 12

70
33

4 | 00

6 Queſtion. At 8 *per centum*, What comes unto for 6 months.

8060 li. 12 s. 0 d. 8

Anſwer. Multiply by 8 li. then for your 6 months take the $\frac{1}{2}$ of the Totall, cut off the two laſt figures of your pounds. Multiply by 12, taking in your odd ſhillings & pence, and you ſhall finde 22 li. 8 s. 5 d. $\frac{12}{31}$ parts of a peny, your deſire.

64484. 16 s.
322|42.08 0
20
8|48
12
96
48
5|76|38.19
100|50.25

7 Queſt. At 8 li. *per centum*, What comes unto for 4 months.

5896. 00. 0 d. 8

Anſwer. Multiply by 8 li. then for your 7 months take $\frac{1}{3}$ and $\frac{1}{4}$ of the Totall, cut off the two laſt figures of your pounds, then multiply by 20 and 12, taking in your odde money, and you ſhall finde 275 li. 13 s. 11 d. $\frac{1}{3}$ your deſire.

7168. 13. 4
11792. 00. 0
275|14. 13. 4
20
2|93
12
190
93
11|2|0|1
10.0|5

8 Queſt.

8 *Queſtion.* At 8 *per centum*, What comes unto for 8 months.

3680 li. 0 8s. 03 d. 8

29443. 04 0
9813. 08 0
9814. 08 0

116 28 16 0
20
5 76
12
152
76

912 122 6
100 50 25

Anſwer. Multiply by 8 li. then for 8 months take $\frac{2}{3}$ of the totall, cut off the two laſt figures of your pounds, then multiply by 20, and by 12, adde in your odde money, and you ſhall finde 116 li. 5 s. 9 d. $\frac{1}{25}$ your deſire.

9 *Queſt.* At 8 li. *per centum*, What comes unto for 9 months.

3684 li. 19 s. 0 d. 8

29479. 12 0

14739. 16 9
7369. 18 0

221 09. 14 0
20
1 94
12
188
94
11 28 14 1
100 50 25

Anſwer. Multiply by 8 li. then for your 9 months take $\frac{1}{2}$ and $\frac{1}{4}$ of the whole ſumme, cut off the two laſt figures of the pounds, then multiply by 29, and by 12, taking in your odde ſhillings & pence, and you ſhall finde 221 li. 1 s. 11 d. $\frac{1}{25}$ which is ſomething above a farthing

10 *Quest.* At 6 ¼. *per centum*, What comes unto for 10 months?

100 li. 0 s. 0 d.
6

600. 0 0
50. 0 0
25. 0 0
675. 0 0
337. 10 0
225. 00 0

Answer. Multiply by 6 li. then take the ½ and ¼ of 100 li. adde all 3 summes together, then for the 10 months take ½ and ⅓ of the Totall, add them together, cut off the two last figures of the pounds, multiply by 20. and 12. adding in your shillings and pence, cutting off the last figures of your shillings & pence, you shall finde 5 li. 12 s. 6 d. your desire.

5 | 62. 10 0
20.
12 | 50
12
100
50
6 00

11 *Quest.*

11 *Queſtion.* At 8 li. *per centum*, what comes unto for 11 months.

Anſwer. Multiply by 8 l. then for 11 months take $\frac{2}{3}$ and $\frac{1}{4}$, from the Totall adde all three ſummes together; cut off the two laſt figures of your pounds. Multiply by 20, and by 12, adding in of your ſhillings & pence, cutting off the two laſt figures of your ſhillings and pence, and you ſhall finde 65 li. 0 s. 7 d. $\frac{12}{25}$ parts of a peny, your deſire.

886 li. 16 s. 0 d.
8
7094 08 0
2364 16 0
2364 16 0
1773 12 0
65 03 04 0
20
0 64
12
128
64
768 34 17
100 50 25

12 *Queſt.* At 8 li. *per Centum*, What comes unto for 12 months.

Anſwer. Multiply by 8 l. cut off the two laſt figures of the pounds, Multiply by 20, and by 12, adding in your ſhillings and pence, cut off the two laſt figures of your ſhillings, and the two laſt of your pence, and you ſhall finde 726 li. 8 s. 11 d. $\frac{12}{25}$ parts of a peny, your deſire.

9080 li. 12 s. 2 d.
8
726 44 17 4
20
8 97
12
11 68 34 17
100 50 25

FINIS.

The third Chapter teacheth of the Order and work of the *Rule of Three* in Broken Numbers after the Trade of Merchants digreſſing ſomething from Mr *Records*, which is comprehended in three Rules.

NOw, that I have ſomewhat intreated of the *Rules of Practiſe*, I will give a few inſtructions, after my ſimple order, for the working of the *Rule of Three* in broken numbers, wherein I ſhall need to ſay the leſſe, becauſe I hope the ſtudious Learner, that hath travelled any thing in the *Grounds of Arts*, is not unfurniſhed of knowledge capable to underſtand me.

But before I deliver any Inſtructions for broken Numbers, I will propone a Queſtion which ſhall be wrought three ſundry waies, thereby to ſhew as it were, three Degrees of Compariſon: how farre the *Rule of Three* in broken, for more ſpeed of work, differeth from the whole, which I rather ſet down for a view, that the ſtudious herein may be more deſirous to attain broken, leaving any more to diſcourſe in Dialogue form, but only to give inſtructions where need is: and in the reſt to put forth the Queſtions, with their Anſwers.

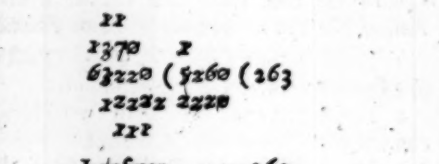
My

My first Question is thus:

If one yard cost 6 s. 8 d. What are 789 worth at that rate?

1 —— 6 s. —— 8 d. —— 789 The first way.
12 80
80 63 120 d.

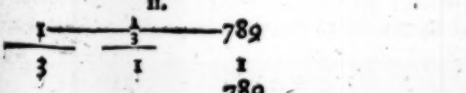
Here the Product of the summe are pence, according to the nature of the middle number.



11
1370 1
63220 (5260 (263
12222 2220
111

I answer —— 263 The second way.
1 —— 6 $\frac{2}{3}$ s. —— 789
—— 20
3 20 ——
15780 s.

Here the Product of the summe are s. according to the nature of the middle number.



1
15786 (5260 (263
3333 2220
li.
1 —— $\frac{1}{3}$ —— 789 The third way.
3 1 1
789

Here

Here the Product is pounds, according to the title of the second number.

1
689 (263
333

I answer, 263 li.

Now that you have seen the three former virtues of the Rule of Three, whose Products have first brought forth pence, next shillings, and lastly pounds; I will deliver three Notes in order following: and with them a dozen Questions, that shall shew the work of the *Rule of Three* in broken Numbers or Fractions.

Note these three well.

1 The first four shall be sundry Questions of a Fraction coming in the second place.

2 The second four shall be of two Fractions coming in the second or third place.

3 The third four of Fractions in all three places.

Notes upon the first Rule for a Fraction coming in the second place.

My first Question is this.

1 Rule. IF one yard cost me 3 s. 4 d. What are 756 worth at that price?

The first variety. In setting down the Question to perform the work, I turn four pence into the part of a shilling, which is $\frac{1}{3}$ and then the Question standeth thus:

1 ——— $63\frac{1}{3}$ ——— 756

To

To the ready working of this Queſtion, and all ſuch other like: my firſt note is this, which take for a generall Rule; That when any one Fraction ſhall come, either in the ſecond or third place, that the Denominator of that Fraction or Fractions, muſt alwaies be brought unto the Number, or Numerator of the firſt place; and thereby multiply the one into the other. A generall Rule.

And this benefit is alwaies gotten by the vertue of bringing the Denominator of the ſecond Numbers Fractions unto the firſt place: For the Fraction in the middle number is now releaſed: and the Product that cometh of the Multiplication, is of the nature and like the denomination of the whole number in the ſecond place which here are ſhillings. Note this.

Whereupon now to work the Queſtion, I bring 3, the Denominator of the Fraction in the ſecond place, unto the firſt Number 1, with a line ſet under thus $\frac{1}{\ }$, and the third under it thus, $\frac{1}{3}$ ſaying, once 3, is 3 my Diviſor: that done, reduce $3\frac{1}{3}$ ſaying, 3 times 3 is 9, and the other 1 over 3 make 10, my ſecond number in the *Rule of Three*, by which 10 I doe multiply my laſt number 756, as appeareth by the work thereof, and it yeeldeth 7560 ſhillings, my Dividend.

Then dividing 7560 by 3, my Diviſor, it yeildeth in Quotient 2520 ſhillings, which maketh 126 pounds, as appeareth here moſt plainly, both by the Example, and the work.

At

At 3 s. 4 d. the yard, What 756 yards?

1 — 3 1/3 — 756

3. 10 10

7590 s.

1	x
756	25200
3333	22220

(126.

I answer 126 li.

Yet otherwise upon the same Question, altering the price now into the proportion it beareth to a pound, for the 3 s. 4 d. is 1/6 part of a pound: which Example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

1 — 1/6 — 756

The second variety.

1 — 1/6 — 756 6 1 1

756 pounds.

As soon as I have carried 6 the Denominator of my middle number unto my first place, as before hath been taught, I pull down 1, the Numerator of 6, with a line under 6, thus, 1/1; and that 1 in custome I pull down in sight; being the figure that I will multiply my third or last number by, according to the tenour of the *Rule of Three*. And because one can neither Multiply nor yet Divide (though here it is set down in form of Multiplication, the rather for your understanding) the Product of the Multiplication according to the Declaration of

of this my first Rule or Note, is converted into the Title of my second number, which here are pounds. Now followeth the Division performed in my Divisor 6, to make an end of that Question.

23
756 (126. which maketh 126 li as before.
666

And thus much for the variety in working that Question.

And now followeth another.

My second Question.

If one yard of Cotten cost $\frac{1}{4}$ d. what 859?

1		8 $\frac{1}{4}$		859
4		33		33
		2577		
		2577		
		28347		

x

x | 3x x	x | 1	li.	s.	d.
28347 (7086.	(590	(29	10	6 $\frac{2}{4}$
4444	2222	220		

This Question was also wrought like the first, and bringeth forth 29 li. 10 s. 6 $\frac{1}{4}$ d. the price of 859 yards.

My

My third Question.

If seven pounds of any thing cost 3 li. 10 s. What comes 987 pounds to?

li.

7 ——— 3 ½ ——— 987

2 ——— 7 7

14 6909

00

241

2347

6909 (493 $\frac{1}{14}\frac{1}{2}$

2444

11

Answer, 493 li—10 s.

Notes upon my second Rule for two Fractions coming in the second and third place.

My first Question is this.

IF one Ell cost 13 s.—4 d. What half a quarter or $\frac{1}{8}$ of an Ell?

Answer. First bring 13 s.—4 d. into the parts of a pound, which is $\frac{2}{3}$, and then will the Question stand thus.

1 ——— $\frac{2}{3}$ li. ——— $\frac{1}{8}$

ITem, for the performance of this work, doe as before was taught in the first Rule: first bring

bring 3 the Denominator of the second Fraction unto your first number 1, setting a line under it thus: $\frac{1}{\_}$, Saying once 3 is 3, that done, bring 8 the Denominator of the third Fraction, setting it under 3, and multiply them together, saying, 3 times 8 maketh 24, which 24 is your Divisor. (Now have you done with the Denominator 8) therefore you shall put a line under thus, $\frac{3}{\_}$. And the like line also under $\frac{8}{\_}$, setting or pulling down under them their own Numerators, that is, 2 under 3, and also 1 under 8, as appeareth in the Example, which Numerators for a generall Rule evermore to be pulled down of custome in sight, to multiply the one by the other, according to the tenour of the *Rule of Three*. Then I multiply the one by the other, saying, once 2 is 2, which signifieth 2 li. being of the nature and like denomination of the middle number, which 2 li. is to be reduced into shillings, otherwise it cannot be divided by my first number 24.

Then dividing 40 by 24, the Quotient bringeth forth $1\frac{2}{3}$. So much is $\frac{1}{3}$ of an Ell worth after that rate. Otherwise, although 2 li. could not be divided by 24, yet it might have been abbreviated to $\frac{1}{12}$ of a pound: which is worth 1 s. 8 d. as before.

li.

1 —— $\frac{2}{3}$ —— $\frac{1}{2}$ (1

3 2 1 2 (6

8 20 4⁰ (1 $\frac{2}{3}$ s.

24 40 24

2 *Queſtion.*

IF one pound of any weight coſt 13 ſhillings 4 pence, what are $\frac{7}{8}$ of the pound worth after that rate?

Anſw. Reduce the 13 ſhillings 4 pence into the parts of a pound: which is $\frac{2}{3}$, and then will the Queſtion ſtand thus:

li.

1 —— $\frac{2}{3}$ —— $\frac{7}{8}$

Item, for the underſtanding of this, if you mark well the laſt Example, this and the reſt lieth open, and needs ſmall inſtruction. For as you did laſt, ſo again, bring the Denominator of the ſecond and third Fraction, unto the firſt figure 1, multiplying the one into the other; which maketh alſo 24, your Diviſor.

Note. Then making a line under 3 thus, $\frac{3}{\ }$ and a line under 8 thus, $\frac{8}{\ }$ and pulling down their Numerators under each figure, that is 2 under 5, and 7 under 8, which as I ſaid before for a generall Rule I pull down of cuſtome in ſight,

ſight, to be the two Numbers, that of duty ought to be multiplied together; which done, I bring 2, being the leſſer figure under 7, multiplying them together, it maketh 14, which are of the nature of the middle number: that is to wit, pounds, which 14 cannot aptly be divided among 24, therefore are reduced into ſhillings, as is plainly to be ſeen in the Example: then 280 ſhillings parted among 24 yeildeth for his Quotient 11 s. 8 d. your deſire, and the juſt price of $\frac{1}{8}$ of an Ell. Otherwiſe 14, though it could not be divided by 24, might by Mediation or Diviſion in broken Numbers have been divided or abbreviated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11 s. 8 d. as before. But my good will and meaning is to aid young beginners: therefore have I reduced the 14 li. into ſhillings, which is the eaſier way.

Now followeth the Example.

1	2	7	1
3	3	8	2
8	2	7	4 (6
24		3	280 (11 $\frac{8}{24}$ s.
		14	214
		20	2
		280 s.	Anſwer, 11 $\frac{8}{24}$ s.

The third Example.

If one yard coſt me 2 s. 6 d. What $345\frac{1}{4}$ yards?

Anſwer. Firſt put 6 d. into the parts of a ſhilling, and then the Queſtion ſtandeth thus:

1 ——— $2\frac{1}{2}$ ——— $345\frac{1}{4}$.

Item, to the ready underſtanding of this, and all ſuch like, according as before hath been declared, bring the Denominators of the ſecond and third Fractions unto the firſt place, multiplying them the one into the other, all which make 8 for the common Diviſor. Then next reduce your ſecond number: ſaying, two times 2 is 4, and 1 is 5. as was taught in the Example aforeſaid. Laſtly, reduce your third number $345\frac{1}{4}$ all into fourths, and they make 1381. which 1381 is to be multiplied by 4. according to the tenour of the *Rule of Three:* which done, maketh 6905 s. and divided by 8, your Diviſor yeildeth in Quotient $863\frac{1}{4}$ s. which maketh in pounds 42 li. 3 s. $1\frac{1}{2}$: and ſo much are the $345\frac{1}{4}$ yards worth at that price.

The ſame Queſtion wrought again by 2 s. 6 d. is now converted into the parts of a pound, and ſtandeth thus:

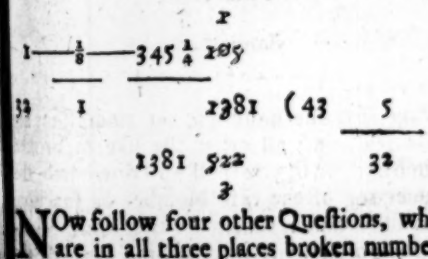
1 ——— $\frac{1}{3}$ ——— $345\frac{1}{4}$.

Item, After I have brought here my ſecond and

and third Denominator unto my first place, and found 32 to be my Divisor; having thus finished my first place with all things unto him belonging (which is meant of bringing and Multiplying the Denominators of the second and third Fractions into him) I then go in hand to see what is to doe in my second place, where presently of custome I pull down my Numerator 1 under 8, being the figure in sight that shall multiply my third number.

Then lastly, I reduce $345\frac{1}{4}$ all into fourths as afore was practised, which maketh 1381, the which 1381 I am to multiply by 1 my second number, they are nothing increased, but by the *Metamorphosis* of my work they are now 1381 li. being of the nature of the middle number, as I have often shewed you, which divided by 32 my Divisor yeildeth 43 li. and $\frac{5}{32}$, which $\frac{5}{32}$ of a li. reduced into known numbers make 3 s. $1\frac{1}{2}$ d. as before.

Example.



NOw follow four other Questions, which are in all three places broken numbers; or whole and broken together.

Item, First, for the finding out of your Diviſor, you ſhall take this for a moſt certain, and generall Rule: That you muſt multiply the Numerator of the firſt number in the Queſtion, by the Denominator of the ſecond: And that Product again, by the Denominator of the third: And the totall thereof ſhall be your Diviſor.

Secondly, for a generall Rule to finde out your Dividend, multiply the Denominator of the firſt number by the Numerator of the ſecond, and the whole thereof by the Numerator of the third. And the totall thereof ſhall evermore be your Dividend.

Now for an Example, I propone this Queſtion, thereby to make my meaning more plain, and to ſhew you, as I have done in the reſt, the manner and order of the work.

If $\frac{2}{3}$ of any weight or meaſure coſt $\frac{1}{6}$ of a pound, or 20 s. what are $\frac{1}{8}$ of the like weight or meaſure worth after that rate?

Example.

$\frac{2}{3}$ — $\frac{1}{6}$ — $\frac{1}{8}$

Item, for the more plainer underſtanding hereof, and all other the like in broken Numbers: Firſt, you ſhall pull down two, the Numerator of the firſt Number or Fraction, with a line under, thus, $3\ \frac{1}{3}$: that done, according as you have learned before, bring 6, the Denominator of the ſecond Fraction, and ſet it under

under two, multiplying the one into the other, which maketh 12. Then lastly, bring 8, the Denominator of the third Fraction, and set it under 12, multiplying that 12 by 8, which amounteth to 96, or else for more brief, multiply 6 by 8 saying, six times 8 makes 48, which 48 set under 2, and multiply the one into the other, it maketh 96, as before. And this 96 is the first number in the *Rule of Three*. That shall alwaies for a most generall Rule be your Divisor.

Secondly, to work for your Dividend, you shall, (as it hath been sufficiently declared before) pull down 5, the Numerator of your second Fraction, and set it under 6, with a line under, thus $\frac{6}{5}$.

That done (as you know) you are to pull down 3, the Numerator of the third Fraction, and set it under 8, with a line under it, thus, $\frac{8}{3}$, multiplying the one into the other, according to the Tenour of the *Rule of Three*; which maketh 15. Then according to my Note, forget not to bring the Denominator of the first Fraction, which is 3 under 15, and multiply them together, which maketh 45, which 45 is your Dividend, and are of the nature of Denomination of the middle number, as I have taught you before: And therefore are 44 li. which aptly cannot be divided by 96. Therefore you shall reduce the 45 li. into s. as you see performed in the Example, which amounteth to 90 s. which divided by 96 your Di-

viſor, it yeeldeth 9 s. and $\frac{16}{20}$ of a ſhilling, which in leſſer terms is $\frac{4}{5}$: which $\frac{4}{5}$ in money maketh $4\frac{1}{2}$ d. and ſo much will the aforeſaid $\frac{1}{8}$ coſt, as by the work following ſhall appear.

The Example.

2	5	3	
3	6	8	
2	5	3	
6		5	3
12		15	96
8		3	9003 ($9\frac{16}{20}\frac{4}{5}$ s.
96		45	96
		20	
		900	

Otherwiſe though 45 could not be divided by 96, yet by Diviſion in broken numbers it might have been abbreviated to $\frac{15}{32}$ of a pound, which reduced into known parts, will make 9 s. $4\frac{1}{4}$ d. as before.

Now my ſecond Example ſhall be the proof of this Queſtion.

If $\frac{1}{8}$ yards coſt $\frac{15}{32}$ of a pound, or 20 ſhillings, What ſhall $\frac{1}{3}$ coſt?

Anſwer. Work as was taught you before, and you ſhall have your deſire.

Here

$\frac{1}{2}$	$\frac{11}{32}$	3
3	15	2
32	2	
96	30	
3	8	
288	240	

Here as appeareth by the work, the Multiplication being ended, 240 is to be divided by 288, which to ſome perchance may ſeem hard, yet notwithſtanding is the work good. Therefore abbreviate 240, by 288, as you ſee here is practiſed: and the end of your abbreviation ſhall come to $\frac{1}{6}$ your deſire, $\frac{240}{288}\ \frac{120}{144}\ \frac{60}{72}\ \frac{30}{36}\ \frac{5}{6}$.

Otherwiſe,	240	120	60	30	5
	288	144	72	36	6

Otherwiſe,	340	40	5
	288	48	6

The third Queſtion:

If $\frac{2}{4}$ Ells coſt 13 s.--d. What 156 $\frac{1}{2}$ Ells?

Anſwer. To work this Queſtion the ſhorteſt way: reduce 13 s.4 d. into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you have ſet down the Queſtion, the Numerator of the firſt Fraction 3 is pulled down under 4, and the Denominators of the other 2 Fractions multiplied

into

into him, which maketh 18, your Diviſor.

Then the Numerator of the ſecond Fraction is pulled down, under 3 in cuſtome now in ſight, ready to multiply my third number, by which is performed as ſoon as the laſt number 156 ½ is reduced into halfs.

Then laſtly, I multiply that Product by 4, the Denominator of the Fraction: it yeeldeth 1504, which I divide by 18, and my Quotient is 139, li. and ⅛ of a pound remaining, which is worth 2 s. 2⅔ d. And ſo much will 156 ½ Ells coſt, as by the work following doth appear.

3	2	156 ½	x	
4	3	313	47	
3	2	2	x752	li.
6		626	2504	(139 ⅛
18		4	2838	
		2504	xr	

The fourth Queſtion.

If 2½ Ell coſt 1⅔ li. what cometh 26 ¼ Ells to?

Item, to the workmanſhip of this Queſtion, firſt reduce your ſecond number in ſaying, three times 1 is 3, and 2 is 5. Then bring the multiplication of the Denominators of the ſecond and third Fractions which maketh 12, and multiply that 2 by 5 your firſt Numerator, and it maketh 60, which is your Diviſor.

Then the Reduction of the ſecond Number, which

which is 5, multiplied by 117 the Product of the laſt numbers-reduction, make 585, which 585 yet reſteth to be multiplied by 2, the Denominator of the Fraction in the firſt place, yeildeth 170, which divided by your Diviſor, 60, yeildeth 19 li. 10 s. as appeareth by the work thereof.

Thus having now touched the 12 Queſtions whereof I firſt pretended, which with diligence and oft practiſe, I truſt are ſufficient to aid the deſirous unto the working of any broken numbers, I will now intreat of divers neceſſary Rules incident unto traffick, as hereafter followeth.

The fourth Chapter teacheth of Loſſe, and Gain, in the Trade of Merchandiſe.

IF one yard coſt 6 s. 8 d. and the ſame is ſold again for 8 s. 6 d. the Queſtion is, what is gained in 100 li. laying out on ſuch commodities?

Anſwer. The *Rule of Three* direct, applyed two manner of waies to doe the ſame: the one is to ſay, If $6\frac{2}{3}$ give $8\frac{1}{2}$, what giveth 100? Multiply and Divide, and look what your Quotient bringeth forth, above your laying out, is the neat gains and ſolution to your Queſtion:

Queſtion: If you follow the work, your ſolution will bring forth 127 li. 10 s. which is 27 li. 10 s. more then your principall, and ſo much is gained in the 100 pounds laying out.

Item, to work it the other way, which I take the neareſt, ſeek the difference betwixt the juſt price and the other price, which is 1 s. 10 d. then ſay by the *Rule of Three*:

If 6 $\frac{2}{3}$ s. gain 1 $\frac{1}{6}$ s. What ſhall 100 li. gain? Multiply and divide, and you ſhall finde 27 li. 10 s. and ſo much is gained in 100 li. laying out.

You may uſe which of theſe two waies you think good.

The proof.

If a yard of Cloth be delivered for 8 s. 6 d. whereupon was gained after the rate of 27 li. 10 s. in 100 li. laying out: The Queſtion is, What the yard coſt at the firſt hand?

Anſwer. Put your gain 27 li. 10 s. to 100 li. all maketh 127 li. 10 s. Then ſay, If 127 li. 10 s. give but a 100 li. what giveth 8 $\frac{1}{2}$ s. Work, and you ſhall finde 6 s. 8 d. the true ſolution to your Queſtion.

Yet another Example or proof upon the firſt Queſtion.

If one yard coſt 6 s. 8 d. The Queſtion is, at what price the ſame is to be ſold again, for to gain

gain 27 li. 10 s. in 100 li. laying out?

Anſwer. Say by the *Rule of Three*, if 100 li. gain 127 li. 10 s. what giveth 6 $\frac{2}{3}$ s? Multiply and Divide, and you ſhall finde 8 s. 6 d. your true Solution.

If one Ell coſt 7 s. 8 d. and be ſold again for 8 s. 6 d. The Queſtion is, What is gained in 20 li. laying out in ſuch commodities?

Anſwer. Seek the difference betwixt the juſt price, and the other price which is 10 d. and then apply the *Rule of Three*, as before is taught, ſaying, If 7 $\frac{2}{3}$ s. give $\frac{5}{6}$ s. what giveth 20 li? Multiply and divide, and you ſhall finde 2 li. 3 $\frac{11}{23}$ s. and ſo much is gained in 20 li. laying out.

The proof alſo by an Example of Loſſe.

A Merchant hath bought Holland-cloth at 8 s. 6 d. the Ell, which proveth not to his expectation, whereupon he is content to loſe 2 li. 3 $\frac{11}{23}$ in 20 li. laying out. The Queſtion is, what price ought to be made of the Cloth, abating this loſſe?

Anſwer. Doe as before in Gains hath been taught, putting 2 li. 3 $\frac{11}{23}$ s. to your 20 li. all together, maketh 22 li. -3 $\frac{11}{23}$. Then ſay by the *Rule of Three*, If 22 li. 3 $\frac{11}{23}$ s. give but 20 li. what ſhall come of 8 $\frac{1}{2}$ s? work, and you ſhall finde 7 s. 8 d. the juſt price that the Ell ought to be ſold for after the rate of this loſſe.

Thus

Thus it appeareth evidently, as in Company the Rule is appliable as well to Gain as Losse.

If 20 $\frac{1}{4}$ yards cost 36 li. $\frac{1}{2}$ 10 s. how shall I sell the same again $\frac{1}{3}$ of the Principall, or to make of 3, 4. which is all one?

Answer. By the *Rule of Three*, If 3 doe give 4, what will 36 $\frac{1}{2}$ give? Multiply and divide, and you shall finde 48 $\frac{2}{3}$ li. Then say again, if 20 $\frac{1}{4}$ yards doe give 48. $\frac{2}{3}$ li as well Principall as Gain, what will 1 yard be worth at that price? Multiply & divide, and you shall finde 2 li. $\frac{46}{243}$

If 1 Ell of Cloth cost me 8 s. 8 d. and afterwards I sell 10 $\frac{1}{2}$ Ells thereof for 5 li. 13 s. 4 d. I would know, whether I win or lose: and how much upon the 100 li. of money.

Answer. See first at 8 s. 8 d. the Ell, what 10 $\frac{1}{2}$ Ells comes to, and you shall finde 4 li. 11 s. and I sold the same for 5 li. 13 s. 4 d. so that I did gain upon the 10 $\frac{1}{2}$ Ells 22 s. 4 d. Then if you would know how much is gained in 100 li. I say by the *Rule of Three*, if 4 li. 11 s. did gain 22 s. 4 d. what will 100 li. gain? Multiply and divide, and you shall finde 24 li. 10 s. 10 d. $\frac{11}{91}$ and so much is gained in the 100 li. of money.

If 12 $\frac{1}{2}$ yards cost me 11 li. 5 s. and I sell the yard again for 16 s. the Question is whether I doe win or lose, and how much in or upon the pound of money?

Answer.

Answer. Look what the 12 $\frac{1}{2}$ yards come to at 16 s the yard, and you shall finde ten pound. But they cost 11 pound 5 shillings. So there is lost upon the whole 1 pound 5 s. Then to know how much is lost in the pound, say by the *Rule of Three*, if 11 $\frac{1}{4}$ pound doe lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and divide, and you shall finde 2 s. 2 d. $\frac{2}{3}$. and so much is lost in the pound of money.

If I sell the 100 weight of any commodity for 4 pound, whereupon I doe lose after 10 pound in the 100 pound, I demand how much I shall lose or gain in the 100 li. if in case I had sold the same for 4 pound 10 shillings.

Answer. Say, if 90 pound yeild 100, how much will 4 give? Multiply and divide, and you shall finde 4 $\frac{4}{9}$. Then say again, if 4 $\frac{4}{9}$ give me 4 $\frac{1}{3}$ what will 100 come to? Multiply and divide, and you shall finde 101 pound $\frac{1}{4}$ which is more then 100 li. by 1 li. 5 s. and so much is gained in the 100 li.

A Merchant hath sold Currans for the sum of 430 li. and he hath gained therein after 10 li. in the 100 li. The Question is, to know how much he gained in all.

Answer. Say by the *Rule of Three*: If 100 li. doe gain 10 li. what will 430 li. gain? Multiply and divide, and you shall finde 43, and so much hath he gained in all.

If

If one yard be worth 28 ½ s. for how much shall 10 yards be sold to gain after 8 li. 6 s. 8 d. in the 100 pound?

Answer. First, adde 8 li. 6 s. 8 d. to 100. Then say, if 100 li. doe give 108 ⅔ s. for principall and gain, what will 28 ⅓ s. principall yeild? Multiply and divide, and you shall finde 30 ⅔ s. Then say again by the *Rule of Three*, if one yard doe give 30 ⅔ s. (which is as well the principall as the gain) what shall 10 yards give? Multiply and divide, and you shall finde 15 li. 8 s. 9 d. And for the same price shall the 10 yards be sold, for to gain after the rate of 8 li. 6 s. 8 d. upon the 100.

A branch or proof out of this Question.

A Merchant hath sold Clothes for 15 li. 8 s. 9 d. and he hath gained in the whole the summe of 1 li. 3 s. 9 d. The Question is to know how much he hath gained in the 100 li?

Answer. To know this, first rebate the gain from the price, and there will remain 14 li. 5 s. 0 d. Then say by the *Rule of Three* direct, If 14 li. ¼ give me 1 li. 3 ¼, what will 100 li. give? Multiply and divide, and you shall finde 8 li. 6 s. 8 d. the effect desired, the proof is apparent in the Question before.

Yet

Yet another branch or proof of the first Question.

If 10 yards be delivered for 15 li. 8 s. 9 d. whereupon was gained after the rate of 8 li. 6 s. 8 d. upon the 100 li. the Question is, What the yard did cost at the first hand?

Answer. First, say by the *Rule of Three*, if 10 with principall and gaine yeild 15 li. 8 $\frac{1}{4}$ s. what shall 1 yeild? Multiply and divide, and you shall finde 30 $\frac{3}{5}$ s. Then say again by the *Rule of Three*, if 108 $\frac{1}{2}$ principall and gain give but 100, what shall 30 $\frac{3}{5}$ of principall and gain yeild? Work, and you shall finde 21 $\frac{1}{2}$ s. And so much did the yard cost at the first peny.

If 1 yard cost 36 s. how much shall 12 yards be sold for, to gain after the rate of 10 li. in the 100?

Answer. First, say, If 100 give 100 li. principall and gain, what will 36 s. give? Multiply and divide, and you shall finde 39 $\frac{1}{5}$ s. Then say again by the *Rule of Three*: If 1 yard of principall and gain yeild 39 $\frac{1}{5}$ s. what shall 12 yards gain? Multiply and divide, and you shall finde 23 li. 15 $\frac{1}{5}$ s. which $\frac{1}{5}$ s. in known number, is 2 $\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gain after the rate of 10 in the 100.

The Proof.

If 12 yards be sold for 23 li. 15 s. 2 $\frac{2}{5}$ d. whereupon

upon is gained after 10 li. in the 100. The Question is, what the yard cost at the first peny?

Answer. First say, If 12 give 23 li. $15\frac{4}{3}$ s. what one yard? Multiply and divide, and you shall finde $39\frac{4}{3}$ s. Then say again by the *Rule of Three*, If 110 li. give but a 100. what shall $39\frac{4}{3}$ s. give? Work, and you shall finde 36 s. the just price of the yard at the first hand.

Item, When one Merchant selleth Wares to another, and he giveth to the Buyer 1 li. 6 s. 8 d. upon the score, or 20 li. The Question is, How much shall the Buyer gain upon the 100 li. after that rate?

Answer. First, adde 1 li. 6 s. 8 d. unto 20 li. and they are $21\frac{1}{3}$. Then say, If 20 li. give $21\frac{1}{3}$, what shall 100 give? Multiply and divide, and you shall finde $106\frac{1}{3}$. So the Buyer getteth after the rate of $6\frac{1}{3}$ li. upon the 100 li.

Gentle Reader, other necessary Questions appertaining to Losse and Gain, you shall have in the eighth Chapter of this Treatise.

The

The fifth Chapter entreateth of Loſs and Gain upon time, wrought by *the double Rule of Three*, or by *the Rule compoſed*: which is contained in four ſpeciall ſelected branches, or Queſtions of divers forms, *each one of them ſpringing from the firſt Queſtion*, and each one of them alſo being a proof to other, &c.

IF one yard coſt me 2 s. 8 d. ready money, and after I ſell the ſame again for 2 s. 10 d. to be paid for it at the end of 3 months: the Queſtion is, What I gain upon the 100 li. in 12 months.

Anſwer. Firſt ſay, if $2\frac{1}{3}$ gain $\frac{1}{3}$, what ſhall 100 li. gain? Multiply and divide, and you ſhall finde $6\frac{1}{4}$ li. Then ſay again, by the *Rule of Three*, if 3 months gain $6\frac{1}{4}$ li. what ſhall 12 months gain? Work, and you ſhall finde 25 li. and ſo much ſhall I gain in 12 months after the rate.

Item, You may alſo work it all at one working by the firſt part of the *Rule of Three* compoſed, ſaying, if $2\frac{1}{3}$ d. in 3 months doe gain $\frac{1}{3}$ of a ſhilling, (which is 2 d.) what will 100 li. gain in 12 months? Which for thy further encouragement, the work of this one Example I

have here put down, to verifie that I affirm in the firſt part of this *Ground of Arts*, that this Rule, and ſo all others, more rejoiceth in *Broken*, then in *Whole*.

s.	months	s.	li.	mo.
$2\frac{1}{2}$	3	$\frac{1}{6}$	100	12
8		1	20	
3 2			2000	
24 72000		5	3	
6 24144 (5000 (2			6300	
144 144 2200			12	
x			72000	

Where the Multiplication and the Diviſion being ended, maketh 25 li. your deſire.

If a yard be delivered for 2 s. 10 d. to be paid at 3 months, whereupon was gained after the rate of 25 li. in the 100, for 12 months, the Queſtion is now, what the yard coſt at the firſt hand?

Anſwer. Firſt ſay, if 12 months gain 25 li. what ſhall 3 months gain? Work, and you ſhall finde $9\frac{1}{4}$ li. Then ſay again the ſecond time, if $106\frac{1}{4}$ li. give but 100, what ſhall $2\frac{1}{2}$ li. give? Work, and you ſhall finde 2 s. 8 d. which is the juſt price that the yard coſt at the firſt hand.

If one yard of Cloth coſt me 2 s. 8 d. ready money, for what term ſhall I ſell the ſame again for 2 s. 10 d. ſo that I might gain after the rate of

of 25 li. upon the 100 li. in 12 months?

Answer. First say, if 2 $\frac{2}{3}$ gain $\frac{1}{2}$, what shall 100 li. gain? Multiply and divide, and you shall finde 6 $\frac{1}{4}$ li. Then say again for the second work: If 25 li. be come of 12 months, what shall come of 6 $\frac{1}{4}$? Work, and you shall finde 3 months, the just term of time that the Cloth ought to be delivered at 22 s. 10 d. to gain 25 li. upon the 100 li. in 12 months.

If one yard cost me 2 s. 8 d. ready money, for what price shall I sell the same again to be paid at the end of 3 months, so that I may gain after the rate of 25 li. in the 100 li. for 12 months?

Answer. First say, if 12 gain 25 li. what shall 3 months gain? Multiply and divide, and you shall finde 6 $\frac{1}{4}$ li. Then say for the second work, if 100 li. give 106 $\frac{1}{4}$, what giveth 3 $\frac{2}{3}$ s? Work and you shall finde 3 s. 10 d. and for that price must the yard be sold to gain after 25 li in the 100 pound for 12 months.

Many other of these Questions I might here have delivered, but for fear the Book would rise to too thick a volume, and so to make the price so much the dearer, whereby it might not be so portable to my Countrey-men, as I wish it. But these 4 I have of purpose framed in this order, having relation one to another, assuring you that what Question soever may be proposed within the compasse of this Rule

you ſhall finde by one of theſe 4, to make a ſolution. And moreover, divers others are yet to be delivered where the Creditor giveth divers daies of payment, which can never be well wrought, nor yet underſtood, unleſſe you can firſt finde by Art the juſt times that all thoſe payments, how different ſoever they be, ought to be paid at once: whereupon firſt I think good here to give ſome inſtructions unto ſuch a Rule, for it is the only aid for the finiſhing of ſuch Queſtions as hereafter ſhall follow.

The ſixt Chapter entreateth of Rules of Payment, which is a right neceſſary Rule, and one of the chiefeſt handmaids that attendeth upon buying and ſelling, &c.

Example.

A Merchant doth owe a ſumme of money, whereof the $\frac{1}{3}$ is to be paid at 6 months, and the $\frac{1}{2}$ at 8 months, and the reſt at a year. If he would pay all at one payment, the Queſtion is, What time ought to be given him?

Anſwer. I have omitted the quantity of the ſumme, for you ſhall underſtand the Rule is appliable,

appliable, and yeildeth a true solution to what summe soever shall be proposed: But now for order sake in teaching, I doe imagine the summe to be 60 li. whereupon the manner of this work is to multiply the proportionate part of the money by the time, as in company. Then 20 being the first payment, and the $\frac{1}{3}$ of 60 which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the Example in the Operation, maketh 2 months: next 30, which is the $\frac{1}{2}$ multiplied by his term 8, yeilds 4 months, then the rest which is 10 li. must needs be abbreviated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 months, produceth $\frac{12}{6}$, maketh 2 months. All which added together, as appeareth in the Operation, maketh 8 months, which is the just time that all those payments ought to be paid at once.

$\frac{1}{3}$ by $\frac{6}{1}$	2 Months.
$\frac{1}{2}$ by $\frac{8}{1}$	4 Months.
$\frac{1}{6}$ by $\frac{12}{1}$	2 Months.

A Merchant hath 800 li. to pay the $\frac{1}{6}$ thereof ready money, the $\frac{1}{4}$ at 2 months, the $\frac{1}{2}$ at 4 months, and the rest at a year. The Question is, if he would pay all at one payment, what time ought to be given him?

Answer. The ready money is never multiplied, then $\frac{1}{4}$ multiplied by 2 months, as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 months, as appeareth here in the operation: But now for the rest of the money you cannot multiply it untill you have sought what proportion it beareth to 800 li. Therefore you must subtract the ready money, the $\frac{1}{4}$ and $\frac{1}{2}$ out of the principall. The rest will bee 66 $\frac{2}{3}$ li. which you must look what part it beareth to the principall, which you shall finde to be $\frac{1}{12}$, the same you must also multiply by his time 12 months, and it yeildeth 1 month, so all make $3\frac{1}{2}$ months, as appeareth in the operation.

$\frac{1}{4}\ \frac{1}{2}$	0 $\frac{1}{2}$
$\frac{1}{4}$	
$\frac{1}{2}\ \frac{4}{1}$	2
$\frac{4}{2}$	
$\frac{1}{12}\ \frac{11}{1}$	1
$\frac{1}{1}\ \frac{2}{2}$	

A Merchant is to pay 1200 li. in three terms, that is to wit, 400 li. at two weeks: and 600 li. at four months: Lastly, 200 li. at five months: The Question is, in what time they ought to be paid at once.

Answer. Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600 you shall finde $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you shall have $\frac{2}{3}$ weeks, more eight weeks: and lastly $3\frac{1}{2}$ weeks, which together maketh 13 weeks, or 3 months, your desire.

A Merchant is to pay 600 pound in three terms, whereof 100 pound is paid present, more 300 li. at 20 daies, and the rest at 5 months, accounting

counting 30 daies to a month. The Queſtion is what time ought theſe payments to be paid at once?

Anſwer. Work, and you ſhall finde 2 months.

The ſeventh Chapter intreateth of Buying and Selling in the Trade of Merchandiſe, wherein is taken part ready money, and divers daies of Payment given for the reſt, and what is won or loſt in the 100 li. forbearance for twelve months more or leſſe, according to the quantity of money, or proportion of time, &c.

A Merchant hath bought Satins which coſt 8 s. the yard ready money: and he ſelleth the ſame again to another man for 10 s. the yard, but he giveth 2 daies for the payment, that is to ſay, 3 months for the one half, and 5 months for the other half. The Queſtion is to know how much the ſeller doth gain upon the 100 li. in 12 months after that rate.

Anſwer. Seek firſt, by the Rules of payment, at what time thoſe two payments ought to be paid at once, and you ſhall finde 4 months, at which time the ſecond Merchant ought

ought to have paid the whole entire payment; And therefore say by the first part of the *Rule of Three* composed: If 8 s. in 4 months doe gain 2 s. what will 100 li. gain in 12 months?

	s.	m.	s.	li.	m.
	8	4	1	10	12
	4			20	
	23			2000	
				2	
1				4000	
16				12	
48000	1 s.			4800	
322	(1500		(75		
3	020				

Multiply and divide, and you shall finde 75 pounds, as appeareth in the Example, and so much doth the first Merchant gain upon the 100 li. in 12 months.

A Merchant hath sold 50 Clothes, at $9\frac{1}{2}$ li. the piece, to be paid the one $\frac{1}{2}$ at 4 months, the $\frac{1}{3}$ at 5 months, and the $\frac{1}{4}$ at 7 months, and the sellers mind is to take no more but after 8 li. in the 100, for 12 months. The Question is now, what the first Merchant gaineth in the sale of these Clothes after that rate.

Answer. First, look what the 40 Clothes come to at that price, and you shall finde 475 pounds. Then secondly, according to your direction in the Rules of payment, seek at what time all the payments are to be performed at once. And you shall finde $4\frac{5}{6}$ months. Then thirdly,

thirdly say, by the first part of the *Rule of Three* composed, If 100 li. in 12 months gain 8 li. what will 475 li. gain in $4\frac{1}{6}$ months? Work, and you shall finde 15 li. and $\frac{11}{36}$ of a pound, which is the neat gains that the first Merchant hath after the rate aforesaid.

A Merchant hath bought Holland at 7 s. 3 d. the Ell ready money, and he selleth the same again, for 8 s. 4 d. the Ell, to be paid $\frac{1}{4}$ part in ready money, more $\frac{1}{3}$ part at 2 months, and the rest at 4 months; The Question is now to know how much the first Merchant doth gain upon the 100 li. in 12 months after that rate?

Answer. According to the direction delivered you in the Rule of payment, the ready money is not to be multiplied. Then working for the other two payments to finde out the true proportion at what time they ought to be paid at once, you shall finde for $\frac{1}{3}$ at 2 months, $\frac{2}{3}$ of a month. And the rest of the money which is $\frac{5}{12}$ multiplied by his term 4 months, yeildeth $1\frac{2}{3}$ months, both which added together make $2\frac{1}{3}$ months, the just time that both the payments ought to be performed at once. And therefore say by the first part of the *Rule of Three* composed, if $74\frac{1}{4}$ in $2\frac{1}{3}$ months doe gain $\frac{11}{48}$ of a pound, what shall 102 li. gain in 12 months after that rate? Work, and you shall finde $78\frac{123}{693}$ li. And so much doth he gain upon 100 l. in 12 months.

A Merchant hath bought 30 Clothes at 6 pounds the piece for ready money: Afterward he selleth 10 of them for 7 li. the piece, for 3 months

months terms: And the other 20 he ſelleth for 8 li. the piece, for 4 months term. The Queſtion is now, what he gaineth upon 100 li. in 12 months?

Anſwer. Firſt finde the value of the 30 Clothes, which amount to 180 li. Secondly, ſeek what the 10 pieces come to at 7 li. and what the 20 pieces come to at 80 li. the one comes to 70, and the other to 60, both which together make 230, which is 50 li. more then they coſt. Thirdly, as I have taught you in the Rule of Payment, proportionate the firſt and ſecond prices unto the proportion they bear unto 230 the Product of their two prices, and you ſhall finde $\frac{7}{23}$ for the firſt, and $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe parts by their times, and you ſhall have $\frac{21}{23}$ and $\frac{64}{23}$, both which together maketh 3 whole months, and $\frac{18}{23}$ of a month, which is the juſt time that both thoſe Payments ought to be paid at once.

Then ſay by the firſt part of the *Rule of Three* compoſed; If 180 li in $3\frac{18}{23}$ months doe gain 50. li. what ſhall 102 gain in 12 months? Multiply and divide, and you ſhall finde $90\frac{10}{11}$ li. and ſo much doth he gain upon 100 li. in 12 months.

A Merchant hath bought Cinnamon which coſt him 9 s the pound ready money. The Queſtion is now, at what price he ought to ſell the 100 weight: to wit, 112 li. to be paid the $\frac{1}{4}$ at 2 months, and the reſidue at the end of 3 months,

months, so that he may gain after the rate of 10 li. upon 100 li. for 12 months.

Answer. Seek first by the Rules of payment at what term both the payments ought to be paid at once, where the $\frac{2}{4}$ multiplied by his term 2 months, maketh $\frac{2}{4}$ months.

Likewise the next payment, which is $\frac{2}{4}$ multiplied by his term 3 months maketh $2\frac{2}{4}$ months, both which added together, maketh $2\frac{4}{4}$ months, which is the time that both the payments ought to be paid at once. Then say by the *Rule of Three*, if 12 months doe give me 10 li. what will $2\frac{2}{4}$ months give? Multiply and divide, and you shall finde $2\frac{2}{24}$ li. Then say again by the *Rule of Three*, If 1 li. cost me 9 s. what will 112 li. cost? Multiply and divide, and you shall finde 51 li. 1 s. Then say once again: If 100 li. doe give $102\frac{2}{24}$, what will $50\frac{8}{3}$ li. give? Multiply and divide, and you shall finde 51 li. 11 s. $1\frac{1}{3}$ d. and for that price ought I to sell 112 pound of Cinnamon to be paid at the two severall payments aforesaid, to gain thereby after the rate of 10 li. upon the 100 li. in 12 months.

Brief

Brief Rules for our hundred weight here at London, *which is after* 112 *pound for the* 160.

ITem, Who that multiplieth the pence that one pound weight is worth by 7, and divideth the Product by 15, ſhall finde how many pounds in money 112 pound weight is worth.

And contrariwiſe, he that multiplieth the pounds that 112 pounds weight is worth by 15, and divideth the Product by 7, ſhall finde how many pence in money the one pounds weight is worth.

Example.

At 10 pence the pound weight, what is 112 pounds weight worth?

Anſwer. Multiply 10 by 7, and thereof cometh 70, the which divide by 15, and you finde 4 $\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li. 13 s. 4 d. after the rate of 10 pence the pound aforeſaid.

At 6 pounds the 112 pounds weight, what is one pound worth?

Anſwer. Multiply 6 by 15, and thereof cometh 90, the which divide by 7, and you ſhall finde 12 $\frac{6}{7}$ d. So much is 1 pound worth when the 112 pounds did coſt 6 pounds.

The

The eighth Chapter intreateth of Tares, and allowances of Merchandiſe ſold by weight, and of Loſſes and Gains therein, &c.

AT 16 pound the 100 Suttle, what ſhall 895 pound Suttle be worth, in giving 4 pound weight upon every 100 for Treat?

Anſwer. Adde 4 unto 100, and you ſhall have 104. Then ſay by the *Rule of Three*, If 104 be worth 16 pounds, what are 895 pounds worth? Multiply and divide, and you ſhall finde 237 li. 13 s. 10 $\frac{8}{13}$ d. and ſo much ſhall the 895 pounds weight be worth.

Item, At 3 s. 4 d. the pound weight, what ſhall 754 $\frac{1}{2}$ pound be worth, in giving 4 pounds weight upon every hundred for Treat?

Anſwer. See firſt by the *Rule of Three* what the 100 pound is worth, ſaying, If one coſt 3 $\frac{1}{3}$ s. what 100? Multiply and divide, and you ſhall finde 16 $\frac{8}{3}$ pounds. Then adde 4 unto 100, and they are 104. Then ſay again by the *Rule of Three*, If 104 be ſold for 16 $\frac{8}{3}$ pounds, for how much ſhall 754 $\frac{1}{2}$ be ſold for? Multiply and divide, and you ſhall finde 120 l. 18 s. 3 $\frac{1}{13}$ d.

And

And for ſo much ſhall the 754 ½ pound be ſold for at 3 ſhillings 4 pence the pound, in giving 4 upon the 100.

Other neceſſary brief Rules there are for the finding of Treats, or caſting up of Cheſts of Sugar, &c. which for that it is a myſterie, I omit: if any lack inſtruction that way, they ſhall finde me ready to pleaſure them.

Item, If 100 pounds be worth 36 s. 8 d. what ſhall 860 pounds bee worth in rebating 4 pounds upon every hundred for Tare & Cloff?

Anſwer. Multiply 860 by 4, and thereof cometh 3440, the which divide by 100, and you ſhall have 34 ⅖ pounds, abate 84 ⅖ from 860 and there will remain 825 ⅕ pounds. Then ſay, by the *Rule of Three*, If 100 li. coſt 36 ⅓ s. what will 825 ⅕ coſt after that rate? Multiply and divide, and you ſhall finde 15 li. 2 s. 8 $\frac{16}{25}$ d. And ſo much ſhall the 860 coſt in rebating 4 pounds upon every hundred, for Tare and Cloff.

Item, Whether doth he loſe more that giveth 4 pounds upon the hundred, or he that rebateth 4 pounds upon the hundred?

Anſwer. Firſt note, that he that giveth 4 pounds on 100, giveth 104 for 100. And he which rebateth 4 pounds upon the 100, giveth the 100 for 96. Therefore ſay by the *Rule of Three*,

Three, if 104 be delivered for 100, for how much shall the 100 be delivered? Multiply and divide, and you shall finde 96 $\frac{4}{13}$, and he which rebateth 4 in the 100, maketh but 96 pounds of 100, so that he loseth 4 pounds in the 100, and the other which giveth 4 pounds upon the 100 loseth but 3 $\frac{12}{13}$ pounds upon the 100 Thus you may see, that he which rebateth 4 pounds in the 100, loseth more by $\frac{1}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100, for Tare and Gloss.

If 100 pounds of any thing cost me 23 shillings 4 pence. The Question is, how I shall sell the pound, to gain after the rate of 10 pounds, upon the 100 pound.

Answer. Say by the *Rule of Three*, if 100 pounds give 100 pounds, what shall 23 $\frac{1}{3}$ s. give? Multiply and divide, and you shall finde 1 $\frac{12}{60}$ pounds. Then say again. if 100 pound be worth 1 $\frac{12}{60}$ pounds, what is one pound worth? Multiply and divide, and you shall finde 3 d. $\frac{3}{33}$. And so much is the pound worth in gaining 10 pounds upon the 100.

Item, A Grocer hath bought C. weight of commodity for 6 li. 10 s. The Question is now to know how many pounds thereof he shall sell for 33 s. 4 d. to gain 20 s. in C. weight.

Answer. Adde 20 s. unto 6 li. 10 s. and they

make 7 li. 10 s. Then say, if 7 ½ pound yeild me 112 pound, what shall 1 ⅔ pounds yeild? Multiply and divide, and you shall finde 24 ½ li. And so many pound ought he to sell to gain 20 s. in his C. weight.

If one pound weight cost 3 s. 4 d. and I sell the same again for 4 s. what is gained in 100 li. of money laid out in that commodity?

Answer. You may say, If 3 ⅓ s. give 4, what will 100 li. gain? But then when you have found, you must subtract 100 li. out of the Product, the rest is your neat gain: or else to produce the neat gain in your work at the first, subtract the just price out of the overprice, as I taught before in the first beginning of Losse and Gain, and your conclusion shall be all one. Multiply and divide by which of the two waies you think good, and you shall finde that he gaineth 20 li. in the 100 li.

Item, If the pound weight which cost 4 s. be sold again for 3 s. 4 d. I demand what is lost in the 100 pounds of money.

Answer. Say, If 4 s. lose ⅓ s. what shall 100 lose? Multiply and divide, and you shall finde 16 li. 13 s. 4 d. and so much is lost upon the 100 of money.

Item, If C. weight of any commodity cost 45 pounds, and the buyer repenting, would lose 5 pounds

pounds in the 100 of money. I demand how the pounds may be sold, his losse to be neither more nor lesse then after the rate aforesaid of 5 by the hundred?

Answer. By the *Rule of Three*, if 100 lose 5, what shall 45 lose? Work, and you shall finde 2 ¼ pound, which rebated from the principall 45, resteth 42 li. 15 s. Lastly say, if 110 yeildeth but 42 li. 15 s. what one pound? Multiply and divide, and you shall finde 7 s. 7 d. $\frac{11}{22}$. And so much is the pound worth after that losse.

A Grocer hath bought three pieces of Raisins weighing 175 ½ pounds, 182 ¼ pounds, 191 pounds: tare for each fraile 2 ¾ pounds, at 25 ½ s. the C. weight. The Question is, what they amount to in money.

I answer, 6 li.——3 s.——4 $\frac{11}{22}$ d.

A Grocer hath bought 3 sacks of Almonds weighing 267 ½ pound, tare 2 pound, 257 ½ pounds, tare 2 ½ pound, 252 pound, tare three pound, at 2 s. 10 ½ d. the pound, what amount they to in money?

I answer, 100 li——12 s.——3 $\frac{3}{11}$ d.

The ninth Chapter intreateth of lengths and breadths of Arras and other Cloths, with other Queſtions incident unto length and breadth.

IF a piece of Arras be 7 Ells and $\frac{1}{4}$ long, and five Ells and $\frac{2}{3}$ broad, how many Ells ſquare doth the ſame piece contain?

Anſwer. Multiply the length by breadth, that is to ſay, $7\frac{1}{4}$ by $5\frac{2}{3}$, and thereof will come $43\frac{11}{12}$ Ells: ſo many Ells ſquare doth the ſame piece contain.

Item, more, a piece of Arras doth contain 22 Ells ſquare, and if the ſame were in length $3\frac{1}{2}$ Ells, I demand how many Ells in breadth the ſame piece doth contain.

Anſwer. Divide 22 Ells by $3\frac{2}{4}$ and thereof cometh $6\frac{12}{13}$: So many Ells doth the ſame contain in breadth.

Item, more, a Merchant hath $3\frac{1}{4}$ Ells of Arras, at $1\frac{2}{3}$ Ells broad, which he will change with another man for a piece of Arras, that is $\frac{7}{8}$ Ells ſquare. The Queſtion is, how many Ells of that ſquareneſſe ought the firſt Merchant to have?

Anſwer.

Anſwer. Multiply the firſt Merchants piece his length by the breadth, and you ſhall finde it containeth 5 $\frac{1}{12}$ ells, which $\frac{1}{12}$ ells you ſhall divide by $\frac{1}{8}$ and you ſhall finde 6 $\frac{4}{21}$ ells, and ſo many ells of that ſquareneſſe ought the latter Merchant to give the firſt.

Item, A Student hath bought 3 $\frac{1}{2}$ yards of broad cloth, at 7 quarters broad, to make a gown, & ſhould line the ſame throughout with Lamb at a foot ſquare each skin: The Queſtion is now, how many skins he ought to have.

Anſwer. Seek firſt the number of yards ſquare that his cloth containeth, which to doe, multiply 3 $\frac{1}{2}$ his length, by 1 $\frac{1}{4}$ his breadth, and you ſhall finde 6 $\frac{1}{8}$ yards ſquare: then ſay by the *Rule of Three*, If one yard ſquare give 9 foot, what ſhall 6 $\frac{1}{8}$? Work, and you ſhall finde 59 $\frac{1}{8}$ skins.

Item, more, a Lawyer hath a rich piece of ſeeling come home which is 24 foot and 3 inches long, and 7 foot and 2 $\frac{1}{2}$ inches high: the Joyner is to be paid by the yard ſquare: The Queſtion is, how many yards this containeth?

Anſwer. Multiply his length by his breadth, that is to wit, 24 $\frac{1}{4}$ foot by 7 $\frac{1}{24}$ foot, and you ſhall finde 174 $\frac{11}{96}$ foot ſquare, which 174 you ſhall divide by 9 (for ſo many foot make a yard ſquare) and you ſhall finde 19 yards 3 foot

and ⅔ of a foot, and so many yards doth this piece hold.

Item, I bought a piece of Holland cloth containing 36 Ells ¾ Flemmish. The Question is, how many ells English it makes.

Answer. You must note, that 5 ells Flemmish doth make but 3 ells English.

Therefore say by the *Rule of Three*, if five ells Flemmish make but three ells of English, how many ells English will 36 ¾ ells Flemmish make? Multiply and divide, and you shall finde 21 ¾, and so many English doth 36 ¾ ells Flemmish contain. The like is to be done of others.

Item, more, I have bought 342 ells Flemmish, of Arras work, at two ells broad Flemmish, and I would line the same with ell broad Canvas of English measure. The Question is, how many ells English will serve my turn?

Answer. For as much as three ells English, are worth five ells Flemmish, therefore put three ells English into his square, in multiplying three by himself, which maketh nine. Likewise multiply the English ell, which is five quarters, every way into himself squarely, and you shall finde 25. Then multiply 342 which is the length of the piece by 2, which is the breadth, and thereof cometh 684, then

say

say by the *Rule of Three*, as before: if 25 ells square of Flemmish measure, be worth 9 ells square of English measure, what are 684 of Flemmish measure? Multiply and divide, and you shall finde 246 $\frac{6}{25}$ ells English.

The same is also wrought by the Backer *Rule of Three*, in seeking the squares contained in the Flemmish ell of two Ells broad (which are 18) and also in seeking the squares contained in the English ell (which are 25) then say by the *Rule of Three*, backward, If 18 quarters require 342 ells, what shall 25 quarters give? Multiply and divide by the *Rule of Three* Reverse, and you shall finde as before 246 $\frac{6}{25}$ ells English?

Item, more, at three shillings four pence the Flemmish ell, what is the English Ell worth after the rate?

Answer. Say, if three quarters give 3 $\frac{1}{3}$ s. what giveth five quarters? Multiply and divide, and you shall finde 5 s. 6 $\frac{2}{3}$ d.

Item, more, at 8 shillings 4 pence the Flemmish ell square, what is the English ell worth after that rate?

Answer. According to the reason of the last Question, consider that a Flemmish ell square is equall to 9 quarters of a yard English, and

an English ell square is equall to 25 quarters of a yard. Therefore say by the *Rule of Three*, if 9 quarters give 1 $\frac{1}{3}$ s. what 25 quarters? Work and finde 23 s. 1 $\frac{1}{9}$ d. And so is the English ell worth.

Item, more, at 6 s. 8 d. the ell square: what shall a piece of Cloth cost that is 7 $\frac{1}{2}$ ells long, and 3 $\frac{1}{4}$ ells broad?

Answer. Multiply the breadth by the length, and you shall finde 24 $\frac{3}{8}$ ells square cost 6 $\frac{1}{2}$, what 24 $\frac{3}{8}$ ells? Multiply and divide, and you shall finde 8 li. 2 s. 6 d. and so much the same piece of cloth cost.

Item, more, a Mercer sold three pieces of Silk To wit 24 $\frac{1}{4}$ 13 $\frac{2}{3}$ and 25 yards, at 9 $\frac{1}{2}$ s the yard, and was glad to receive in part of payment again, a cloth containing 34 $\frac{1}{2}$ yards at 7 $\frac{1}{2}$ s. the yard. The Question is now, what the Debtor is in the Creditors debt? Work, and you shall finde he oweth the Mercer 22 li, 3 s. 2 $\frac{1}{4}$ d.

The

The tenth Chapter intreateth of reducing of Pawns of Geans into Englisſh yards.

NOte that 100 Pawnes doe make 26 yards, whereupon three Pawnes $\frac{11}{13}$, doe make one yard, and one Pawn after the rate and proportion is $\frac{11}{30}$ of a yard.

In 4563 Pawns of Geans, how many yards Englisſh?

Anſwer. Say by the *Rule of Three*, if a hundred Pawns doe make 26 yards, what will 4563 Pawns make? Multiply and divide, and you ſhall finde 1186 yards $\frac{12}{30}$: So many yards doe 4563 Pawns make.

Otherwiſe take ſome other number at your pleaſure, as ten pawns, which is the $\frac{1}{10}$ part of 100, then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{6}{10}$, and then ſay alſo by the *Rule of Three*, if ten Pawns give $2\frac{6}{10}$ yards, what will 4563 Pawns give? Work, and you ſhall finde 1186 $\frac{12}{30}$ yards, as before.

More, at 2 s. 6 d. the Pawns of Geans, what will

will the English yard be worth after the rate?

Answer. Say by the *Rule of Three*, if $\frac{11}{10}$ of a yard cost $\frac{1}{2}$ s. what one yard? Multiply and divide and you shall finde 9 s. $7\frac{1}{13}$ d.

More, if $346\frac{1}{3}$ Pawns cost 30 li. 13 s. 4 d. sterling, what is that the English yard after the rate?

Answer. Say by the *Rule of Three*, if $346\frac{1}{3}$ Pawns cost $30\frac{2}{3}$ pounds, what are $3\frac{11}{12}$ Pawns worth? (for so many Pawns make a yard) Multiply and divide, and you shall finde $\frac{2332}{27839}$ parts of a pound, which in known numbers is worth 6 s. 9 d. $\frac{4228}{9059}$.

The eleventh Chapter intreateth of Rules of Loan and Interest, with certain necessary Questions and Proofs incident thereunto, &c.

Item, I lent my friend 326 pounds for five $\frac{1}{2}$ months simply without any interest, upon condition, to have the like courtesie again when I need. But when I came to borrow, he could spare mee but 149 pound 8 shillings 4 pence. The Question is now, how

how long time I ought to have the use thereof, to countervail my friendship before time shewed him?

Answer. Say by the backer *Rule of Three*, if 326 pounds give 5 $\frac{1}{2}$ months, what time will 149 $\frac{4}{12}$ pounds give? Multiply and divide, and you shall finde 12 months, and so long time ought I to use his money.

The Proof.

Item, lent my friend 149 li. 8 s. 4 d. for 12 months: The Question is now, how much money he ought to lend me again for 5 $\frac{1}{2}$ months to recommend my friendship shewed him?

Answer. Say by the Backer or Reverse *Rule of Three*, If 12 months give 149 $\frac{1}{12}$ what shall 5 $\frac{1}{2}$ months give? Work, and you shall finde 326 pounds, and so much ought he to lend me to requite my gentlenesse or good turn.

Two other Branches, yet more, for proof out of the same Question.

ITem, lent my friend 149 li. 8 s. 4 d. for 12 months, to have the like friendship again when I need. And coming to borrow of him, he very courteously took me 326 li. (for that he could well then spare the same) The Question is now, how long I ought to occupy it, not usurping friendship, but in his due time to restore it again.

Answer.

Answer. Say by the *Rule of Three* reverse, If 149 $\frac{1}{12}$ pounds give twelve months, what shall 326 pounds give? Multiply and divide, and you shall finde, that at 5 $\frac{1}{2}$ months term, I ought to restore it again.

Proof.

Item, Lent my friend 326 pounds for five $\frac{1}{2}$ months. The Question is now, how many pounds he ought to lend me for twelve months to recompense this pleasure again.

Scholar. Work by the *Rule of Three* reverse, as you have done before, and you shall finde 149 li.——8 s.——4 d.

Again four other selected Questions, of Loan and Interest, all out of one branch, and each one also a necessary Question, and a particular proof to other.

Item, Lent my friend 430 pounds at Interest for three months; to receive after the rate of 8 pounds in the 100 pounds for twelve months. The Question is what the interest cometh to?

You may, if you please, work it at two workings by the *Rule of Three* direct, in saying, if 12 months

months give 8 pounds, what giveth three months? Multiply and divide, and it giveth 2 pound.

Then for the ſecond work ſay: If a hundred pound yeild 2 pounds, what yeildeth 430 li? Multiply and divide, and you ſhall finde 8 li. 12 s. and ſo much comes the loan of 430 li. to for 3 months after the rate of 8 pound in the hundred pounds of 12 months.

Otherwiſe wrought thus by the *Rule of Three* at twice alſo.

If 100 pounds give 8 pounds, what giveth 430 pounds? Multiply and divide, and you ſhall finde 34 pounds $\frac{4}{5}$. Then again for the ſecond work ſay: If 12 months give 34 pounds $\frac{4}{5}$, what giveth three months? Work and finde 8 li. 12 s. as before.

Otherwiſe yet at one working: By the firſt part of the Rule of five numbers forward, in ſaying if 100 pounds in twelve months, gain 8 pounds, what ſhall 430 pounds gain in three months? Multiply the firſt by the ſecond for your Diviſor, and the other three, the one into the other for the dividend, and you ſhall finde 8 pounds 12 ſhillings, as aforeſaid.

The Proof.

Item, A friend of mine received of me 8 pound 12 ſhillings for the Intereſt and Uſe of 430 pounds for three months term: The Queſtion is

now

now, what he took in the 100 pound for 12 months after that rate?

Answer. For most brief, say by the first part or Rule of five Numbers forward: If 430 pounds in three months did pay 8 li. 12 s. what doth 100 pounds in 12 months take after the rate? Work and you shall finde 8 pounds, and so much he took upon the 100 pounds for 12 months.

A third Question and proof also by the Backer Rule of five Numbers.

ITem, I lent my friend 430 pounds to receive for the Interest thereof, after the rate of 8 pounds in the 100 for 12 months. The Question is now how long time my friend ought to give the use thereof, that it may be returned with 8 li. 12 s. gains.

You may work it, if you please, by the Rule of Three direct at twice, in saying: If 100 li. yeild 8 pounds, what yeildeth 430 pound? Multiply and divide, and finde 34 pound and $\frac{4}{10}$.

Then again for the second work say, if 34 $\frac{4}{10}$ pounds, give 12 months, what giveth 8 $\frac{1}{2}$ li? Multiply and divide, and you shall finde three months, and so long time ought my friend to use it to return with 8 li. 12 s. gain.

Otherwise at one working by the Backer Rule

Rule of 5 numbers, in ſaying, if 100 pounds in 12 months doe gain 8 pounds, how much time ſhall 430 pounds be a gaining of 8 pounds 12 ſhillings? Multiply the firſt and the ſecond into the laſt for your Dividend, and the third and fourth multiply together for your Diviſor, and then divide, and you ſhall finde 3 months, the juſt time that my friend ought to uſe it to return it with 8 li. 12 s. gain.

A fourth derived Queſtion out of this Branch, which is a proof of this laſt, and alſo of the other two going before.

Item, How much money ought a Merchant to deliver after 8 pounds in the 100 for 12 months, that in 3 months he may gain 8 pound 12 ſhillings?

Anſwer. You may alſo if you pleaſe, work it by the Golden Rule of Three at twice: firſt ſaying, If three months give $8\frac{3}{5}$ pound, what twelve months gain? You ſhall finde $34\frac{2}{5}$: Then ſay again, If 8 pounds be come of 100 pounds, what ſhall come of 34 li. 8 s? Work, and you ſhall finde the Anſwer to the Queſtion, which is 430 pounds, and ſo much ought the Merchant to deliver.

But moſt briefly it is anſwered by the Backer Rule of 5 numbers, where I argue thus, ſaying:

If

If 100 li. be twelve months a gaining of 8 li. then but for three months term only to take 8 pound 12 shillings must needs be a good round summe to work it, set your numbers thus: 100——12——8——s——8½ multiplying the first into the second, and also by 43 the Product of the fifth, for your Dividend, and the third and fourth together with 5, the Denominator of your Fraction for your Divisor: then divide, and you shall finde as before 430 pounds: the true solution to your Question.

The twelfth Chapter intreateth of the making of Factors, which is taken in two sorts.

THe first is, when the estimation of the Factor is taken upon the sending of the Merchant, as if the estimation of his person be ¼ it is understood that he shall have ¼ of the gain, the Merchant the other ¼.

The other sort is, when the estimation of his making is out of the sending of the Merchant, as if the order and agreement between them were such, that the Merchant shall put in 800 li. and the Factor for his making shall have ¼: nevertheless he shall have but ½ of the gain

of

or profit, for the $\frac{1}{4}$ of 800 is 200 (for the eſtimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 pound, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, under ſuch condition, that the ſaid Factor ſhall have the $\frac{1}{4}$. And after certain time they find in profit 124 lib. 6 s. 8 d. I demand how much the Merchant ſhall have hereof, and how much ought the Factor to have?

Anſwer. *When the eſtimation of the Factor is out of the ſending of the Merchant, it maketh*

li.	s.	d.		
99	9	4	for the	Merchant.
24	17	4		Factor.

But if that his eſtimation be at the ſending of the Merchant, then it maketh

li.	s.	d.		
93	5	0	for the	Merchant.
31	1	3		Factor.

For the Merchant is then to have $\frac{1}{4}$, *and the Factor* $\frac{1}{3}$.

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 lib.

to have the $\frac{1}{3}$ part of the profit: I demand now for how much his perſon is eſteemed, when the ſame is counted upon the ſending of the Merchant.

Anſwer. *According to the Tenour and order before preſcribed in the firſt Rule, that is if his eſtimate be $\frac{1}{4}$ he ſhall have the $\frac{1}{4}$ of the gain. Therefore ſay by the* Rule of Three direct: *If $\frac{1}{4}$ taken put in 400 pound, what is the eſtimate, or putting in of $\frac{1}{3}$ taking? Multiply and divide, and you ſhall finde 320 pounds, and ſo much is the perſon of the Factor eſtimated.*

Otherwiſe.

To finde the eſtimation of the perſon of the Factor, you ſhall conſider, that ſeeing it was agreed between them, that the Factor ſhould take the $\frac{1}{3}$, then the Merchant ſhall have the reſidue, which are $\frac{4}{3}$: wherefore the gain of the Merchant unto that of the Factor is in ſuch proportion as 5 unto 4. Then if you will know the eſtimation of the perſon of the Factor, ſay, If 5 give 4, what will 400 give? Multiply and divide, and you ſhall finde 320 pound. And ſo much is the perſon of the Factor eſteemed to be worth.

Other conditions then theſe aforeſaid, may alſo be between Merchants and Factors, without reſpect either of ſending, or not ſending of the Merchant, where moſt commonly the eſti-

mation of the body of the Factor is in ſuch proportion of the ſtock which the Merchant layeth in, as the gain of the ſaid Factor is unto the gain of the Merchant. As thus, if a Merchant do deliver into the hand of his Factor 400 pound, and he to have half the profit, the perſon of the ſaid Factor ſhall be eſteemed to be worth 400 pound: and if the Factor do take but $\frac{1}{3}$ of the gain, he ſhould have but $\frac{1}{2}$ ſo much of the gain as the Merchant taketh, which muſt have $\frac{2}{3}$, wherefore the perſon of the Factor is eſteemed but the $\frac{1}{2}$ of that which the Merchant layeth in, that is to ſay, two hundred pound.

And if the Factor did take the $\frac{2}{5}$ of the gain, then the Merchant ſhall take the reſidue which are $\frac{3}{5}$, wherefore the gain of the Merchant unto the Factor is then in ſuch proportion as 3 unto 2: whereupon if you will then know the eſtimation of the perſon of the Factor, ſay, If 3 give 2, what ſhall 400 give? Work, and you ſhall finde 266 $\frac{2}{3}$ pounds. And ſo much is the perſon of the Factor eſteemed to be worth.

And if the Merchant ſhould deliver unto his Factor 400 pound, and the Factor would lay in 80, and his perſon, to the end he might have the $\frac{1}{4}$ of the gain, I demand how much ſhall his perſon be eſteemed?

Anſwer. *Abate* 80 *from* 400, *and there will remain* 320. *And at ſo much ſhall his Perſon be eſteemed.*

A Merchant hath delivered unto his Factor 900 pounds to govern in the Trade of Merchandize, upon condition that he shall have the $\frac{1}{3}$ of the gain, if any thing be gained, and also to bear the $\frac{1}{3}$ of the losse, if any thing be lost. Now I demand how much his person was esteemed at?

Answer. *Seeing that the Factor taketh the* $\frac{1}{3}$ *of the gain, his Person ought to be esteemed as much as* $\frac{1}{2}$ *of the stock which the Merchant layeth in: that is to say, the* $\frac{1}{2}$ *of* 900 *Pound, which is* 450. *The reason is, because* $\frac{1}{3}$ *of the gain that the Factor taketh is the* $\frac{1}{2}$ *of the* $\frac{2}{3}$ *of the gain that the Merchant taketh, and so the Factor his Person is esteemed to be worth* 450 *Pounds.*

A Merchant hath delivered unto his Factor 600 li. and the factor layeth in 250 li. and his person. Now because he layeth in 250 li. and his person, it is agreed between them, that he shall take the $\frac{2}{3}$ of the grain. I demand for how much his person was esteemed?

Answer. *For as much as the Factor taketh* $\frac{1}{3}$ *of the gain, he taketh* $\frac{2}{3}$ *of that which the Merchant taketh, for* $\frac{2}{3}$ *are the* $\frac{2}{3}$ *of* $\frac{2}{1}$. *And therefore the Factors laying in ought to be* 400 li. *which is* $\frac{2}{3}$ *of* 600 *pound that the Merchant laid in. Then subtract* 250, *which the Factor did lay in, from* 400 *pound which should have been his whole stock, and there remaineth* 150 *pound for the estimation of his person.*

More, a Merchant hath delivered unto his factor 800 pound, upon condition that the factor

Factor ſhall have the gain of 160 li. as though he laid in ſo much ready money: I demand what portion of the gain the ſaid Factor ſhall take?

Anſwer. *See what part the* 160, (*which the Factor laid in*) *is of* 960, *which is the whole ſtock of their company, and you ſhall finde* $\frac{1}{6}$: *And ſuch part of the gain ſhall the Factor take.*

But in caſe, that in making their Covenants, it were ſo agreed between them, that the Factor ſhould have the gain of 160 *Pound of the whole ſtock which the Merchant layeth in, that is to ſay, of the* 800 *Pound: then ſhould the Factor take* $\frac{1}{5}$ *of the gains, for* 160 *is* $\frac{1}{5}$ *of* 800 *Pound.*

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandize, which is diſtinct into ſeven Rules, with divers other neceſſary queſtions incident thereunto.

The firſt Rule.

TWo Merchants, willing to change their Merchandize the one with the other: The one hath 24 broad clothes at 10 li. 10s. the piece: The other hath Mace at 12 ſhillings the pound. The queſtion is, how many pounds of Mace he ought

to give for his Cloth, to ſave himſelf harmleſs, and be no loſer?

Anſwer. *Seek firſt by the* Rule of Three *what the 24 Clothes coſt at 10 pound 10 ſhillings the piece, and you ſhall finde 252 pound: Then to finde the quantity of Mace, ſay again by the* Rule of Three, *If 12 ſhillings buy one pound, what ſhall 252 pound buy me? Work, and you ſhall finde 420 pound of Mace: And ſo many pound ought he to give for his Clothes.*

The Proof.

Two barter. The one hath 420 pounds of Mace at 12 s. the pound, to barter or change broad Clothes, at 10 pounds 10 ſhillings the piece. The queſtion is, how many broad Clothes he ought to give for all his Mace?

Anſwer. *Firſt ſay, If one coſt 12 ſhillings, what 420? you ſhall finde 5040 s. Then ſay again, If 10½ pounds give one Cloth, what ſhall 5040 ſhillings give? Work, and you ſhall finde 24 Clothes, your deſire.*

The ſecond Rule.

Two change Merchandize for Merchandize: The one hath Pepper at two ſhillings four pence the pound, to ſell for ready mony. But in barter he will have no leſs then three ſhillings the pound. And the other hath Hol-

land

land at five ſhillings ſix pence the Ell ready money. The queſtion is now, at what price he ought to deliver the Ell in the barter to ſave himſelf harmleſs.

Anſwer. *Say by the* Rule of Three direct: *If* 2 $\frac{1}{3}$ *ready money give* 3 s. *in barter, what ſhall* 5 $\frac{1}{2}$ *give in barter? You ſhall finde* 7 $\frac{1}{14}$ s. *and at that price ought the ſecond Merchant to ſell his Holland in barter.*

The Proof.

Two barter. The one hath Holland at 5 s. 6 d. the Ell to ſell for ready money. And in barter he will have 7 $\frac{1}{14}$ s. The other hath Pepper at 2 s. 4 d. the pound, to ſell for ready money. The queſtion is now, how he ought to ſell in barter?

Anſwer. *Say by the* Rule of Three direct, *If* 5 $\frac{1}{2}$ *ready money give* 7 $\frac{1}{14}$ s. *in barter; what ought* 2 $\frac{1}{3}$ *to take in barter? Multiply and divide, and you ſhall finde* 3 *ſhillings, your deſire.*

The third Rule.

Two barter. The one hath cloth of Arras at 30 s. the Ell ready money, but in barter he will have 35 $\frac{1}{2}$ s. And the other hath white Wines which he delivered in barter for 16 pounds the Tun. The queſtion is now, what his Wines coſt the Tun in ready money.

Anſwer. *Say by the* Rule of Three direct, *If* 35 ½ s. *in barter give but* 30 s. *ready money, what did* 16 *pound in barter coſt? Work, and you ſhall finde* 13 li. 10 s. $\frac{12}{71}$. *And ſo much coſt his Wines for a Tun ready money.*

The proof.

Two barter Merchandize for Merchandize: The one hath white Wines at 13 li. 10 s. $\frac{12}{71}$ s. the Tun to ſell for ready money: But in barter he delivered it for 16 pounds. The other, to make his match good and ſave himſelf harmleſs, delivereth Arras at 35 ½ s. the Ell. The queſtion is now, what an Ell of his Arras coſt in ready money?

Anſwer. *Say by the* Rule of Three directed: *If* 16 *pounds in barter, give but* 13 li. 10 $\frac{12}{71}$ s. *in ready mony, what ſhall* 35 ½ s. *yield in barter? Work, and you ſhall finde* 30 s. *your deſire.*

The fourth Rule.

Two barter: The one hath Kerſeyes at 14 *pounds the Piece ready money: But in barter he will have* 18 *pounds: and yet he will have the part of his over-price in ready money. And the other hath Ginger at eight groats the pound to ſell for ready money. The queſtion is, how he ought to deliver the Ginger by the pound in barter to ſave himſelf harmleſs, and make the barter equall.*

Anſwer.

Anſwer. *Item*, for the working of this queſtion, and ſuch other the like, you muſt underſtand, if the party over-ſelling his wares, require to have alſo ſome portion in ready money, as $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{4}$ &c. Then ſhall you firſt rebate the ſame demanded part, whatſoever it be, from the over-price, and alſo from the juſt price. And thoſe two numbers that ſhall remain after the ſubtraction is made, ſhall be the two firſt numbers in the *Rule of Three*. And the juſt price of the ſame Merchandize ſhall be the third number, which by the operation of the *Rule of Three direct*, ſhall yield you a true ſolution, how, and at what price you ſhall overſell that your Merchandize, to ſave your ſelf harmleſs, and make the barter equall.

Example.

Take the $\frac{1}{2}$ (of eighteen) which is the over-price of his Cloth, which $\frac{1}{2}$ of eighteen is ſix, which you muſt ſubtract from 18, there reſt 12. And alſo abate it from 14, which is the juſt price of the Cloth, and there remaineth 8, which 8 and 12 are the two firſt numbers in the *Rule of Three*. Then take eight groats, or 2 $\frac{2}{3}$ ſhillings for the third number. Then ſay by the *Rule of Three direct*. If eight pounds give 12 pounds, what ſhall 2 $\frac{2}{3}$ s. give?

14	18
6	6
8	12

Mul-

Multiply and divide, and you ſhall finde 4 s. And for ſo much ſhall the ſecond Merchant ſell his Ginger, or his commodity in barter, to ballance the ſame equall.

The Proof.

Two barter; the one hath fine Kerſeys, at 14 *Pounds the piece ready money: But in barter he will have* 18 *Pounds: and yet he will have the* $\frac{1}{3}$ *part of his over-price in ready mony. And the other hath Ginger, which he having cunning enough to make the barter equall, delivered in barter for* 4 s. *the* li. *The queſtion is now, what his Ginger coſt him ready money?*

Anſwer. After you have made the ſubtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14 (as before was taught you:) then will there remain 8 and 12 for your two firſt numbers in the *Rule of Three.* Then ſay, If 12 give 8, what ſhall come of 4 the over-price of the pound of Ginger? Multiply and divide, and you ſhall find 2 s. 8 d. your deſire.

Two Merchants barter Merchandize, for Merchandize. The one hath Devonſhire *whites at* 7 li. 13 s. 4 d. *the piece ready money: but in barter he doth them away for* 8 li. 3 s. 4 d. *and yet he will have the* $\frac{1}{3}$ *part of his price in ready money. And the other hath Cottens at three Pounds the piece ready money. The queſtion is now, at what price he ought to ſell or exchange his Cottens in barter*

barter to ſave himſelf harmleſs, and make the barter equall?

li.	ſ.	d.	li.	ſ.	d.
7	13	4	8	3	4
2	14	$5\frac{1}{3}$	2	14	$4\frac{1}{3}$
4	88	$10\frac{1}{3}$	5	8	$10\frac{2}{3}$

Anſwer. Firſt ſeek the $\frac{1}{3}$ part of 8 li. 3s. 4d. which is 2 li. 14 s. $5\frac{1}{3}$ d. which rebated from 8 li. 3s. 4d. there reſteth as appeareth by the Example aboveſaid, 5 li. 8s. $10\frac{2}{3}$ d. which is $\frac{2}{3}$ parts of 8 li. 3s. 4d. alſo rebated from 7 lib. 13s. 4d. there reſteth 4 li. 18s. $10\frac{2}{3}$ d. the two firſt numbers in the *Rule of Three*, and the three pounds, which is the neat price of the piece of Cotten, is the third number: Then ſay by the *Rule of Three direct*, as was taught before. If 4 li. 18 s. $10\frac{1}{3}$ d. give 5 li. 8 s. $10\frac{2}{3}$ d. what ſhall three pounds give? Multiply and divide, and you ſhall find three pounds 6s. $\frac{21}{49}$ d. The juſt price that he ought to deliver his Cottens in barter.

The fifth Rule.

Two Merchants will change Merchandize for Merchandize. The one hath Kerſeys at 43 s. *the piece to ſell them for ready mony. And in barter he will ſell them for* 56 s. 8 d. *and he will gain after ten Pounds upon the* 100 *Pounds. And*

yet

yet he will have the $\frac{1}{3}$ *of his over-price in ready money. The other hath Flax at* 3 d. *the Pound ready money. The queſtion is now, how he ſhall ſell the Pound of his Flax in barter?*

Anſwer. See firſt at 10 pound upon the 100 pounds what the $56\frac{2}{3}$s. cometh to, in ſaying, (by the *Rule of Three direct*) If 100 pounds give 110 pounds, what $56\frac{2}{3}$ s? Multiply and divide, and you ſhall finde 3 pound 2 ſhillings 4 pence, of which the $\frac{1}{3}$ that he demandeth in ready money, is 1 pound 11 ſhillings 2 pence; the ſame 31 s, 2 d. abated from 40 ſhillings, and alſo from 56 s. 8 d. there will remain 8 s. 10 d, and 25 s. 6 d. for the two firſt numbers in the *Rule of Three*, and 3 pence the price of the pound of Flax for the third number. Then multiply and divide, and you ſhall finde $8\frac{1}{4}$ d. And for ſo much ſhall he ſell the pound of Flax in barter.

The ſixth Rule.

Two are willing to exchange Merchandize: The one hath Norwich *Grograns at* 25 s. *the Piece ready money: and in barter he wil have* 30 s. *and he will have the* $\frac{1}{4}$ *part of his over-price in ready money. The other hath* Norwich *Stockings at* 40 s. *the dozen to ſell for ready money. But in as much as the firſt Merchants Grogranes are no better, he would deliver them ſo to ballance the barter, that he may gain* 10 *Pounds in the* 100 *Pounds. The queſtion is now, how he ſhall ſell his*

his hoſe the dozen in barter, according to his requeſt.

Anſwer. Say, if 100 give 110 li. what ſhall 40 s. give, which is the juſt price of the dozen of ſtockings? Multiply and divide, and you ſhall finde 44 s. Then take the $\frac{1}{4}$ of 30 s. which is 7 s. 6 d. and ſubtract it from 25 s. and alſo from 30 s. and there will remain 17 s. 6 d. and 22 s. 6 d. for the two firſt numbers in the *Rule of Three*, and 44 ſhillings, which is the juſt price (with his gain in the dozen of ſtockins) for the third number. Then multiply and divide, and you ſhall finde 56 s. $6\frac{4}{7}$ d. and for ſo much he is to ſell his dozen of ſtockins in barter.

The ſeventh Rule.

Two Merchants will change their Merchandize one with the other: The one hath 720 Ells of Cambrick at 5 s. the Ell to ſell for ready money, but in barter he requireth 6 s. 8 d. And yet notwithſtanding, he loſeth by it after 10 pounds upon the hundred pounds, wherenpon he requireth one half of his over-price in ready money: and the other Merchant having skill enough to make the barter equall, delivered Engliſh *Saffrons at 30 s. the pound. The queſtion is now, what his Saffrons coſt the pound in ready money.*

Anſwer. You muſt firſt ſeek what is loſt upon the 100 pound, which to do, you may ſay, (if you pleaſe) If 100 pound loſe 10, what ſhall

ſhall 6 3/5 loſe? Work, and you ſhall find 3/5 (or 8 d.) which muſt be rebated from 6 s. 8 d. ſo reſteth 6 s. ſtill. Or you may ſay, If 100 pound give me but 90 pounds, what ſhall 6 s. 8d. give? Work this way either, and you ſhall find alſo as before directly in your quotient 6 s. your deſire. Then are you next to caſt up what the 720 Ells of Cambrick cometh to at 6s. 8d. the Ell, and you ſhall finde 240 pounds: the 1/2 whereof the Cambrick Merchant will have in ready money (which is 120 pounds:) Nexly, you muſt caſt what the Cambrick cometh to after his loſs in the 100 pound, which as you found, is but 6 s. an Ell, and you ſhall finde 216 pounds: Now muſt you ſubtract his ready money (which is 120 pounds in all) out of 240 pound, and alſo out of 216 pound, and there will remain 120 pounds, and 96 pounds for your two firſt numbers in the *Rule of Three*, and 30 ſhillings is the over-price of your Saffron for the third number: Then multiply and divide, and you ſhall finde 24 ſhillings. And ſo much did his Saffron coſt in ready money.

Two Merchants barter; the one hath fifty Cloths to put away for ready money at 11 *pounds the Cloth, and in barter putteth them away for* 13 *pounds, taking Holland Cloth at* 20 d. *the Flemmiſh Ell, which was worth no more but* 18 d. *The queſtion is now, what Holland payeth for the Cloth, and what he winneth or loſeth by the bargain.*

Anſwer.

Anſwer. Fifty Clothes at 11 pounds the Cloth cometh to 550 pounds, and put away at 12 pounds the piece, maketh 600 pound. Then to finde what Holland payeth for the Cloth, ſay by the *Rule of Three direct*, If 20 d. buy one Ell, what 600 pounds? Work, and you ſhall finde 7200 Ells. Now to finde the eſtate of his gain or loſſe, you muſt ſeek what his 7200 Ells cometh to at 18 d. the Ell: Work by the Rule of proportion direct, and you ſhall finde 540 pounds, which is not ſo much as his Clothes were worth in read money by tenne pounds: and ſo much loſt the firſt Merchant by his Exchange.

A Venetian *hath in* London 100 *pieces of ſilk, to put away for ready money at* 3 li. *the piece. But in barter he delivered them for* 4 li. *the piece, taking wools of a Felmonger at* 7 li. 10 s. *the C. weight, which was worth no more but ſix pounds, the C. ready money. The queſtion is now, what wools payeth for the ſilkes, and which of them winneth or loſeth by the barter?*

Anſwer. A hundred pieces of Silk at 3 li. is in all 300 lib. and at 4 lib. is 400 lib. Then to find what Wools payeth for the Silk, ſay by the *Rule of Three direct:* If 7 $\frac{1}{2}$ buy me 100 weight, what 400 pound? Work, and find 53 $\frac{1}{3}$ C. weight of wooll. Now to finde the eſtate of their gain and loſs, caſt up his wooll at 6 lib. the C. (for ſo much they were worth ready money) and you ſhall finde 320 pound, which is 20 pound more then the Silkes were

to

to be ſold for ready money, whereby the *Venetian* gained 20 pounds by the Barter.

A Merchant hath 53 ⅓ weight of Wooll at 6 pounds the C. to ſell for ready money, but in barter he will have 7 pounds 10 s. and another doth barter with him for Silke, which are worth three Pounds a piece ready money. The queſtion is now, how he ought to deliver his Silkes the piece in barter, and how many payeth for the Wools.

Anſwer. Say by the Rule of Proportion, (or by the *Rule of Three direct*) If 6 pounds for C. weight ready money, yield me 7 li. 10 s. what will 3 li. yield, which is the juſt price of a piece of Silk in barter, to make the Truck equall? Work, and finde 3 li. 15 s. the price of a piece of Silk in barter: then ſay, If 3 li. 15 s. require one piece of Silk, how many pieces of Silk are bought with 400 pound, which is the value of 53 ⅓ C. weight of Wooll, at 7 li. 10 s? Work by the *Rule of Three direct*, and you ſhall finde 160 pieces of Silk and ⅓ of a piece, and ſo many of Silk pay for the Wooll, and neither party hath advantage of other.

Two men will change Merchandize, the one with the other. The one of them hath beer at 6 s. 8 d. the barrell, to ſell for ready money, but in barter he will ſell the barrell for 8 s. and yet he wil gain moreover after 10 pound upon the 10 pounds. And the other hath white Spaniſh *Wooll at 2 s. the Roye, to ſell for ready money. The queſtion is now, how he ſhall deliver the Roye of Wooll in barter, to ſave himſelf harmleſſe.*

Anſwer.

Answer. Say, if $6\frac{2}{3}$ s. which is the just price of the barrell of Beer, be sold in barter for 8 s. for how much shall 20 s. (which is the just price of the Rove of Wool) be sold in barter? Work by the *Rule of Three* direct, and you shall finde 24 s. Then for because the first Merchant will gain after 10 li. upon the 100 li. he maketh his 100 li. 110 li. And therefore say by the *Rule of Three*, If the second Merchant of 110 li. doe make but 100 li. how much shall he make of 24 s? Multiply and divide, and you shall finde 21 s. 9 d. $\frac{2}{11}$ of a peny. And for so much shall he sell the Rove of Wool to be delivered in barter, to the end the first Merchant may give 10 in the 100.

Two Merchants will change their Commodities the one with the other. The one of them hath white Paper at 4 s. the Ream, to sell for ready money. And in barter he will doe it away for 5 s. and yet he will gain moreover after the rate of 10 pounds upon the 100 pounds. And the other hath Mace at 14 s. 6 d. the pound weight to sell in barter. Now I demand what the pound did cost in ready money.

Answer. Say if 5 s. (which is the over price of the Paper in barter) become of 4 s. the just price, of how much shall come $14\frac{1}{2}$ shillings, which is the surprise of the pound of Mace in barter? Multiply & divide, and you shall finde $11\frac{1}{9}$ s. Then for because the first Merchant of Paper will gain after 10 upon the 100, Say, if

100 doe give 110, what ſhall 11 $\frac{2}{3}$ ſhillings give? Work and you ſhall finde 12 s. 9 $\frac{11}{17}$ d. and ſo much did the pound of Mace coſt in ready money.

The fourteenth Chapter entreateth of exchanging of money from one place to the other.

EXchange is no other thing, then to take or receive money in one City, to render or pay the value thereof in another City, or elſe to give money in one place, and receive the value thereof in another, at a term of certain daies, months, or fairs, according to the diverſity of the place.

But this practiſe chiefly conſiſteth in the knowledge of the Money or Coyns in divers places, of which for thy benefit (after a few Examples given to the Introduction of this work) I will ſet down certain Notes of the diverſity of the common and uſuall Coyns in moſt places in Chriſtendome for traffick.

And firſt I will begin at *Antwerp*, where they uſe to make their accounts by *Deniers de groſſe*, that is to ſay, pence Flemmiſh, whereof 12 doe make 1 s. Flemmiſh, and 20 s. doe make one pound *de groſſe*.

Item,

Item, A Merchant delivered at *Antwerp*, 400 pounds Flemmiſh to receive in *London* 20 s. ſterling, for every 23 s. 4 d. Flemmiſh: The Queſtion is now, how much ſterling money is to be received at *London* for 400 li. Flemmiſh?

Anſwer. Say by the *Rule of Three*, If 23 $\frac{1}{3}$ Flemmiſh give 20 s ſterling, what 400 li. Flemmiſh? Work, and you ſhall finde 342 l. 17 s. 1 $\frac{2}{7}$ d. and ſo much ſterling ſhall I receive in *London*, for the ſaid 400 li. *Flemmiſh*.

Otherwiſe alſo wrought by Rules of Practiſe in taking the $\frac{1}{7}$ of the Flemmiſh money delivered, and abating the ſame from the principall, the reſt is Engliſh money, as before.

li.	s.	d.
400	0	0
57	2	10 $\frac{2}{7}$
342	17	1 $\frac{2}{7}$ ſterling.

A Merchant at *London* delivered 200 li. ſterling for *Antwerp*, at 23 s. 5 d. Flemmiſh the pounds ſterling: The Queſtion is, how much he muſt receive at *Antwerp*?

Anſwer. Say by the *Rule of Three*, if 1 li. ſterling give 25 s. 5 d. Flemmiſh, what 200 li. ſterling? Work, and thou ſhalt finde 234 li. 3 s. 4 d. So many pounds Flemmiſh ſhall he receive at *Antwerp* for the ſaid 200 li. ſterling.

Otherwiſe by Practiſe.

1—13—5—200

3 s.—4 d. 33——6——8

1 d. ——16——8

maketh ſterling——————2. 4 li.—3 s.—4 d.

In *London* 20 li. ſterling is delivered by Exchange for *Antwerp*, at 23 s. 9 d. Flemmiſh the pound ſterling: The Queſtion is, at what rate the Flemmiſh money ought to be returned to gain 4 li. upon the 200 li. ſterling at *London*.

Anſwer. Firſt, ſay by the *Rule of Three* direct: If 1 li. ſterling give 23 $\frac{1}{4}$ Flemmiſh, what 200 li. ſterling? Multiply and divide, and you ſhall finde 237 li. 10 s. The which to return to gain 8 li. ſterling in *London*, ſay by the backer Rule, If 200 li. ſterling require the exchange 23 s. 9 d. Flemmiſh, what the exchange to make 208 li. ſterling? Work by the Rule, and finde 22 s. 10 $\frac{1}{2}$ d. Flemmiſh, the effect in the Queſtion required.

If I take up money at *Antwerp* after 19 s. 4 d. Flemmiſh, to pay for the ſame at *London*, 20 s. ſterling, and when the day of payment is come, I am forced to return the ſame money again in *London*, to pay my Bill of Exchange: ſo that for 20 s. which I take up here at *London*, I muſt pay 19 s. 6 d. at *Antwerp*, I demand whether

ther I doe win or loſe, and how much in or upon the 100 li. of money?

Anſwer. Say by the *Rule of Three*: If 19 $\frac{1}{2}$ give 19 $\frac{1}{3}$ what will 140 pounds give? Multiply and divide, and you ſhall finde 99 li. 2 $\frac{120}{117}$ s. which being abated from 100 pounds, there will remain 17 s. $\frac{-12}{117}$, and ſo much I doe loſe upon the 100 pounds of money.

If I take up at *London* 20 s. ſterling to pay at *Antwerp* 22 s. 4 d. and when the day of payment is come, my Factor is conſtrained to take up money again at *Antwerp*, wherewith to pay the aforeſaid ſumme, and there he doth receive 23 s. 4 d. Flemmiſh, for the which I muſt pay 20 s. at *London*: The Queſtion is now, whether I doe win or loſe, and how much upon the 100 li. of money after that rate.

Anſwer. Say by the Rule of Proportion, If 22 $\frac{1}{3}$ s. give 23 $\frac{1}{3}$ s. what will 100 pounds give? Multiply and divide, and you ſhall finde 104 pounds 9 ſhillings $\frac{27}{67}$, from the which abate 100 pounds, and there will remain 4 pounds 9 ſhillings $\frac{12}{67}$, and ſo much is there gained upon the 100 pounds of money.

In *Antwerp* is delivered 200 pounds Flemmiſh by exchange for *London*, at 20 ſhillings ſterling for every 23 ſhillings 4 pence, Flemmiſh: The Queſtion is, at what rate the ſame is to be returned to gain 10 li. upon the 100 li. Flemmiſh in *Antwerp*.

Anſwer. Firſt, ſay by the *Rule of Three*, if 23 $\frac{1}{3}$ Flemmiſh give 20 s. what ſhall 200 l. gain? Work, and you ſhall finde 171 l. 8 s. 6 $\frac{4}{7}$ d. Then ſay again by the *Rule of Three* direct, if 171 li. 8 s. 6 $\frac{4}{7}$ d ſterling, give me 210 li. Flemmiſh, what ſhall 20 s. ſterling give? Work, and you ſhall finde 24 s. 6 d. Flemmiſh. And at the ſame rate ought the ſame to be returned at *Antwerp*, to gain 10 li. upon the 100 Flemmiſh.

A Merchant of *Antwerp* delivereth 234 li. 3 s. 4 d. Flemmiſh, to receive at *London* 200 li. ſterling: The Queſtion is now, how the exchange goeth after this rate?

Anſwer. Say by the *Rule of Three* direct, if 200 give 20, what 234 $\frac{1}{6}$? Multiply and divide, and you ſhall finde 23 s. 5 d. And for ſo much goeth the exchange.

Item, the exchange from *London* into *France*, is not like as it is to *Flanders*, but it is delivered by the French Crown, which is worth 50 Soulx Turnois the piece.

Whereupon alſo you muſt note, that in *France* they make their accounts by Franks, Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx maketh one pound Turnois, which they call a Liure or Frank. But the Merchants to make their Accounts, doe uſe French Crowns, which is currant among them for 51 Soulx Turnois. But by exchange it is otherwiſe, for it is delivered but for 50 Soulx Turnois the Crown, or as the taker

up of the money can agree with the deliverer. And note that this △ Character repreſenteth the Crown by exchange, and is ever 50 Soulx Turnois or French money.

A Merchant delivereth at London 240 li. ſterling, after 5 s. 6 d. the Crown, to receive at *Paris* 50 Soulx Turnois for every Crown. I demand how much Turnois or French money payeth the Bills for the ſaid 240 li. ſterling.

Anſwer. Say by the *Rule of Three*, If 5 ½ s. ſterling give me 50 s. Turnois, what ſhall 240 li. ſterling give? Reduce the pounds into ſhillings, then multiply and divide, and you ſhall finde 2181 Liures, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ Turnois, and ſo much payeth the Bills at *Paris*, for the 240 li. ſterling.

A Merchant delivereth at *Roan*, or elſewhere in *France*, 1430 li. or Franks, the which Frank or pound is 20 Soulx, or a pound Turnois to receive in *London* 6 s. 4 d. ſterling for every △ of 50 Soulx Turnois. The Queſtion is, how much ſterling money I ought to receive at *London* for my 1430 li. Turnois.

Anſwer. Say, if 2 ½ li. give me 6 ⅓ s. what will 1430 give me? Work, and you ſhall finde 3622 ⅔ s ſterling, which maketh 181 li. 2 s. 8 d. and ſo much money is to be received at *London*, for the ſaid 1430 Liure Turnois, after 6 s. 4 d. for every △ of 50 Soulx.

In *London* is delivered 200 li. ſterling by exchange for *Paris*, at 5 s. 9 d. the △ of 50 Soulx

Turnois: The Queſtion is, at what price the ſaid △ is to be returned to gain 6 li. upon the 100 li. ſterling at *London*.

Anſwer. Firſt, ſay (by the *Rule of Three* direct) if $5\frac{1}{4}$ s. ſterling give 50 Soulx Turnois; what ſhall 200 li. ſterling give? Work, and you ſhall finde 1739 Franks or Liures, $2\frac{14}{23}$ Soulx. Then the which to return and gain 6 li. upon the 100 li. in *London*, ſay by the *Rule of Three* direct, if 1739 Franks $2\frac{14}{13}$ Soulx yeild 212 li. what the △ of 50 Soulx? Work and finde 6 s. $1\frac{1}{30}$ d. the effect required in the Queſtion.

A Merchant delivered in *London* 160 li. ſterling, to receive in *Biskay* for every 5 s. 6 d. one Ducat of 374 Marvides: The Queſtion is, how many Marvides ought I to receive at *Biskay*?

Anſw. Say, if $5\frac{1}{2}$ s. ſterling give 374 Marvides: what ſhall 160 li. ſterling give? Multiply and divide, and you ſhall finde 217600 Marvides, and ſo many I ought to receive at *Biskay* for my 160 li. ſterling.

A Merchant delivered in *Baion*, 4000 Marvides to receive in *London* 5 s. 8 d. ſterling for every Ducat of 374 Marvides: The Queſtion is now, how much ſterling money payeth the Bills of Exchange for the ſaid 4000 Marvides?

Anſwer. Say, if 374 Marvides make one Ducat, what 4000 Marvides? Multiply and divide, and finde, $106\frac{178}{187}$.

Then ſay again, if 1 Ducat give $5\frac{2}{3}$ s. what giveth $106\frac{178}{187}$ Ducats? work & find 30l. 6s. $\frac{114}{187}$.

Other-

Otherwise it is wrought more brief at one working, as in the last Question before, in considering that 5 s. 8 d. containeth one Ducat, or 374 Marvides. Therefore say by the *Rule of Three*, If 374 Marvides give $\frac{2}{3}$ s. what 4000 Marvides? Work, and you shall also finde in your quotient 30 $\frac{14}{361}$ s. And so many pounds sterling is to be received for the 4000 Marvides.

In *London* 200 li. delivered by exchange for *Vigo*, 374 Marvides the Ducat of 5 s. 10 d. sterling, maketh 256457 $\frac{1}{7}$ Marvides: the which to return and gain 10 li. upon the 100 li. in *London*, say by the *Rule of Three* direct, if 220 li. require 256457 $\frac{1}{7}$ Marvides: what 5 s. 10 d? Work, and finde 340 Marvides, the price of every Ducat in return, which is the effect in the Question required.

These may seem sufficient for instructions.

NOtwithstanding for thy further aid and benefit, hereafter follow six speciall and most brief *Rules of Practise*, for *English*, *French*, and *Flemmish* money:

1	teacheth	How to turn Flem to English sterling.
2		How to turn English sterling to Flem.
3		How to turn Flemmish to French.
4		How to turn French into Flemmish.
5		How to turn sterling into French.
6		How to turn French into sterling.

The

The fifteenth Chapter entreateth of the ſaid ſix Rules of brevity, and of valuation of Engliſh, Flemmiſh, & French money, and how each of them may eaſily be brought to others value.

How briefly to reduce pounds, ſhillings, and pence Flemmiſh into pounds, ſhillings, and pence Engliſh ſterling.

1 Rule. IT is to be noted, that 7 li. Flemmiſh maketh but 6 li. ſterling: 7 s. Flemmiſh maketh 6 s. ſterling: and 7 d. Flemmiſh 6 d. ſterling: ſo that 7 yeildeth but 6. Wherein is evident that then is loſt $\frac{1}{7}$, (if it may be ſo called) when it is reduced into Engliſh money: wherefore to know how much 233 li. 13 s. 4 d. Flemmiſh maketh Engliſh, you muſt ſubtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which reſteth after this ſubtraction, is the ſumme required: ſo that 233 li. 13 s. 4 d. Flemmiſh, maketh 200 li. 5 s. 8 $\frac{4}{7}$ d. ſterling.

Example

Example.			*Another Example.*		
li.	s.	d.	li.	s.	d.
233	13	4	311	0	0
233	7	7 2/7	344	8	6 6/7
200	5	8 4/7 Ster.	266	11	5 3/7

To reduce pounds, ſhillings, and pence Sterling, into pounds, ſhillings, and pence Flemmiſh. 2 Rule.

Note that a pound ſterling maketh 1 li. 3 s. 4 d. Flemmiſh: that is, 1 1/3 li. 1 s. ſterling maketh 1 1/6 s. Flemmiſh, and 1 d. ſterling maketh 1 1/6 d. *Flemmiſh.* So that there is gained (if it may be ſo called) 1/6 of the ſumme being thus reduced to Flemmiſh, for of 6/6 is made 7/6, which is one whole and 1/6. Then to know how much 237 li. 7 s. 6 d. ſterling maketh Flemmiſh, ſubtract from your ſterling, the 1/6 of the whole ſumme, and adde it to the ſame ſumme, and it maketh 276 li. 18 s. 4 d. which is the ſumme required.

Example.				*Another Example.*		
li.	s.	d.		li.	s.	d.
237	7	6	Ster.	337		
1/6 39	11	3		1/6 56	3	4
276	18	9	Flem.	393	3	4

To

3 Rule. *To reduce pounds, ſhillings, and pence Flemmiſh, into pounds, ſhillings, and pence French.*

Ye ſhall note, that the equality of Flemmiſh and French money is this, that is to ſay, the pound Flemmiſh, maketh 7 li. $\frac{1}{3}$. French, or Turnois. 1 s. Flemmiſh maketh 7 $\frac{1}{3}$ s. French, and a groat Flemmiſh, maketh 7 $\frac{1}{3}$ d. French.

Wherefore to know how much 143 li. 4 s. 9 d. Flemmiſh maketh French, ye muſt multiply the whole number twice by 6, beginning at pence, and ſo forward, and the Product of your ſecond multiplication divide by 5, ſo the work is finiſhed. Or multiply the ſaid ſumme by 7, and take out of it $\frac{1}{3}$, adding it to the Product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 li. 4 s. 9 d. Flemmiſh, maketh 1031 li. 6 s. 2 $\frac{1}{3}$ d. French or Turnois.

Example.

li.	s.	d.	
143	4	9	Flem.
		6	
859	8	6	
		6	
5156	11	0	Fren.
$\frac{1}{3}$ 1031	6	2 $\frac{2}{5}$	Fren.

The ſame otherwiſe.

li.	s.	d.	
134	4	9	
		7	
1002	13	3	
$\frac{1}{3}$ 28	12	11 $\frac{1}{5}$	
1031	6	2 $\frac{2}{3}$	

Another

Another Example.

143 li. Flem.
6
858
6
⅔ 5148 French.

Or thus.

143
7
1001
⅓ 28——12
1029 li.—12

1029 li. ⅓ or 12 s. French.

To reduce pounds, ſhillings, and pence, French, into pounds, ſhillings, and pence, Flemmiſh. 4 Rule.

Multiply 233 li. 8 s. 4 d. French by 5, and divide the Product twice by 6, that is, the ſaid number by 6, and the Product or Quotient again by 6, and the Quotient of this ſecond Diviſion is the thing required. So that 233 li. 8 s. 4 d. French, maketh 32 li. 8 s. 4 ⅓ d. Flemmiſh.

Example.

li.	s.	d.	
233	8	4	Fren.
		5	
1167	1	8	
⅙194	10	3 ⅓	
⅙32	8	4 ⅓	Flem.

Another Example.

li.	s.	d.	
758			French.
5			
3765			
⅙627	10		
⅙104	11	8	Flem.

To

5 Rule. *To reduce pounds, shillings, and pence Sterling, into pounds, shillings, and pence, French, or Turnois.*

The pound Sterling maketh 8 li. 8 s. French, that is to say, 8 $\frac{2}{3}$ li. the shilling maketh 8 $\frac{4}{5}$ s. and the peny 8 $\frac{2}{5}$ d. French. Wherefore to know what 221 li. 13 s. 4 d. Sterling maketh French, ye must multiply your whole summe by 42, that is, by 7, and the Product of it by 6, and divide this second Product by 5, and that is the summe required.

Otherwise, multiply the summe Sterling by 8, and adde twice to the Product $\frac{2}{5}$, and it shall produce the summe required. So that both waies 231 li. 13 s. 4 d. Sterling, maketh 1946 pound French, as here under followeth.

Example.

Sterling.

li.	s.	d. Ster.
231	13	4
		6
1390	0	0
		7
$\frac{4}{5}$ 9730	0	0

French. $\frac{2}{3}$ 1946 — 0 — 0 Fren.

The same otherwise.

li	s.	d.
231	13	4
		8
1853	6	8
46	6	8
46	6	8
1946	0	0

Another

Another Example.

753
6
4518
7
31626
⅓6325——4 Fren.

Ster.

The ſame.

753
8
6024
½ 150——12
½ 150——12
6325——4 French.

Sterling.

To reduce pounds, ſhillings, and pence, French, *into pounds, ſhillings, and pence, Sterling.* 6 Rule.

To know how much 1256 li. 12 s. 6 d. French maketh in ſterling money: multiply the ſumme by 5, and divide the Product by 7 & 6 at twice, and the laſt Quotient ſhall be the thing required, that is to ſay, 1256 li. 12 s. 6 d. maketh 149 li. 11 s. 11 4/7 d. ſterling.

Example.

li. s. d. Fren.
1256——12——6
5
6283——2——6
⅛1047——3——9
⅑149——11——11 4/7 Ster.

Another Example.

li. s. d.
2531——0——0 French.
5
12655
½2109——3——4
⅐301——6——2 4/7 Ster.

Note,

Note, that when any money is given by exchange at *London* for *Roan* at 71 ½ d. or rather 71 ⅐ for the Crown of 50 s. French; there is neither gain nor loſſe: for it is one money for another, accounting 8 li. 8 s. French, for 1 li. ſterling. So the giver loſeth the time of payment, which is about 15 daies, and he that taketh it, hath the gain of the ſame.

They of *Roan*, that put forth or take money by exchange for *London*, ought to have like conſideration.

Item, When any man giveth at *London* 64 d. ⅓. or rather 65 ⅐ d. to have at one of the Fairs of *Lions* a Crown *de Marc*, he that ſo giveth the money, loſeth the time, and he that taketh it, gaineth the ſame: for 62 d. ⅐. is equall in value to 45 s. French. He that putteth or taketh money at *Lions* for *London*, ought to conſider the ſame.

Item, when any deliver in *Antwerp* 75 d. to receive at *Lions* a Crown *de Marc*, he that putteth it forth, loſeth the time, and he that taketh it gaineth the ſame. For 75 groats Flemmiſh, is equall in value to 45 s. French.

Thus for this time I make an end of the Praciſe of Exchange, and the inſtructions thereunto belonging, and according to my promiſe: yet further to gratifie ſuch as are deſirous to know the common Coyns uſed for traffick among Merchants in theſe Cities following, a brief declaration of their Moneys, and the reckonings, and account of them is added.

The

The sixteenth Chapter containeth a Declaration of the valuation and diversity of Coyns of most places of Christendome for Traffick; And the manner of Exchange in those places from one City or Town to another: which known, is right necessary for Merchants, by means whereof they doe finde the gain or losse upon the Exchange.

Item, for as much as the greatest diversity of money of Exchange is at *Lions*; therefore I will begin duly of the money of that place.

At *Lions* they use Franks, Soulx, and Deniers Turnois. A Frank maketh 20 Soulx, and 1 Soulx 12 Deniers; but the Merchants to keep their Books of Accounts, do use French crowns of the mark at 45 Soulx the piece, and doe divide it into 20 Soulx, 1 Soulx is 12 Deniers.

Item, a mark of Gold maketh 65 △ of the mark, which serveth for exchange, and divide it into 8 ounces, the ounce into 24 pence or deniers, the denier into 24 grains, and so the summe or whole by imagination or guesse.

△ This mark standeth for a Crown.

Also at *Lions* there are four Fairs in a year, at the which they doe commonly Exchange, which are from 3 months to 3 months.

At *Geans* they uſe the Soulx: one Ducat maketh 3 pound.

At *Naples* they uſe Ducats, Taries, and Grains; the Ducat maketh 5 Taries, and one Tary 20 Grains: but they take 6 Ducats which maketh 30 Taries for the ounce.

A Ducat maketh 10 Carlins, and a Carlin 10 Grains, ſo that 2 Carlins make a Tary, and 100 Grains make a Ducat.

At *Rome* they uſe the Ducats of the Chamber; one Ducat is worth 12 Guillis, and one Guillis 10 Soulx.

At *Venice* they uſe Ducats currant at 124 Soulx a piece, or 24 Deniers, and one Denier maketh 32 Picolis.

At *Palerme* and *Meſſina* they write, after Ounce, Tary, and Grains, and 1 Ounce is worth 6 Ducats of 30 Taries, and 1 Tary is 20 Grains, and 1 Grain 6 Picolis, 1 Ducat is alſo worth 24 Carlins.

At *Millan* they uſe li. s. d. of Ducat Imperials, and Δ of exchange is worth 4 li.

At *Lucques*, *Florence*, and *Ancone*, they uſe the Δ of Gold: in Gold the French Crown is worth 7 li. but at *Boloigne* 3 li. 10 s.

At *Barcelone* they uſe the Soulx: the Ducat of Exchange is worth 22 Soulx.

At *Valence* and *Saragoſſe* they uſe the Liver, Soulx, and Denier: the French Crown of exchange is worth 20 Soulx, and one Soulx is 12 Deniers.

At the Fairs of *Caſtile* they uſe the Marveides

veides, the Ducat is worth 375 Marveides.

At *Lubone* they uſe the Rayes, one Ducat of Exchange is worth 400 Rayes.

At *Norenburg*, *Frankeford*, and *Auguſt* in *Germany*, they uſe the Krantzers, whereof 60 make a Floren.

At *Antwerp* they uſe li. s. d. de Gros, and they exchange into the Denier de Gros, to wit, our Engliſh peny.

At *London* they uſe the li. the s. the d. ſterling, and they exchange in pence ſterling.

The exchange of Lions *at ſundry places.*

Item, at *Lions* there is exchange in three ſorts, at the Cities and Towns following.

Firſt, they deliver at *Lions* one Mark to have or receive at *Naples* almoſt 41 ½ Ducats, at *Venice* 70 Ducats currant, at *Rome* 63 Ducats of the Chamber, at *Lucques* and *Florence* 65 △ of Gold, at Millan 82 △.

And contrariwiſe, at the ſaid Cities aforeſaid they doe give ſo much of money, to have a Mark at *Lions*.

Secondly, they give at *Lubone* one of △ of Mark of 45 Soulx Turnois a piece, to have at *Geans* almoſt 68 Soulx at *Palerme* and *Meſſine* almoſt 24 Carlins, at *Barcelone* 22 Soulx, at *Valence* or *Saragoſſe* 20 Soulx, at the Fair at *Caſtile* 350 Marveides, at *Lubone* 360 Rayes in *Antwerp* 57 Deniers de Gros, and at *London*, 76 d. ſterling.

And contrariwise, they give in the said Cities almost as much of their money to have a French Crown of the mark at *Lions*.

Thirdly, they doe give at *Lions* a △ of the Sun to have almost 93 Krentzars at *Frankeford*, *Ausburg*, *Noremberge*, or other Cities in *Almain*.

Also at *Lions* only they doe pay, they change the $\frac{2}{3}$ in Gold, and $\frac{1}{3}$ in money, or else all in money, in giving $1\frac{1}{2}$ for the hundreth.

Changes at Naples, *and other Towns.*

Item, at *Naples* they give or deliver almost 112 Ducats, to receive at *Rome* 100 Ducats of the Chamber at the old value.

Through *Lucques* and *Florence* they deliver 100 Ducats Carlins, to receive there almost 86 △ of Gold.

Through *Palerm* and *Messine*, one Ducat of 5 Tary, to receive there almost 164 grains.

Through *Millain*, one Ducat to receive there almost 90 Soulx.

Through *Geans*, one Ducat to receive there almost 65 Soulx. The whole summe to be paid within ten daies after the sight of the Bill of Exchange.

Also at *Naples*, they deliver one Ducat to receive in *Antwerp* almost 67 d. or Deniers *de Gros*, within two months. At *London* almost 60 d. sterling in three months. At *Barselone* almost 20 Soulx within two moneths.

At

At *Valence*, almoſt 18 Soulx within two months. At *Lisbone* 333 Raies within 3 months: and at the Fair at *Caſtile* almoſt 340 Marvides, at the ſame Fair.

Change of Venice *to other places.*

At *Venice* they deliver 100 Ducats currant to receive in *Almain* almoſt 140 Florens at 60 Krentzers the piece.

At *Lucques* and *Florence* almoſt 108 ∆ of Gold in 10 daies.

Likewiſe at *Venice* they deliver a Ducat currant to receive at *Palerm* and *Meſſine* almoſt 21 Carlins: at *Millan* almoſt 93 Soulx: at *Geans* almoſt 69 Soulx, the whole at 10 daies end.

Of the Pair *or* Pari.

As touching the Exchange, it is neceſſary to underſtand or know the *Pair*, which the Italians call *Pari*, which is no other thing then to make the money of the change of one City or Town, to or with the money of another, by means whereof they doe finde the gains or loſſe upon the Exchange.

Example.

Item, having received Letters of credit of one of *Antwerp*, that the ∆ of the Sun is there worth 7 Soulx: The Queſtion is, what the ſame is worth at *London*, when the *Pair* of Exchange goeth for 23 ſhillings?

Anſwer. Say, if 23 give but 20, what giveth 7? Work, and finde 8l. 1, $\frac{1}{3}$ d. and ſo much is the ∆ of the Sun worth at *London*.

The ſeventeenth Chapter containeth alſo a Declaration of the diverſity of the Weights and Meaſures of moſt places of Chriſtendome for traffique. At the end of which diſcourſe are two Tables, the one for Weight, and the other for Meaſure, proportionate and reduced to and equality of our Engliſh Meaſure and Weight, by the aid whereof, the ingenious may eaſily by the *Rule of Three*, convert the one into the other at pleaſure, &c.

AT *London*, and ſo all *England* thorow, are uſed two kindes of Weights and Meaſures, as the *Troy* weight, and the *Haberdepoiſe*. From the *Troy* weight is derived the proportion and quantity of all kinde of drie and liquid Meaſures, as Pecks, Buſhels, Quarters, &c. Wherewith is bought and ſold all kinde of grain and other Commodities mete by the Buſhell. And in liquid, Ale, Beer, Wine Oyle, Butter, Honey, &c. Upon theſe grounds and Statutes is bread made, and ſold by the *Troy* weight: and ſo is Gold, Silver,

Silver, Pearl, pretious Stones, and Jewels. The leaſt quantity of this *Troy* weight is a grain: twenty four of theſe grains make a peny-weight, twenty penny-weights an Ounce, and twelve ounces a pound, two pounds or pints of this weight maketh a quart. And ſo aſcending into bigger quantities, is produced the Meaſures whereby are ſold our other naturall ſuſtenance: *viz:* Ale or Beer, with all other neceſſary commodities, as Butter, Hony, Herings, Eels, Sope, &c. All which laſt before rehearſed, though their Meaſures (wherein they are contained) be framed and derived from the *Troy* weight, yet are they in traffique with divers Commodities, as Lead, Tinne, Flax, Wax, with all other commodities, both of this Realm, and of other forain Countries whatſoever, bought and ſold by the *Haberdepoiſe* weight after ſixteen ounces to the pound, and 112 pound to the hundred weight. And to every hundred is allowed but 12 pound weight at the common-beam. From hence is alſo derived the weight of *Suffolke* Cheeſe, which containeth thirty two Cloves, 8 pound to a Clove, and weigheth in all 256 pounds. And alſo the Barrell of *Suffolke* Butter is, or ſhould be of like weight with the weight of Cheeſe, *viz:* 256 pounds. More 14 of theſe pounds make a Stone, and 26 Stone containeth a Sack of Engliſh Wool: Forain Wools, to wit, French, Spaniſh, and Eſtrich, is alſo ſold by the pound, or C. weight, but moſt

See further of theſe Weights and Meaſures in Reduction beginning pag. 133.

commonly by the Rove, 25 pound to a Rove: other commodities of Tale, are bought and ſold by the C. fiveſcore to the C. Except headed ware, to wit, Cattell, Nails, and Fiſh, which are ſold after ſixſcore to the C. There are alſo two other ſorts of meaſures, to wit, the Ell and the Yard. By the Ell is uſually mete Linen cloth, as Canvas, &c. And by the yard Silks, woollen clothes, &c.

Antwerp.

At *Antwerp* are alſo two ſorts of weights, their Gold and Silver weight, and their common weight. Gold and Silver is weighed by the mark, the mark is 8 ounces, the ounce 20 Eſterlings, and the Eſterling 32, as our grains. The Goldſmiths divide that into ſmaller, but not the Merchants. The proof of Gold is made by Karects, whereof 24 maketh a mark of fine Gold, the Karect is 24 grains: the proof of the money is made by Deniers: 12 Deniers is 1 s. fine, that is, a mark of fine Silver: the Denier alſo is divided into 24 grains, and the grain into four quarters.

Item, 100 marks in *Antwerp* Troy weight, maketh at *Lions* 103 marks, 2 ½ ounces, and 20 grains, 23 p. At *Noremberg* 103 marks 2 ½ ounces, 2 Quints, 3 Deniers; at *Frankeford* 105 marks: at *Auſburg* 104 marks, 3 ounces, 1 Quint: at *Venice* 103 marks, 1 ounce, 7 Deniers, 18 grains: at *London* 66 pounds.

The

The Mark of gold or ſilver at *Antwerp*, Troy weight, which is 8 ounces, maketh 7 ½ ounces common weight, with which all other Merchandiſe is weighed. So that the Troy weight is greater then the common weight by 6 ¼ in the C. By this weight of Troy, they alſo weigh Musk, Amber, Pearl, &c.

All Silks are bought at *Antwerp* by the *Burges* Ell, which is greater then the common meaſure, by which they retail by 2 in the hundred. Their common Ell is ¼ of our yard, and ⅕ of our Ell.

Lions.

At *Lions* is uſed three ſorts of weight, whereof the firſt is the common Town weight, with which they weigh all kinde of Spicery, and divers other Merchandiſe. The ſecond is called *Geneva* weight which is 8 in the C. greater then the common weight, with which they weigh Silks, &c. The third is French weight, called commonly the Mark weight, and the 100 pounds thereof maketh 106 ¼ li. *Geneva*, and 114 ¼ of their common weight: with which French weight, is weighed all things that paid Cuſtome or Toll.

At *Lions* is alſo uſed two ſorts of Ells or Aulnes. The one wherewith they meaſure groſſe clothes, as Canvaſſe, and ſuch like. The other is called the French Ell or Aulne, with which they meaſure all other kinde of Merchandiſe, whereof 7 common Town Ells maketh 11 ordinary French Ells.

Roan.

Roan.

At *Roan*, 6 ½ Muides of Salt, being the meaſure of the place, make a C. at *Armviden* in *Zeland*, and the C. of *Bronage* meaſure of *Armviden*, maketh at *Roan* 11 Muides, 30 Muides make a Laſt of Corn, and 16 a Laſt of Oats, 100 pound weight there, maketh at *London* 114 ½, and 190 ¼ at *Antwerp*. And 200 Ells make at *London* 115 ¼.

Noremburge.

A 100 pound weight at *Noremburge* maketh at *London* 111 ¼; at *Antwerp* 107 ½, and 100 Ells at *Noremburge* make at *London* 75 ½, at *Antwerp* 95 ½, &c.

Lisbone.

The C. weight at *Lisbone* maketh 4 Roves, every Rove 32 pounds, ſo that their C. weight is 128 pounds, and their pound containeth 14 Ounces, and 100 pounds of their weight maketh at *London* 113 ⅛.

Their Silk, Cloth of Gold, and Woollen is meaſured with a meaſure which they call a Cubit, containing about ¼ of a Varre of *Caſtile*. Howbeit their common Meaſure is called a Varre, which maketh five Palms, and containeth 1 ¼ of a Varre of *Caſtile*, our Ell of *London* is equall with the Varre of *Lisbone*.

All

All kinde of Merchandiſe brought from *Flanders*, *Koan*, or *Britain*, payeth at *Lisbone*, as a duty or cuſtome to the King, 20 in the C. which they call the tenth in Merchandiſe, and the other tenth in money.

Note alſo, that all kinde of Merchandiſe coming to *Lisbone* by land, payeth leſſe in cuſtome, then that that cometh by water.

Civill.

The Rove of *Civill* is 30 pound, 4 Roves make their C. weight, which is 120 pounds. The 100 pounds of *Civill* maketh at *London* 102 pounds. Their other common meaſure is a Varre, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* be two ſorts of weight, the one called *la Groſſe*, the other *la Suttle*; with the groſſe is weighed all kinde of great wares, and with the ſmall all kinde of ſpicery, and ſuch like: 96 pounds of groſſe weight there, maketh at *London* 100 pound, and 100 pounds of ſpicery there, without any tare or allowance, make at *London* 94, and with tare 65.

Their owne common Meaſure are Braces, whereof 100 make at *London* 55 ½ Ells, at *Antwerp* 92 ½, &c.

Florence.

Florence.

At *Florence* the 100 lib. weight maketh at *Aquila*, for Saffron 110 and 145 pounds of *Florence*, make at *Roan* but 100 pounds the weight of *Florence*, and that of *Lucque* is all one.

Their other meaſures are Braces, whereof 100 maketh at *Antwerp*, *Burges* meaſure, 81 $\frac{1}{3}$ Ells, 100 Braces there, make at *London* 49 Ells, &c.

Lucque.

The *Lucque* Sattens commonly ſold at *Lions* by weight, and 133 $\frac{1}{3}$ pounds maketh at *Lions* 100 pound, ſo that 1 pound $\frac{1}{3}$ maketh at *Lions* but one pound.

Their other meaſures are Braces, whereof 100 of them make at *London* 50 Ells; at *Antwerp* 83 $\frac{1}{3}$ Ells, &c.

Aquila.

At *Aquila*, their 100 pounds maketh at *London* 71 $\frac{1}{4}$, their 136 $\frac{1}{3}$ pounds of Saffron maketh at *Geneva* but 100, and 11 li. of *Geneva*, maketh 15 li. at *Aquila*.

Valentia.

At *Valentia* be two ſorts of weights, a great and a ſmall. The C. weight or great weight con-

containeth four Roves, the Rove 36 li. ſo the C. great weight is 144 li. and the C. weight ſmall containeth but 120 pounds, and is alſo parted into four Roves, which is 30 pounds to a Rove. By the ſmall is ſold the ſcarlet grain, with all other kinde of Spicery, and by the great is ſold wooll, with all ſuch like groſſe wares. The 1 ⅓ pounds of Silk at *Valentia*, maketh at *Lions* one pound *Geneva* weight. The charge of great Merchandiſe at *Valentia* containeth 432 pound, and in ſmall wares 360 pounds.

The weight here and at *Barſellone* is all one.

Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Danſicke.

At *Danſick* or *Spruceland* the rule is, that whoſoever buyeth any merchandiſe there, buyeth it by the ſhip-pound, which is 320 li. 20 Liſpounds make a ſhip-pound, and the Liſpound containeth 16 pound, which ſhip-pound of *Danſick* maketh at *Antwerp* 266 ⅔ li. Their 100 li. weight maketh at *London* 86, &c.

Their other common meaſures are Ells, whereof 100 maketh at *London* 72 ¼, and at *Antwerp* 120 ½ Ells.

Toulhouſe.

At *Toulhouſe* 6 cabes of Woad maketh a charge, two ciſterns of corn-meaſure, and all kinde

kinde of grain maketh a charge, the Ciſtern weigheth 160 l. weight of that place. Their 100 in weight, maketh at *London* but 91 ¼ pound.

Geans.

At *Genna* or *Geans*, 100 li. of their weight maketh at *London* 71 ¼, and at *Antwerp* 68 ½; 100 li. weight at *Genna*, maketh at *Venice*, to wit, Suttle 106 li.

Their other common Meaſures are Palms, whereof 100 make at *London* 20 ⅓ Ells, and at *Antwerp* 34 ⅓.

The reſt are ſupplied in two Tables, which hereafter followeth: whereby the ingenious may gather his deſire.

The Table of the agreement of the Weights of divers Countreys, the one with the other, being reduced to an equality, as followeth:

112 li. weight at *London*, make at

Antwerp	107 ⅛
Frankeford	099
Colen and *Auburg*	102 ¼
Noremburg	100 ⅓
Roan	098
Paris	102 ¼
Lions	118 ½
Deep	100 ¼
Geneva	090 ½
Towlouſe	122 ¼
Rochell	124 ¼
Marſellis	124 ¼
Sivill, &c.	109 ¼

112 li. weight at *London*, make at

Venice groſs weight	105 ½
Venice ſuttle weight	166 ⅔
Aquina	157 ½
Vienna	089 ½
Preſlaw	134 ¼
Leipſig	101 ¼
Danſick	129 ¼
Lubeck	097 ½
Barcellon	144 ½
Lisbone	099
Geans	157 ½

The

The other Table of agreement of Measures of divers Countreys reduced unto an equality, by the aid whereof you may with the use of the *Rule of Three*, convert either more or lesse of any one Measure unto the other.

60 Ells, or 75 Yards at *London* make at

Antwerp	100	Ells.
Noremburg	104 1/2	
Frankeford	125	
Leipsig	125	
Preslaw	125	
Dansick	183	
Vienna in *Austria*	87	Aulnes.
Lions in *France*	60 1/4	
Paris in *France*	57	
Roan in *Normandy*	52	
Lisbone	60	Varres.
Sivil in *Spain*	81	
Castile in *Spain*	81	
Methera Isles	62	
Venice	108	Braces.
Lucques	120	
Florence	122 1/2	
Millan	138	
Rome	90	Canes.
Geans	288 6/13	Palms.

The

The eighteenth Chapter of treateth Sports and Paſtimes, done by number.

IF you would know the number that any man doth think or imagine in his minde, as though you could divine, bid him triple it, or put twice ſo much more to it as it is, which done, ask him whether it be even or odde; if he ſay odde, bid him take one to it, to make it even, and for that one, keep one in your minde. Now after he hath taken one to it to make it even, bid him give away half and keep the other half for himſelf, which when he hath done, bid him triple that half, and again, after he hath tripled it; ask him whether it be even or odde: if he ſay odde, then bid him take one to make it even again, and for that laſt one, keep two in your minde: now after he hath made his number even, bid him caſt away the one half, and keep the other ſtill, from which half that he keepeth, cauſe him ſubtilly to put away or give you nine out of his number, as oft as he can, and for each nine that he giveth you, keep four in minde, and thereunto joyn the three which I had you keep, and you ſhall have your deſire.

Example

Example.

Imagine he thought 7, the triple whereof is 21, and becauſe it is odde he is to take 1 to make it even, which firſt 1 given, is for you to keep in minde. Then the half of his 22 being caſt away, he reſerveth ſtill 11, which after you have bid him triple, it maketh 33: then in giving of him 1 again to make it even, upon the laſt 1 reſerve in your minde, then his half of 34 maketh 17; from whence he can give you 9 but once. Therefore that yeelding to you 4, and the 3 that you keep, make 7 your deſire.

Another kinde of Divination, to tell your friend how many pence of ſingle pieces, reckning them one with another, he hath in his purſe, or ſhould think in his minde.

WHich to doe, firſt bid him double the pieces he hath in his purſe, or the number he thinketh (if he participate his number or ſecrecy unto ſome one friend that ſitteth by him that can but multiply, and adde never ſo little: if their number be great, then ſhall they work as you bid them ſo much the ſurer.)

Now after he hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 alſo; which done, bid him tell you the juſt ſumme of his laſt multiplication, which ſumme the giver thinketh it nothing available, becauſe it is ſo great above his pretended imagination: yet thereby ſhall you preſently with the help of Subtraction tell his propoſed number.

The Rule is this.

Imagine he thought 17, double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplied by 5, as here is practiſed, it yeilded 195, which 195 is the ſumme delivered you in the work: then for a general Rule you ſhall evermore cut off the laſt figure toward your right hand, with a daſh of your pen, as here is performed, as a figure nothing available unto your work, and then rebate 2 from your firſt figure, after 5 is cut off, and the reſt ſhall evermore be your deſire, as by this Example doth appear.

17	
2	
—	
34	
5	
—	
39	
5	
—	
195	
2	
—	
17	

Another of a Ring.

IF in any company you are diſpoſed to make them merry by manner of divining, in delivering a Ring unto any one of them, which after you have delivered it unto them, that you will abſent your ſelf from them, and they to deviſe after you are gone, which of them ſhall have the keeping thereof, and that you at your return will tell them what perſon hath it, upon what hand, upon what finger, and what joynt: Which to doe, cauſe the perſons to ſit down all in a row, and to keep likewiſe an order of their fingers: now, after ye are gone out

out from them to ſome other place, ſay unto one of the lookers on, that he double the numbers of him that hath the Ring and unto the double bid him adde 5, and then cauſe him to multiply the Addition by 5, and unto the Product bid him adde the number of the finger of the perſon that hath the Ring. And laſtly, to end the work, beyond that number towards his right hand, let him ſet down a figure ſignifying upon which of the joynts he hath the Ring, as if it be upon the ſecond joynt, let him put down 2. Then demand of him what number he keepeth, from the which you ſhall abate 250, and you ſhall have three figures remaining at the leaſt. The firſt toward your left hand, ſhall ſignifie the number of the perſon which hath the Ring, the ſecond or middle number ſhall declare the number of the finger; and the laſt figure towards your right hand ſhall betoken the number of the joynt.

Example.

Imagine the ſeventh perſon is determined to keep the Ring upon the fifth finger, and the third joynt: firſt double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplied by 5, yeildeth 95, unto which 95, adde the number of the finger, and it maketh 100: and beyond 100 toward the right hand, I ſet down 3 the number of the joynt, all maketh 1003, which is the number that is to be de-

livered you, from which abating 205, there reſteth 753, which prefigureth unto you the ſeventh perſon, the fifth figure, and the third joynt.

But note, that when you have made your ſubtraction, if there doe remain 0, in the place of tens, that is to ſay, in the ſecond place, you muſt then abate 1, from the figure which is the place of the hundreds, that is to wit, from the figure which is next your left hand, and that ſhall be worth 10 tenths, ſignifying the tenth finger, as if there ſhould remain 803, you muſt ſay, that the ſeventh perſon upon his tenth finger, and upon his third joynt, hath the Ring.

Another of three Dice.

IF a man doe caſt three Dice, you may know the points of one of every of them. For if you cauſe him to double the points of one Die, and to the double to adde 5, and the ſame ſumme to multiply by 5, and unto the Product adde the points of one of the other Dice; and behinde the number towards the right hand, to put the figure which ſignifieth the points of the laſt Die, and then ask what number he keepeth, from which abate 250, and there will remain three figures, which doe note unto you the points of every Die.

Another of things hidden.

IF three divers things are to be hidden of three

three divers persons and you to divine, which of the three persons hath the three divers things, doe thus: Imagine the three things to be represented, *A*, *B*, *C*. Then secondly, keep well in your minde which of the persons you mean to be the first, second, and third. Then take 24 Counters or Stones, and your three things, and give *A* to the party whom you imagine to be your first man, and therewithall give him one of your 24 Counters in his hand, and *B* unto your second man, and therewithall 2 Counters, and *C* unto your third man, and therewithall 3 Counters: and leave the rest, which are 18, still among them: which done separate your self from them, and afterwards bid them change the things among them as they shall think good: which done, after they are agreed, bid him that hath such a thing, as before you have represented by *A*, for every Counter that he hath in his hand, to take up as many more. And for him that hath *B*, for every one in his hand to take up two. And for him that hath *C*, for every one in his hand to take up 4, and the rest of them to leave still upon the board. These 3 things, and the three persons being fully printed in your minde, come to the Table, and you shall evermore finde out of these 6 numbers, 1, 2, 3, 4, 5, 6, or 7. If therefore one remain still upon the board, then have they made no exchange, but keep them still as they were delivered unto them. So that the first man hath *A*, the second

B, and the third man *C*. But if 2 remain, then the first man hath *B*, your second man *A*, and your third man *C*. The rest of the work and the order thereof are here apparent by the Table following.

	1	*A*		1	*B*
1	2	*B*	5	2	*C*
	3	*C*		3	*A*
	1	*A*		1	*C*
2	2	*B*	6	2	*A*
	3	*C*		3	*B*
	1	*A*		1	*C*
2	2	*B*	7	2	*B*
	3	*C*		3	*A*

Another divination of a number upon the Casting of two Dice.

FIrst let the Caster cast both the Dice, and mark well the number: then let him take up one of them, it maketh no matter which, and look what number it hath in the bottom, and adde all together: then cast the Die again, and keep in his minde what all together maketh: then let the Dice stand, and bring seven with you, and thereunto adde the rest of the pits that you see upon the upper side of the Dice, and so many did the Caster cast in all.

FINIS.

An Appendix concerning the Resolution of the *Square* and *Cube* in Numbers, to the finding of their side,

By *Ro. Hartwel.*

A Figurate Number is a number made by the multiplication of one number or more by another. A figurate number what.

The sides of a figurative number, are the numbers by whose multiplication it is made. The sides of a figurate number what.

A Figurate number is two-fold. a— { Plain. / Solid.

And it is { Of one Multiplication :— / Or consequently of many : } as a { Plain. / Solid.

And in each { Both Æquilater. / And Inæquilater.

A figurate number made of one multiplication, by two sides or numbers, multiplied together, is called a plain figurate number.

For every number made by the mutuall multiplication of two numbers, may be called a Plain, because it bringeth forth a right-angled Parallelogramme, according to his Unites disposed in length and breadth, the sides whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5 is called a Plain, and A plain figurate number.

the ſides thereof are 4 and 5 as here.

* * * * *
* * * * *

Becauſe the Unites thereof diſpoſed in length and breadth, as the ſides doe expreſſe, doe bring forth an inæquilater Parallelogram, for that the numbers, or ſides are inequall.

* * * * *
* * * * *

By like reaſon 36 made by multiplication of 6 by 6, is called an *Æquilater plain*, for the ſides thereof 6 and 6 are equall.

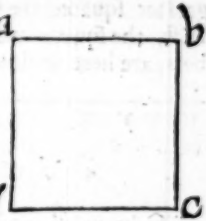
Moreover one and the ſame plain number may have many ſides, as the plain number 24, hath ſides 4 and 6 : 3 and 8 : 2 and 12. For it is produced from the mutuall multiplication of theſe numbers : whereupon for the invention of the ſides, to wit, in inæquilater Plains, it is needfull to give one of the ſides by which the plain it ſelf divided, the other ſide is made known. As the plain 48 being divided by the ſide 8, the Quotient 6 is the remaning ſide. Notwithſtanding another reſolution & inquiſition doth happen in the ſides of the *Æquilater plains*.

An Æquilater plain or Quadrat what.

An *Æquilater plain* is a number made by two equall ſides, or by any number multiplied by it ſelf. It is vulgarly called a Square or Quadrat; by the Arabians *Zenſus*, it is commonly expreſſed by this note z, by us *q*.

A Quadrat or Square in Geometry is called a right-lined plain figure, made by four equall right lines, and ſo many right Angles, and every one of the lines is called the ſide of the Quadrat,

drat, as this figure *a b c d* whose side is *a b*, or *b c*, as also *c d*, and *a d*.



To the similitude hereof, that number is called a Quadrate, which is made by the multiplication of two equall numbers, or of one in it self, of which manner 36 is made, by 6 multiplied in it self, or by the mutuall multiplication of 6 and 6. For if 36 Unites be placed in plain form, it bringeth forth a perfect Geometricall Quadrat, having in every side 6 Unites as here.

* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *

The number whereof the Quadrat is produced by multiplication in it self, is called the side or root of the Quadrat.

The side or root of a number what.

Concerning the Extraction of the Quadrat or square Root.

THerefore to finde the Quadrat Root, or the side of any Quadrat number, is to search a number which brought or multiplied in it self, maketh the number propounded: concerning the finding whereof, as it is requisite that the sides (being lesser then 10) of the squares under an hundred should be gathered by the Table of Multiplication: so the sides of the greater

greater ſquares are to be ſought out by Art. Firſt, the ſquares whoſe ſides are ſimple numbers, are here ſet down as you ſee.

The Roots.	1	2	3	4	5	6	7	8	9
The ſquares.	1	4	9	16	25	36	49	64	81

The knowledge of a ſquare is by finding out his ſide expreſſed by a whole number.

Although the finding out of the ſide of a ſquare be applied to each number given, as to a ſquare, yet ſquare numbers only have a ſide to be expreſſed by a certain number of Unites, or by rationall numbers, the other are to be expreſſed but only in power. The ſides are commonly called Roots by a Metaphoricall phraſe.

The Root or ſide of a ſquare is to be found by the *Theorem* following.

1. If the odde degrees of a ſquare number being marked from the right toward the left hand with points, you ſubduct from the number given, the particular ſquare of the laſt period, ſetting the ſide thereof alone by it ſelf.

2. Then going on, if you divide the remainder (if there be any) with the figure going before it, by the double of the ſide ſet alone by it ſelf.

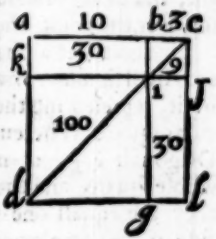
3. And multiply the Quotient, found out (being placed by the ſide, which was firſt ſet alone by it ſelf, and alſo before the doubled number

on

on the right hand, by both the numbers (namely by the double number, and the Figure ſet by it ſelf) being counted as one Diviſor, ſubducting the products from the given number, and then renue this laſt work of Diviſion ſo many times as there are pricks remaining, the ſide of the ſquare ſhall be found out.

This artificiall device is taken out of the 4 *p.* 2 of *Euclide*. Where by demonſtration it is proved, that if a right line be cut into two Segments, howſoever the ſquare of the whole line is equall to the ſquares of the Segments, and the two right-angled figures made of the Segments as in the figure annexed the two Diagonals, *k g*, & *b f*, are the ſquares of the Segments, *a b*, and *b c*. Alſo the complements *b k*, and *f g*, are the right-angled figures made by multiplying the line *a b*, by *b c*.

To extract the ſquare Root.



The ſelf-ſame parts are to be found in any ſquare number. As for Example, let the number be 169, whoſe ſide is 13. This ſide being divided into two pieces, 10 and 3, multiplying each piece by it ſelf once, namely, 10 by 10, and 3 by 3: then multiply one by another, as 10 by 3, and 3 by 10, ſo ſhall you have four plain numbers, whereof two are ſquare, as here you ſee.

The firſt Example.

There-

Therefore as the ſquare 169 is made by adding together of theſe four plain numbers, ſo by ſubducting them ſeverally it is reſolved.

	10	3
	10	3
	100	
	30	
	30	
	9	
	169	

Firſt therefore I mark each odde place with points, becauſe the particular ſquares are to be found in the odde places. Then for ſo much as the unity ſtanding under the firſt point next the left hand, and repreſenting the laſt period, is both a ſquare and the ſide of a ſquare: that figure therefore being ſet alone in the Quotient, and being ſubducted from the unity ſtanding over the point, there remaineth nothing.

This unity ſet alone by it ſelf in the Quotient, ſhall ſignifie 10, when another figure is ſet by it, repreſenting the ſide of ſome other particular ſquare. Whereupon I ſay, that the greater Diagonall *k g*, is now ſubducted from the whole ſquare, and the ſide of it *k i*, or *a b*, (for they are equall one to another) and alſo the ſide of the complement is found out.

This is the firſt ſtep to this Reſolution.

Moreover, I double the figure found out, becauſe being doubled, it is the ſide of both the complements taken jointly together, namely *k i*, and *g i*. Then ſetting 2 the doubled number under 6, I divide 6 (which in this place is as much

much as 60, and repreſenteth both the complements) by 2, the Quotient is 3, repreſenting the other ſide remaining of the complement; namely, *if* for *b c*, which number I ſet in the Quotient and count it for the Segment remaining of the right line given. Wherefore becauſe this number 3 is the ſide of the remaining Diagonall, that is to ſay, of the leſſer ſquare *b f*, therefore being ſet by the Diviſor on the right hand, and multiplied by it ſelf, and alſo by the Diviſor, it bringeth forth three plain numbers, namely, the ſquare *b f*, and the two complements *a i*, and *i l*, which being ſubducted from the numbers ſtanding over them, there remaineth nothing.

The Example is thus:

169	(12	Which is all one, as if you had put down the numbers found out in this manner: —
123		
3		
69		

169	
100	The greater Diagonall.
60	The complements twofold.
9	The leſſer Diagonall.
169	

The ſubtilty of this invention is illuſtrated by many examples. The ſecond Example.

Let the ſquare given be 1764. This number being marked with two points, telleth us, that the ſide thereof is to be written with two Figures. First,

First therefore beginning at the point on the left hand, I ſeek the ſide of the laſt period, namely, 17. But for ſo much as it is no ſquare number, I take 4 the ſide of the next leſſer ſquare, which I ſet alone by it ſelf in the Quotient, and then multiply it by it ſelf, the Product is 16, which being ſubducted from 17, there reſteth 1. Moreover I double the ſide found out, the Product is 8, I place this doubled number under 6, and by it divide 16 ſtanding above it, the Quotient is 2, which muſt be ſet by 4. This Quotient 2 muſt be ſet before the Diviſor 8, on the right hand under the point, and then muſt it be multiplied both by it ſelf, and into 8, the product is 164, which being ſubducted from the figures ſtanding over them, there remaineth nothing: whereby I gather that the number given is a juſt ſquare.

The Example ſtandeth thus:

x

1764 (42

1682

x

164

1764 *The Collection.*

The ſame manner of working is to be followed in greater ſquare numbers given, ſaving that the former part of the work is to be uſed but

but once, but the latter part is to be followed ſo many times as there are points remaining excepting the laſt.

As in 5 47 56, I ſay, that the ſide of the ſquare next unto 5 is 2: therefore 2 being ſet in the Quotient, and multiplied by it ſelf, makes 4, and taken from 5, the remainder is 1. Moreover I double the Quotient the product is 4, which I ſet under the next Figure toward the right hand, and thereby divide 14, the Quotient is 3, which three being ſet both in the Quotient, and alſo before the Diviſor toward the right hand, I multiply both the numbers by it, the Product is 129: this being ſubducted from 147 ſtanding above it, the remainder is 18. But becauſe there is yet one point remaining, with which I have not medled; I therefore again double all the whole Quotient, for in this caſe I muſt take 23 for the ſide of one former ſquare, and generally in great numbers, when I light upon more particular ſquares then two, I muſt eſteem them but as two, and take the ſides which are firſt found out, but as the ſides of one only ſquare. Therefore twice 23 is 46: by this I divide 185, the number to be ſet in the Quotient is 4, which number alſo muſt be ſet before the Diviſor on the right hand: then muſt 464 be multiplied by 4: the Product is 1856, this Product being ſubtracted from the numbers ſtanding over it, there remaineth nothing. The Example ſtandeth thus:

The third Example.

Note.

218

. . .

54756 (234
44364
129

4
1836
54756 *The Collection.*

See also the Examples following.

. . . .

10942864 (3308.

Therefore out of this invention is this consectarie.

4 Example of a Surd number. The number whose side cannot be expressed by whole numbers, is not a square number.

Such are all prime numbers, and the squares themselves excepted) all other compound numbers. For if in them you desire to finde out the square side, you shall labour in vain, because they are not squares, for to the whole numbers arising in the Quotient, there will be some Fraction adjoyned, whereby it cometh to passe, that the number of the side is not to be expressed by a true number, and it is commonly called a Surd number.

Notwithstanding, if you adjoyne to the side found out, the number remaining, taking his denomination from the double of the side augmented

mented by an unity, you ſhall finde the next ſide that may be like to the ſide of a ſquare.

As if from 40 you take the neareſt ſquare, to wit, 36. the remainder is 4. Here therefore the ſide ſought for of the ſquare exceedeth not the ſide found out by an unity, but either by one, or more parts of ſome whole number: wherefore I double 6, the ſide found, and adde an unity to it being doubled, the totall is 13, this number I ſet under 4 the remainder, and ſay that the ſide of 40 demanded as neere as may be, is $6\frac{4}{13}$: the Denominator of the Fraction being added to the greateſt ſquare in the number given, namely, unto 36, maketh the next greateſt ſquare above it, namely 49, whoſe ſide is 7. But this ſurd ſide, to wit, $6\frac{4}{13}$, multiplied by it ſelf, maketh $39\frac{144}{169}$, which are not juſt equal to 40, the given number.

Judge the like concerning the reſt which are not ſquares.

Thus much concerning plain figurate numbers, but eſpecially ſuch as are ſquare numbers.

Concerning ſolid figurate Numbers.

A Solid figurate Number is made of two multiplications by three numbers, or ſides, multiplied together, admitting length, breadth, and thickneſſe.

A ſolid figurate number.

Therefore every number made by the mutuall multiplication of three numbers, may be called a ſolid, becauſe it bringeth forth a right-angled *Parallelipipedon*, diſpoſed according to his Unities in length, breadth, and thickneſſe, the ſides whereof are the three multiplying numbers. As the number 30 made by the mutuall multiplication of 2, 3, and 5, is called an *Inequilater ſolid number*, and the ſides thereof are 2, 3, and 5; becauſe the Unites thereof diſpoſed by a certain diſtance one from another, in length, breadth, and depth: as the ſides doe expreſſe, doe bring forth in reſemblance an Inequilater Parallelipipedon, for that the numbers or ſides are inequall.

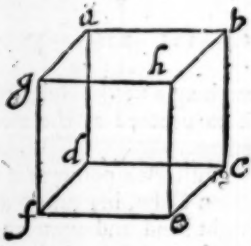
By like reaſon 216 made by multiplication of 6 by 6, and the Product thereof by 6, is called an Equilater ſolid, for the ſides thereof 66 and 6 are equall.

An Equilater Solid or Cube.

An Equilater, is a number made by three equall ſides, or by any number multiplied by it ſelf, and that Product again by the foreſaid number. And it is called an Equilater and Equiangled Parallelipipedon or Cube, and is commonly repreſented by us thus C.

A Cube in Geometry is a right-angled Parallelipipedon having ſix equall ſurfaces, and 8 ſolid Angles, and 12 ſides, as this figure following, *a. b. c. d. e. f. g. h.* whoſe ſide is *a b*, or *a d*, alſo

alſo *b c*, or *c d*, either *c e*, or *e f*, likewiſe *c h*, or *h g*, alſo *g f*, or *d f*, or *d a*, and *g a*.



The number whereof the Cube is produced by Multiplication in it ſelf twice is called the ſide or root of the Cube, which being found out in whole numbers, the Cube is known. The ſide or root of Cube.

Concerning the Extraction of the Cubick Root.

THerefore every Cube in numbers hath ſuch a ſide as may be expreſſed in whole numbers, but in magnitudes it is not alwaies ſo, as indeed in magnitudes there are many things not to be expreſſed in whole number. Now for as much as the ſide of any Cube under 1000, is a ſimple figure, it is neceſſary, before we undertake to finde out the ſide of any great number, to know what Cube is made of each ſimple figure, and what is the ſide of any Cube leſſer then 1000, as I have here ſet them down.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	226	343	512	729

But in ſearching out the ſides of greater Cubes we are to proceed as the *theorem* following teacheth us.

If you diſtinguiſh with points as it were into periods, the given Cube, beginning at the firſt figure on the right hand, and omitting each two figures continually, and firſt of all ſubduct the particular Cube of the laſt period from the given number, ſetting the ſide thereof in the Quotient: and then ſet triple of the Quotient under the figure next following the former point, on the right hand, and the ſquare of the Quotient being tripled beneath it one degree more toward the left hand: and afterward divide the number above written by the triple of the ſquare, ſetting the Quotient by it ſelf, and then multiply the Diviſor by the Quotient found out, and the tripled ſquare by the ſquare of the Quotient, and the Quotient cubically, ſubducting the products (ſo orderly added together, that each figure may anſwer the numbers whereof it was multiplied) from the number given, and renue this laſt manner of diviſion ſo many times as there are points remaining, the ſide of the Cube ſhall be found out.

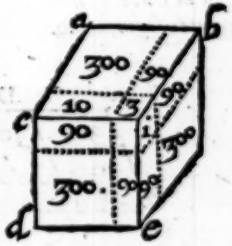
This

This artificiall device is drawn out of that Theorem, which *Ramus* made, imitating that of *Euclide* concerning Square numbers in this manner.

The extraction of the Cubick side or Root.

If a right line be cut into two ſegments, the Cube of the whole line ſhall be equall to the Cubes of the ſegments, & the two ſolid firgures comprehended three times under the ſquare of his ſegment, and the ſegment remaining.

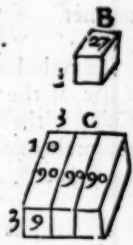
As the line *c i*, which is 13, is cut into two Segments, 10 and 3, therefore the Cube of the whole line, namely 2197, is equall unto the Cubes of the Segments, namely unto 100, and 27 alſo to the two-fold Solids or Parallelipipedons thrice taken,



whereof three have like ſoliditie, the ſolidity of each of the three leſſer is 90, being made of the Square of the Segment 3, that is to ſay of 9, multiplied by the other Segment 10. Theſe three Parallelipipedons joyntly taken together, make 270. But of the three greater Parallelipipedons each containeth 300, being made of 100, the Square of the greater Segment 10, multiplyed by the leſſer Segment 3,

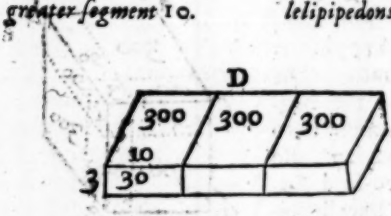
and they being taken joyntly together, make 900.

The Cube of the leſſer ſegment 3.



The Cube of the greater ſegment 10.

The 3 leſſer Parallelipipedons.



The three greater Parallelipipedons.

The Cube therefore hath eight particular ſolids in number, which are made of the parts of the number given, namely of 10 and 3 in this manner. Firſt, let there be four plain numbers made, each part being multiplied by it ſelf, and one by another.

10	3
10	3
	9
	30
	30
	100

If again I multiply the Plains by the ſame parts, there will ariſe 8 ſolids, as you ſee here,

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	*All theſe being added*
90	300	*together, are equall to*
90	300	*the Cube of the whole,*
300	1000	*to wit,* 2197.

The firſt Example to extract the Cubick Root.

Therefore the ſame way that is kept in making the Cube, is alſo to be followed in reſolving the Cube: As for Example, I marke the Cube given points in this manner, 2197.

Then I ſubduct the particular Cube of the number ſet under the laſt point: but for ſo much as that number is no Cube, I take the neereſt to it, namely, an unity, which alſo I ſet in the Quotient. This unity in the number given, is 100, but in the Quotient it is but 20, the unity ſubducted from 2, the remainder is 1, which muſt be written over the number

given. So that the greater Cube A, is to be ſuppoſed to ſubducted from the number given.

This is the firſt ſtep of this work,

1
2197 (1
1

After I triple the Quotient found out (that is to ſay, I multiply it by 3) this triple repreſenteth the three ſides (joyntly taken together) of the three leſſer ſolids marked with C, I place the tripled number under 9. Again, I multiply the Quotient ſquare-wiſe, and triple the Product, which maketh likewiſe 3. This Product reſembleth the three ſquare ſides (taken joyntly together) of the three greater Solids, marked with D; I place the Product on a degree lower toward the left hand underneath 1. with it I divide 11, which written above it, the quotient is 3. This Segment or Quotient 3, being multiplyed by 3, the Diviſor maketh 9, which in reſpect of the place wherein it ſtandeth, is 900, and repreſenteth the three greater Solids marked with D, taken joyntly together. Furthermore the ſame Quotient being multiplied ſquare-wiſe, maketh 9, and multiplied afterward by the tripled number ſtanding under 9, it maketh 27, which in reſpect of the place wherein it ſtandeth, is 270, and repreſenteth the 3 leſſer Solids marked with C. Laſt of all, the

ſame

ſame Quotient multiplied Cubically, breedeth the leſſer Cube B. Theſe 3 products therefore being added together, and the total ſubducted from the Numbers ſtanding over it, there remaineth nothing, which importeth the given number is a Cube.

The example is as you ſee.

1	2197 (13
2197 (13	1000 *The greater Cube.*
1 3	3
3	3
9 Or thus :	900 *The greater Parallelipipe.*
27	270 *The leſſer Parallelipipedons.*
27	27 *The leſſer Cube.*
2197	2197

The matter may be explained by many examples.

The ſecond example of the Cubick root.

Let the ſide of the given Cube 16387064, be ſought out, contrive it therefore (as it were) into certain periods with points. Then firſt of all ſearch out the ſide of the Cube next to the left hand: But for as much as 16 is no Cube, take 2 the ſide of the next Cube under it, that is to ſay, of 8, and ſet in the Quotient, and ſubduct 8 the Cube thereof from 16, there remaineth 8. The firſt work is not to be renued throughout the whole Number, but the Rules following muſt be repeated as often as there are points remaining.

The

The first step to finde out the Root, is in this manner.

8

. . .

16387064 (2

8

Moreover, triple the Quotient now found out and the Product is 6, which is to be placed under 8, namely, under the figure following the next prick toward the right hand. Then multiply the Quotient by this tripled number (or which is all to one purpose, square the Quotient, and then triple the Product) it maketh 12, set that number in a lower place one degree neerer the left hand, and make it the Divisor: divide 83 by 12, observing this rule in choosing your Quotient, that it be no greater, then that the numbers afterward produced by multiplication may not exceed the numbers standing over it. So that here you shall take 1 in 8, but 5 times. Afterward by this number 5, multiply the Divisor 12, and by the square of 5, multiply the tripled number 6, and last of all multiply 5 cubically: so shall you produce three numbers, namely, 60, 150, 125, to be described in such sort as you see. These numbers added together, and subducted from 8387, the remainder is 762.

The second step to finde out the Root, in this manner.

8762

8762

16387064 2

6

12

60

150

125

7625

And becauſe there is yet one point remaining, this laſt manner of Diviſion muſt be wrought again.

Firſt, therefore I triple the Quotient, the Product is 75, which muſt be ſo placed, that the firſt figure thereof, namely 5, may ſtand under 6, the ſecond under the 0. Again, multiply the Quotient by this tripled number (or which is all one, ſquare the quotient, and triple the product) it maketh 1875, which muſt be the Diviſor, whoſe firſt figure namely 5, muſt be placed under 7, the laſt figure of the tripled number. Then ſee that 1 may be contained in 7, many times, but I can take it but 4 times, I ſet 4 in the Quotient and multiply the Diviſor, by 4, the Product is 7500, afterward I ſquare 4, it maketh 16, which I multiply by the tripled number 75, the Product is 1200. Laſt of all I multiply 4 cubically, it maketh 64, theſe Products added all toge-

together, make 762064, which number being ſubducted from the Cube given, there remayneth nothing, whereby I gather that the number given is exactly Cubicall.

The third ſtep to finde out the ſide is in this manner,

762

16387064 (254

75

1875

7500

1200

64

762064

Behold alſo the example following.

The third example of the Cubick root.

6141250000 (850

Another manner of working.

HItherto the Princely high-way to find out the ſide of the Cube hath been declared.

But there are moreover certain other wayes alſo bending thereto, and leaning to the ſame principles, whereof this is one.

The ſecond form.

Having found out in the Table of ſimple Cubes, the firſt figure repreſenting the ſide of the Cube contained in the number ſtanding under the firſt Point on the left hand, ſet it in the Quotient

Quotient, and ſubduct the particular Cube of that figure as you did before: then ſquare that figure, and triple that ſquare, the Product ſhall be the Diviſor, the firſt figure whereof ſhall be ſet under that figure which is on the right hand next of all to the point (now examined) before going.

See how many times the Diviſor is contained in the number written over it, and multiply the Diviſor in the Quotient, and ſubduct the Product from the Dividend: yet here you muſt take heed, that you take not a greater Quotient then that the Products made afterward thereby may be ſubducted from the number given. 2.

The ſubduction being done, triple the firſt figure which was ſet in the Quotient, and adde to the triple the laſt number which was ſet in the Quotient on the right hand of the Product. 3.

This total multiplied by the ſquare of the figure laſt found out, and ſet down the Product ſo, that the firſt figure thereof toward the right hand may ſtand under the point next before going on the ſame hand, and finally, ſubduct the ſame from the number given. 4.

The fourth Example of the Cubick Root.

As in 804357. The particular Cube, namely, 729 being taken from the number ſtanding under the laſt period upon the left hand, there remaineth 75357, the ſide of that particular Cube being 9 I ſet in the Quotient. Then I ſquare that ſide, it maketh 81, and triple the ſquare, the Product 243 is my Diviſor, which I ſet under the given number, ſo that 3 may ſtand under 3 with

with this Diviſor, divide the number ſtanding over it, you ſhall finde 2 to be contained in 7 three times. Therefore I ſet 3 in the Quotient, and multiply the Diviſor by it, the Product is 729, which being ſubducted from 753, the remainder 24.

The Induction is thus.

2

754

. .

104357 (93

243

729

Moreover I triple 9, the Product is 27, by which on the right hand I ſet 3 the Quotient laſt found out, the total is 273.

This number I multiply by 9 the ſquare of the Quotient laſt found out, the Product ſhall be 2557, which being ſubducted from the ſupetiour number, there remaineth nothing.

The Induction is thus.

24

. .

8104357 (93

732

9

2457

Another

Another manner.

THe ſelf-ſame work may be diſpatched another way, a little differing from the former in this manner:

The figure in the Quotient, being found out by ſubducting the particular Cube, and alſo the ſecond figure in the Quotient being found by diviſion, let the totall Quotient be tripled, and let the tripled number be multiplied by the former figure in the Quotient. Then let the Product be multiplied again, by the latter figure found out, and let a Cypher be ſet on the right hand of that Product. Laſt of all, ſet the Cube, of the latter figure found out, be added to this product, and let the total ſumme be ſubducted from the number given. As in 373248. The third form.

The firſt induction is in this manner.

30

373248 (7

343

Moreover I ſquare the ſide found out, it maketh 49, and triple the ſquare, the Product is 147, which ſhall be the Diviſor, by this I divide 302, the number written over it, the Quotient is 2. Now I triple the total Quotient 72, it maketh 216, and multiply this triple by 7, the former figure in the Quotient, the Product is 1512. I multiply this Product alſo by 2, the latter figure of the Quotient, and ſet a Cypher on the right hand of it, ſo as it maketh 30240, unto this number laſt of all I adde 8, the Cube of The fifth Example.

of the latter figure found out, the total is 30248, which being subducted from this figure above it, there remaineth nothing.

The second Induction is thus.

$$\begin{array}{r} 30 \\ 373\,248\ (72 \\ \hline 147 \\ \hline 30248 \end{array}$$

All the points of the number given being examined, if any thing remain, it signifieth the number given is no Cube: wherefore the true side of it cannot be exactly given in numbers. Yet if it please you to sift out the neerest side that may be, by the first kinde of reduction of mixt numbers, you shall reduce the number given unto a cubical fraction of a greater denomination, and afterward seek out the cubical side of that fraction.

To finde the neerest cubick root in a surd number.

For example sake, because 120 is no Cube, therefore let it be reduced into sixty Cubical parts, after this manner. Multiply 60 Cubically in it self, it maketh 216000, by this being taken for the Denominator of the fraction, multiply 120 the number given, the product is 2592000 whose Cubicall side is $\frac{205}{60}$ that is $4\frac{11}{12}$ the neerest to the true side that can be.

For the extraction of all sorts of roots, the table of Logarithmes set forth by Mr. *Briggs* are most excellent, and ready.

FINIS.

A Table of Board and Timber measure more perfect then ever hath been made; shewing also the Squares between 4 and 37 from quarter to quarter: Calculated by Robert Hartwell.

Board measure.	Inches & quarters.	Squares.	Timber measure.	Board measure.	Inches & quarters.	Squares.	Timber measure.
36.0.0	4	16	08.0.2	16.0.0	9	81	21.3.3
33.8.8	1	18	06.0.0	15.5.6	1	85	20.3.3
32.0.0	2	2	86.4.0	15.1.6	2	90	19.2.0
30.3.1	3	2	78.5.4	14.7.7	3	95	18.1.8
28.8.0	5	20	69.1.2	14.4.0	10	100	17.2.8
27.4.3	1	27	64.0.0	14.0.5	1	105	16.4.6
26.1.8	2	35	57.6.0	13.7.5	2	110	15.7.1
25.0.4	3	38	52.3.6	13.3.9	3	115	15.0.2
24.0.0	6	36	48.0.0	13.0.9	11	121	14.2.8
23.0.4	1	39	44.3.0	12.8.0	1	126	13.7.1
22.1.5	2	42	41.1.4	12.5.3	2	132	13.0.9
21.3.3	3	45	38.4.1	12.2.7	3	138	12.5.2
20.5.7	7	49	35.2.6	12.0.0	12	144	12.0.0
19.8.6	1	52	33.2.3	11.7.5	1	150	11.5.1
19.2.0	2	56	30.8.6	11.5.3	2	156	11.0.7
18.5.8	3	60	28.8.0	11.2.9	3	162	10.6.6
18.0.0	8	64	27.0.0	11.0.7	13	169	10.2.2
17.4.6	1	68	25.4.1	10.8.7	1	175	9.8.7
16.9.4	2	73	24.0.0	10.6.7	2	182	9.4.6
16.4.6	3	76	22.7.5	10.4.7	3	189	9.1.4

Board measure.	Inches & quarters	Squares.	Timber measure.	Board measure.	Inches & quarters	Squares.	Timber measure.
10.2.8	14	169	8.8.1	6.8.6	21	441	3.2.2
10.1.1	1	203	8.5.1	6.7.7	1	451	3.8.2
9.9.3	2	210	8.2.3	6.6.6	2	462	3.7.3
9.7.6	3	217	7.9.6	6.6.1	3	473	3.6.5
9.6.0	15	225	7.6.8	6.5.4	22	484	3.5.7
9.4.5	1	232	7.4.3	6.4.7	1	495	3.4.9
9.2.9	2	240	7.2.0	6.4.0	2	506	3.4.1
9.1.4	3	248	6.9.7	6.3.3	3	517	3.3.3
9.0.0	16	256	6.7.5	6.2.6	23	529	3.2.7
8.8.6	1	267	6.5.4	6.1.9	1	540	3.2.0
8.7.3	2	272	6.3.5	6.1.2	2	552	3.1.3
8.6.0	3	280	6.1.6	6.0.6	3	564	3.0.6
8.4.7	17	289	5.9.8	6.0.0	24	576	3.0.0
8.3.5	1	297	5.8.1	6.9.4	1	588	2.9.4
8.2.3	2	306	5.6.4	5.8.8	2	600	2.8.8
8.1.1	3	315	5.4.8	5.8.2	3	612	2.8.2
8.0.0	18	324	5.3.3	5.7.6	25	625	2.7.6
7.8.9	1	333	5.1.8	5.7.0	1	637	2.7.1
7.7.8	2	342	5.0.5	5.0.5	2	650	2.6.5
7.6.8	3	351	4.9.2	5.5.8	3	662	2.6.0
7.5.8	19	361	4.7.8	5.5.4	26	676	2.5.5
7.4.8	1	370	4.6.7	5.4.8	1	689	2.7.0
7.3.9	2	380	4.5.5	5.4.3	2	702	2.4.7
7.2.9	3	390	4.4.3	5.3.8	3	715	2.4.1
7.2.0	20	400	4.3.2	5.3.3	27	729	2.3.8
7.1.1	1	410	4.2.1	5.2.8	1	742	2.3.2
6.0.2	2	420	4.1.1	5.2.3	2	756	2.2.7
6.9.3	3	431	4.0.1	5.1.8	3	767	1.2.4

Board measure.	Inches & quarters.	Squares.	Timber measure.	Board measure.	Inches & quarters.	Squares.	Timber measure.
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.9	4.3.0	2	1120	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1136	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.5.0
4.9.2	1	855	2.0.2	4.2.0	1	1174	1.4.7
4.8.8	2	871	1.9.8	4.1.7	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1210	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1237	1.4.0
4.7.2	2	930	1.8.6	4.0.5	2	1247	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1280	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	974	1.7.7	3.9.7	1	1313	1.3.1
4.5.7	2	98	1.7.5	3.9.4	2	1331	1.2.9
4.5.3	3	1000	1.7.2	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	37	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.2	2	1056	1.6.3	3.8.4	2	1416	1.2.2
4.4.0	3	1072	1.6.1	3.8.2	3	1425	1.2.0

The uſe of this former Table.

IF upon a Scale or Ruler you divide one inch into ten equal parts or primes, and again by Diagonals, and Parallel-lines, you ſubdivide each of them into ten equal parts or ſeconds, with your compaſſes, you may take a more exact running meaſure for board and timber, then by any other means whatſoever, and ſo place the ſame, or this Table if you will, upon any ruler.

Alſo by means of the columns of Squares, you may readily finde a ſquare equall to any Parallelipipedon, or piece of timber, which is thicker then it is broad. *As for Example:* ſuppoſe a piece of timber to be ten inches thick, and nine inches broad: if I multiply thoſe ſides one by another, they will produce 290, then ſeeking the column of ſquares for 290, which I finde not, but I finde 280 the neareſt number to 290, to ſtand againſt 17: therefore I ſay ſeventeen inches *fere*, will make a ſquare equal to ſuch an unlike ſquared piece, then looking in the column of timber meaſure againſt 17, you ſhall finde that five inches, nine primes, or $\frac{2}{10}$ and eight ſeconds, or $\frac{12}{100}$ of an inch in length, of that piece will make a foot of timber.

Likewiſe

Likewiſe for board meaſure, you may finde how much in length of breadth of board muſt be in one foot.

By the like means, ſuppoſe *for example* that a board, appointed to be meaſured, is fifteen inches $\frac{1}{4}$ broad, if I deſire to know how much in length thereof will make a foot; I ſeek in the columns that ſtand under Unites and quarters, for $15\frac{1}{4}$, and alſo againſt the ſame in the column under the title of Board meaſure, where I finde nine inches one prime, or tenth of an inch, and four ſeconds, or hundreds of an inch will make a foot at that breadth: The like may be practiſed for any other breadth of board whatſoever.

Certain Tables ſhewing the Intereſt of any ſumme of mony whatſoever unto 40 years; how much Annuities reſpited or forborn cometh unto. And for buying or ſelling of Annuities for the ſaid time; and alſo the ſame in reverſion after any number of years unto 30. what they may be worth in preſent ready mony, by R. C. and now diligently corrected and amended by *Robert Hartwell*.

Definition of Intereſt.

PRincipall, is the ſumme from which the Intereſt is reckoned.

2. Intereſt is the ſumme reckoned for the lending or forbearance of the Principall for any termes or time.

3. Intereſt ſimple is that which is counted from the Principall onely.

4. Intereſt compound is that which is counted from the Principal, together with the Arrerage.

5. Intereſt profitable is that which is added to the principall.

6. Intereſt Damageable is that which is to be ſubtracted from the principall.

The uſe *per annum* of	1 li.	is	2 s.
	10 s.		12 d.
	5 s.		6 d.
	2 s. 6 d.		3 d.
	12.		1 d. $\frac{1}{3}$ of a peny.

A Table shewing what 1 li. *with interest, and interest upon interest after* 10 *in the* 100 *comes to every year under* 41 *years. As followeth:*

years	li.	s.	d.
1	1	2	0
2	1	4	2
3	1	6	7
4	1	9	3
5	1	12	2
6	1	15	5
7	1	18	11
8	2	2	10
9	2	7	1
10	2	11	10
11	2	17	1
12	3	2	9
13	3	9	0
14	3	15	11
15	4	3	6
16	4	11	10
17	5	1	1
18	5	11	2
19	6	2	3
20	6	14	6

li.	s.	d.	years
7	8.	0	21
8	2	9	22
8	19	1	23
9	16	11	24
10	16	8	25
11	18	4	26
13	2	2	27
14	8	5	28
15	17	3	29
17	8	11	30
19	3	10	31
21	2	3	32
23	4	6	33
25	10	11	34
28	2	0	35
30	18	0	36
34	0	0	37
39	8	1	38
41	2	10	39
45	5	2	40

By the former Table, if you desire to know what 1 li. cometh to with interest, and interest upon interest after 10 in the 100, for any number of years unto 40. Look in the row, or margent (over which is written Years) and against it on the right hand close unto it in the row or margent of pounds, shillings, and pence, (which is titled thus, li. s. d.) you shall finde your desire.

Example.

I would know what 1 li. with interest, and interest upon interest cometh to in 7 years?

I look in the row of years for the number 7. And against it on the right hand I finde 1 li. 18 s. 11 d. Also what it cometh unto in 13 years. I seek among the years for 13, and against it I finde 3 li. 9 s.

Again, for 21 years, I look for 21 among the years, and I finde 7 li. 8 s. 0 d. But if you would know for a greater summe then 1 li. Then multiply your summe by that summe of 1 li. in the Table for any of those years, and you shall easily finde it. As thus, I would know what 10 li. cometh to for 7 years with interest, &c. I see that 1 li. cometh to 1 li. 18 s. 11 d. in that time. Then say I that 10 li. must be ten times as much in that space, which is 19 li. 9 s. 2 d. Also of 10 li. in 13 years, I see that 1 li. in that time cometh unto 3 li. 9 s. Then must 10 li. be ten times as much in that space, which

is

is 34 li. 10 s. Alſo what 10 li. cometh to in 21 years. I finde firſt that 1 li. in that ſpace cometh to 7 li. 8 s. Then I ſay 10 muſt be ten times as much, which is 74 li. Laſtly, I would know what 100 li. cometh to in ſeven years, I ſee it muſt be 100 times as much as 1 li. cometh to in that ſpace, which is 194 li. 11 s. 8 d. Hereby you ſee the common ſaying is not true, that 100 li. doth double it ſelf in ſeven years, for it wants thereof 5 li. 8 s. 4 d. But in eight years 100 li. cometh to 210 li. 8 s. 4 d. which you ſee is more then double it ſelf by 10 li. 8 s. 4 d. And in this ſort may any that can but caſt with Counters, or indeed by memory finde the increaſe of any ſumme whatſoever for any of the number of years in the foreſaid Table, after they have found what 1 li. cometh unto for that time, as before is ſpecified.

A Table ſhewing if 1 li. *annuity to endure for any number of years, under* 41, *be all reſpited or forborn, untill the laſt payment grow due, and then all be received together, with intereſt, and intereſt upon intereſt after* 10 *in the* 100 per annum, *what they will amount unto by any of the ſaid number of years. As followeth:*

years	li.	s.	d.
1	1	0	0
2	2	2	0
3	3	6	2
4	4	10	10
5	6	2	1
6	7	14	3
7	9	9	8
8	11	8	8
9	13	11	7
10	15	18	8
11	18	10	7
12	21	7	8
13	24	10	5
14	27	19	5
15	31	15	6
16	35	18	11
17	40	10	10
18	45	11	11
19	51	3	2
20	57	0	6

li.	s.	d.	years
64	0	0	21
71	8	0	22
79	10	10	23
18	9	11	24
98	6	11	25
109	3	7	26
121	1	11	27
134	4	2	28
148	12	7	29
164	9	10	30
181	18	10	31
201	2	9	32
221	5	0	33
245	9	6	34
271	0	5	35
299	2	6	36
330	0	9	37
364	0	10	38
401	8	11	39
442	11	10	40

By this Table you may know what any Annuity being reſpited or forborn for any number of years unto 41, with intereſt upon intereſt, after ten in the 100 will come unto: firſt ſeeking in the Table, what 1 li. will come unto, in that time, and that being found to multiply it by the ſumme you deſire to know.

Example.

Firſt, I would know what 1 li. Annuity being forborn or reſpited for 14 years cometh unto.

I look in this laſt Table (which is for purpoſe) and I find 27 li. 19 s. 5 d.

Again, what 1 li Annuitie reſpited for 21 years commeth to; I look in the ſaid Table for 21 years, and I find 64 li. Alſo the like for 1 li. for 30 years reſpited. I look, and finde it to be 164 li. 9 s. 10 d. as by the ſaid Table may appear. Now for greater Annuities, as 30 li. *per annum*, reſpited or forborn, what it amounteth to in 16 years. I ſeek firſt for 1 li. in this laſt Table before for 16 years, and againſt it I find 35 li. 18 s. 11 d. Then ſay I, that 30 li. *per annum* being reſpited for that time, will come to 30 times as much, which is 1078 li. 7 s. 6 d. Alſo if there be an Annuity of 45 li. due and unpaid for 12 years, I look in the ſaid Table what 1 li. cometh to, 12 yeares being reſpited, and I finde it is 21 li. 7 s. 8 d. Then I conclude that

5 li.

5 li. muſt be forty five times as much, which is 962 li. 5 s.

Laſtly, I have an Annuity of 50 li. *per annum*. which hath been behind for 16 years, and muſt be anſwered unto me with intereſt, and intereſt upon intereſt, all at one payment, what ſhall or ought I to receive in all, at the 16 years end?

I ſeek what 1 li. comes unto in that time (as before taught) and I finde 35 li. 18 s. 11 d. Then muſt my 50 li. *per annum* forborn for that time, come to fifty times as much, which is 1797 li. 6 s. 10 d. And thus may you finde any other ſumme great or ſmall, for any number of years contained in the aforeſaid Table, without the help of Arithmetick, if you can but uſe your Counters, or by memory count well.

A Table shewing if 1 li. *Annuity (to endure for any number of years unto* 41*) be to be sold for present ready money, how much ought that ready money to be, reckoning* 10 per 100 per annum *abating interest, and interest upon interest. As followeth:*

years	li.	s.	d.
1	0	18	2
2	1	14	8
3	2	9	8
4	3	3	4
5	3	15	9
6	4	7	1
7	4	18	4
8	5	6	8
9	5	15	2
10	6	2	10
11	6	9	9
12	6	16	3
13	7	2	0
14	7	7	4
15	7	12	1
16	7	16	5
17	8	0	5
18	8	4	0
19	8	7	3
20	8	10	3

li.	s.	d.	years
8	12	11	21
8	15	5	22
8	17	7	23
8	19	8	24
9	1	6	25
9	3	0	26
9	4	8	27
9	6	1	28
9	7	4	29
9	8	6	30
9	9	7	31
9	10	6	32
9	11	4	33
9	12	2	34
9	12	10	35
9	13	6	36
9	14	1	37
9	14	7	38
9	15	1	39
9	15	6	40

This Table before laſt ſpecified is very neceſſary and commodious for all Gentlemen or others, that ſhall have cauſe to buy or ſell Annuities or ſuch like, for by this they ſhall know what they doe, whether they demand, or take too little or too much, after the rate of ten in the 100, by which proportion all theſe Tables are ruled.

As for example, I am to buy an Annuitie of 16 li. *per annum*, for 12 years and am demanded for it ready mony 120 li. I would know, if I give this rate, whether I give too much or too little, according to the proportion of 10 in the 100 *per annum*, &c.

I look in the Table laſt before what 1 li. is worth for 12 years, and I find againſt 12 this ſumme 6 li. 16 s. 3 d. Now I ſay that 16 li. Annuity for that time, and after that proportion cometh to 16 times as much, which is 100 li. So that I ſee the party demanded of mee 11 li. too much after the rate of 10 in the 100 *per annum*, and therefore I muſt draw him to a lower price or leave it.

Again, I am offered an Annuity of 20 li. *per annum* of 14 years for 130 li. I would know if I give it whether I give too much or too little, according to the proportion aforeſaid.

I ſeek firſt what 1 li. Annuity is worth for 14 years, and I find in the ſaid laſt Table 7 li. 7 s. 4 d. Then ſay that the Annuity of 20 li. *per annum*, will come to 20 times as much, and will be worth 147 li. 6 s. 8 d. according to the

the proportion before mentioned: and is more then his demand by 17 li. 6s. 8d. So that I ſee if I accept of it, I ſhall have a good bargaine. And thus may you know readily by looking in your Table, and finding what 1 li. is worth for any time therein contained, how much any greater ſumme will come unto, if you multiply it by that ſumme of 1 li. as before is ſufficiently ſhewed.

But ſuppoſe this, I have 300 li. ready mony, and would beſtow the ſame for a valuable Annuity anſwerable thereunto according to the proportion aforeſaid. I would know what Annuity to endure 21 years this 300li. will buy?

I look in the former Table what 1 li. Annuity will coſt for that time, and I find 8 li. 12s. 11 d. Then I ſay by the Rule of proportion. If 8 li. 12 s. 11 d. will buy 1 li. Annuity for 21 years: what Annuity ſhall 300 li. buy or be worth for that time? I reduce the ſummes to the leaſt denomination (which is pence) and I find 34 li. 10 s. 9d. And after this manner (by the help of this Rule) may you find all other ſummes for any time contained in the foreſaid laſt Table.

A Table shewing what 1 li. *in reversion for any number of years under* 31 *is worth in ready money, the buyer staying untill the thing be falne in hand.*

years	li.	s.	d.	li.	s.	d.	years
1	6	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	3	29
15	0	4	9	0	1	1	30

This

This laſt Table differeth, and is contrary to the other three before-mentioned: For whereas the others increaſed more and more according to the number of years ſpecified, this doth grow and diminiſh leſſe and leſſe, as the number of years increaſeth. *As for Example:*

There is a Tenement, the Fee ſimple whereof after 7 years will be worth 40 li. what am I to give for it in ready money, now ſtaying untill it fall in hand?

To know this I look in this laſt Table for 7 years, and againſt it I finde 10 s. 3 d. So that a thing that after 7 years will be worth 1 li. is worth now in ready money but 10 s. 3 d. Then ſay I, that the foreſaid Tenement (which after 7 years will be worth 40 li.) is now worth forty times 10 s. 3 d. which is 20 li. 10 s.

Again, there is a Farm which after 9 years will be worth the Fee-ſimple 420 li. what is it now worth in ready money, ſtaying untill it fall in hand?

I look in the ſaid Table what 1 li. is worth in reverſion after 9 years, and I finde 8 s. 5 d. Then ſay I, that the Farm of 420 li. ſo long in Reverſion, will be now worth in ready money, 420 times as much, which is 176 li. 15 s.

Laſtly, there is a Lordſhip to be ſold, the Fee-ſimple whereof after 14 years will be worth 7500 li. I would know what the ſame is now worth in ready money for the Reverſion.

I look in this laſt Table for 14 years, and againſt it I finde 5 s. 3 d. ſo much 1 li. is worth in reverſion after 14 years. Then ſay I, that 7500 li. is worth no more in reverſion for that time then 7500 times 5 s 3 d. which is 1968 li. 15 s. And after this manner may you finde out any other ſumme whatſoever. And though ſome men of their own experience can aym (as they think) neer enough the mark to ſerve their own turns: yet I dare undertake they ſhall never ſo exactly doe it, nor juſtifie what they doe, as if they did it by Art.

FINIS.

New Tables of Intereſt at 8 *per cent. per annũ*, exactly calculated for 30 years, by *Robert Hartwell*, with neceſſary Queſtions for the uſe of them.

The firſt Table expreſſing the increaſe of 1 li. principal, put out and forborn for any number of years under 31, at 8 per cent. per annum.

yers	li.	s.	d.	q.
1	1	1	7	0
2	1	3	3	3
3	1	5	2	1
4	1	7	2	2
5	1	9	4	2
6	1	11	8	3
7	1	14	3	1
8	1	17	0	0
9	1	19	11	3
10	2	3	2	0
11	2	6	7	2
12	2	10	4	1
13	2	14	4	2
14	2	18	8	3
15	3	3	5	1

li.	s.	d.	q.	yers
3	8	6	0	16
3	14	0	0	17
3	19	0	0	18
4	6	3	3	19
4	13	2	2	20
5	0	8	0	21
5	8	8	3	22
5	17	0	0	23
6	6	9	3	24
6	16	1	2	25
7	7	11	0	26
7	19	9	0	27
8	12	6	2	28
9	6	4	0	29
10	1	3	0	30

The deſcription and uſe of the Tables of Intereſt at 8 *per annum*, being profitable.

The firſt of them.

THeſe Tables conſiſt of four Columns, in the firſt and fourth whereof is written over the head, Years, and under the firſt number of years deſcending from 1 to 15, likewiſe in the fourth the number of years deſcending from 16 to 30. And againſt every year in the ſecond Column, toward the right hand the pounds, ſhillings, pence & farthings, which one pound, or 20 s. principal will amount unto, being put forth and forborn for the number of years ſet againſt it; (but the pounds, ſhillings, pence, &c. in the third Column belongeth to the years ſet in the laſt Column.)

1 Example.

Let it be required what 1 pound or 20 s. being put forth and forborn for 12 years, ariſeth to at 8 *per centū*, *per annū*, intereſt upon intereſt.

Seek in the firſt Column under the title of Years, for 12 the number of years propoſed in the Queſtion, and right againſt it toward the right hand in the ſecond Column, you ſhall finde 2 li.-10 s.-4 d.-1 q. which is the principal and increaſe thereof due for the time required.

2 Example.

If 100 li. be put forth for 17 years according to the ſame intereſt, I demand what it will amount to in that time?

Look in the Column under the title of years for

for 17, and right againſt it towards the left hand in the Table is found 3 li.-14 s.-0 d.-2 q. which is the increaſe of 1 li. by which you may thus gather the increaſe of 100 li. or any other ſumme; a hundred times 3 li. is 300 li. then a hundred times 14 s. is 70 li. both which added together doe make 370 li.-0 s.-0 d. which is the increaſe of 100 li. put forth & forborn 17 years, the ſolution to the Queſtion.

li.	s.	d.	q.
300	0	0	0
70	0	0	0
370	0	0	0

3 Example.

Suppoſe 60 li. be put forth for 19 years according to that rate, what wil it increaſe to in that time?

Seek 19 under the title of Years, and againſt it toward the left hand is found 4 li.-6 s.-3 d.-3 q. now ſay ſixty times 4 li. is 240, and ſixty times 6 s. is 360 s. or 18 li. & ſixty times 3 d. is 180 d. or 15 s. and ſixty times 3 q. is 3 s. 9 d. all which added together make 258 li. 18 s. 9 d. the increaſe thereof demanded.

li.	s.	d.	q.
240	0	0	0
18	0	0	0
0	15	0	0
	3	9	0
258	18	9	0

The second Table shewing what 1 li. *annuity or yearly rent is worth at the end of any number of years under* 31, *being forborn, at* 8 per centum, per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	yers
1.	1.	0.	0.	0	30.	6.	5.	3.	16
2.	2.	1.	7.	0	33.	15.	0.	0.	17
3.	3.	4.	6.	0	37.	9.	0.	0.	18
4.	4.	10.	1.	1	41.	8.	11.	0.	19
5.	5.	12.	2.	3	45.	15.	2.	3.	20
6.	7.	6.	8.	0	50.	8.	5.	2.	21
7.	8.	18.	5.	1	55.	9.	1.	2.	22
8.	10.	12.	8.	3	60.	17.	10.	1.	23
9.	12.	9.	9.	0	66.	15.	3.	2.	24
10.	14.	9.	8.	3	73.	2.	1.	1.	25
11.	16.	12.	10.	3	79.	19.	1.	0.	26
12.	18.	19.	6.	2	87.	7.	0.	0.	27
13.	21.	9.	10.	3	95.	6.	1.	2.	28
14.	24.	4.	3.	2	113.	19.	3.	3.	29
15.	27.	3.	0.	2	118.	5.	7.	3.	30

The use of the second Table, whose disposition is altogether like the former according to the title thereof, being profitable.

1 *Ex-*

1 *Example.*

There is a Leaſe worth 28 li. *per annum*, to endure 14 years, I demand what it will riſe unto at the end of thoſe years, being all forborn with the intereſt upon intereſt at the rate preſerved in this Table.

Look in the third Table for 14 years, againſt which toward the right hand, you ſhall finde 24 li.-4 s.-3 d.-2 q. Now multiply 28 li. by 24, there ariſeth 672 li. then 28 li. by 4 s. yeldeth 112 s or 5 li. 2 s. Again 28 li. by 3 d. proceeth 84 d. or 7 s. Finally, 28 by 2 q. yeelden 56 q. or 1 s. 2 d. All which added together make 678 l. 0 s. 2 d. to be received at the end of 14 years, the ſame rent or annuity being reſpited.

li.	s.	d.	q.
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	0	2	0

2 *Example.*

If 60 li. yearly rent or annuity be forborn 20 years: I demand how much it will increaſe at the end of the ſaid term?

In the Table I finde that 1 li. in 20 years will ariſe to 45 l.-15 s.-2 d.-3 q. therefore 60 l. in the like term will yeeld ſixty times as much; which I will reckon thus: ſixty times 45 li. is 2700 li. ſixty times 15 s. is 900 s. or 45 li. ſixty times 2 d. is 120 d. or 10 s. laſt of all, ſixty times

3½ is 180 farthings, or 3 s. 9 d. all which together amount unto 2745 li-13 s.-9 d. the value thereof to be received at the end of the term.

li.	s.	d.	q.
2700	0	0	0
45	0	0	0
	10	0	0
	3	9	0
2745	13	9	0

3 Example.

The yearly rent of 1 li-13 s.-4 d. being behinde and unpaid the ſpace of 7 years, at the end of which term the Tenant is compelled to pay the ſame with the intereſt thereof according to the above-named rate; I demand what the payment ought to be?

The increaſe of 1 li. yearly rent anſwering to 7 years, is 8 li. 18 s. 5 d. 1 q. which for 6 li. rent is to be taken ſix times, which ariſeth to 53 li. 10 s 7 d. 1 q. now becauſe 13 s. 4 d. is two thirt parts of 1 li. therefore I take ⅔ of 8 li. 18 s. 5 d 1 q. which is the increaſe of 1 li. forborn for 7 years, that is 5 li. 18 s. 11 d. 2 q. which together make 59 li. 9 s 6 d. 3 q. the ſumme to be received, as was required.

li.	s.	d.	q.
53	10	7	1
5	18	11	1
59	9	6	3

The

The third Table declaring what 1 li. *due at the end of any number of years under* 31 *is worth ready money at* 8 per centū, per annū.

yers	li.	s.	d.	q	li.	s.	d.	q.	years
1	0	18	7	0	0	0	10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	0	0	3	11	2	21
7	0	11	8	1	0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	3	0	2	8	1	26
12	0	7	11	1	0	2	6	0	27
13	0	7	4	0	0	2	2	3	28
14	0	6	9	2	0	2	1	3	29
15	0	6	3	3	0	1	11	3	30

This

This third Table is diſpoſed as the firſt, the uſe according to the Title thereof, being damageable.

1 Example.

Suppoſe there is 750 li. due to be payed at the end of 9 years, the Creditor would ſell this debt for preſent money, what ought that money to be at the rate deſcribed in the Table?

Seek in this third Table for 9 years at the left ſide of the Table, and right againſt it toward the right hand, you ſhall finde 10 s. which multiplied or taken 750 times, yeeldeth 7500 s. which is 375 li. the value of that debt in preſent money.

2 Example.

There is a Leaſe worth 500 li. after the end of 7 years; what is it worth preſent money, according to the rate deſcribed in the Table, ſtaying till it fall?

I ſeek in the Table for the 7 years, and right againſt it I finde 11 s.-8 d. now I multiply 5500 by 11, it yeeldeth 5500 s. or 275 li. then 500 times 8 d. maketh 4000 d. which is 16 li. 13 s. 4 d. which added together is 291 li. 13 s. 4 d. the value of the Leaſe to be paid before it fall in hand.

li.	s.	d.	q.
275	0	0	0
16	13	4	0
291	13	4	0

The

The fourth Table expressing what 1 li. *yearly rent or annuity for any number of years not exceeding* 30 *is worth ready money at* 8 per centum, per annum.

years.	li.	s.	d.	q.
1.	0.	18.	6.	0
2.	1.	15.	7.	3
3.	2.	11.	6.	2
4.	3.	6.	2.	3
5.	3.	19.	10.	1
6.	4.	12.	5.	1
7.	5.	4.	1.	2
8.	5.	14.	11.	0
9.	6.	4.	11.	1
10.	6.	14.	2.	1
11.	7.	2.	9.	1
12.	7.	10.	8.	2
13.	7.	18.	0.	3
14.	8.	4.	10.	2
15.	8.	11.	2.	1

li.	s.	d.	q.	yers
8.	17.	0.	1.	16
9.	2.	0.	0.	17
9.	7.	5.	1.	18
9.	12.	0.	3.	19
10.	16.	4.	1.	20
10.	0.	4.	0.	21
10.	4.	0.	0.	22
10.	7.	5.	0.	23
10.	10.	6.	3.	24
10.	13.	5.	3.	25
10.	16.	2.	1.	26
10.	18.	8.	1.	27
11.	1.	0.	0.	28
11.	3.	2.	0.	29
11.	5.	1.	3.	30

The fourth Table is disposed altogether as the former, and the use thereof in like sort being damageable.

1 Example.

There is an annuity or rent of 20 s. *per annum* to endure 25 years, it is required what it is worth ready money?

Look in the Table for 25 years, and right against it you shall finde 10 li.-13 s.-5 d.-3 q. which is the solution.

2 Example.

What is the Lease of certain Land valued at 140 li. *per annum*, to begin presently and endure 18 years, worth ready money?

Search in the Table for 18 years, the term named in the Question, and right against it toward the left hand you shall finde 9 li. 7 s. 5. 1 q. which expresseth that 1 li. rent to be bought for that term is worth so much; therefore that summe 140 times is the value required. Now 140 times 9 li. is 1260, and 140 times 7 s. is 980 s. or 49 li. likewise 140 times, 5 d. is 700 d. or 2 li. 18. s. 4 d. and 140 farthings is 2 s. 11 d. all which added together make 1312 li. 1 s. 3d. for the value of the said Lease paying no rent.

li.	s.	d.
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3 Ex-

3 Example.

A Leaſe taken for 21 years at 13 li. 6 s. 8 d. *per annum*, which after 5 years expired, the Tenant is deſirous to give a fine, and bring the rent down to 8 li. *per annum*, for the reſt of the term, the demand is what fine is to be payed?

Subtract 5 years from 21, the remain 16, is the time unexpired; likewiſe from the preſent rent abate 8 li. the reſt will be 5 li-6 s-8 d. now the drift of the Queſtion is, what 5 li-6 s-8 d. yearly rent or annuity to indure 16 years is worth preſent money.

The value of 1 li. rent or annuity anſwering to 16 years is 8 li-17 s-0 d-1 q. Now five times 8 li. is 40 li. and five times 17 s. is 4 li-5 s. and five times 1 q. is 1 d-1 q. and becauſe 6 s-8 d. is $\frac{1}{3}$ of 1 li. I take $\frac{1}{3}$ of 8 li-17 s-0 d-1 q. which is 2 li-19 s-0 d. all which added together, make 4 li-4 s-1 d-q. which is the fine that ought to be paid to bring the rent to 8 li. *per annum*.

li.	s.	d.	q.
40	0	0	0
4	5	0	0
2	19	1	1
47	4	1	1

The fifth Table declaring what yearly rent or annuity of 1 li. *ready money will purchase for any number of years under* 31 *at* 8 per cent. per annum.

yers	li.	s.	d.	q	li.	s.	d.	q.	years
1	0	1	7	0	0	2	9	3	16
2	0	14	0	2	0	2	8	3	17
3	0	9	8	2	0	2	8	0	18
4	0	7	6	0	0	2	7	0	19
5	0	6	3	0	0	2	6	2	20
6	0	5	4	3	0	2	5	3	21
7	0	4	9	2	0	2	5	1	22
8	0	4	4	0	0	2	4	3	23
9	0	4	0	0	0	2	4	1	24
10	0	3	8	2	0	2	4	0	25
11	0	3	6	0	0	2	0	3	26
12	0	3	3	3	0	2	3	1	27
13	0	3	1	3	0	2	3	0	28
14	0	3	0	1	0	2	2	3	29
15	0	2	11	0	0	2	2	2	30

In the fifth Table the Numbers and Columns are all disposed as the former Tables, and needeth no further explanation but only Examples.

1 Example.

The Table declareth at first sight what yearly rent or annuity 1 li. ready money will purchase for any term in the Table expressed.

But

But if the ready money be above 1 li. then if any value or rent ſet down in this Table, be multiplied by the number belonging to the years in Queſtion, the Product will ſhew what yearly rent or annuity that ready money will purchaſe for the time propoſed.

2 *Example.*

A certain man hath 750 li. to purchaſe an Annuity to endure 27 years, ſo as it may yeeld him the like profit, as if it were put out according to the rate in the Table expreſſed, it is required what that annuity ought to be?

Becauſe the annuity is to endure 27 years, ſeek out the value or rent ſet againſt 27 years, in this fift Table, which is 2 s-3 d-1 q. now this being the Annuity of 20 s. ready money will purchaſe for that term, it muſt be multiplied by 750 li. as followeth, becauſe 2 s. is the tenth part of 20 s therefore take the tenth part of 750 li. which is 75 li. which ſet firſt down, then 750 times 3 d. is 9 li-7 s-6 d. which ſet under the former, laſt of all 750 q. is 15 s-7d-ob. All which added together, produce 85 li-3s-1d-ob. the yearly Annuity required.

li.	s.	d.	q.
75	0	0	0
9	7	6	0
	15	7	2
85	3	1	3

Deo ſoli laus, omnis honor & gloria tribuatur. Amen.